Simulation Studies of Emittance Growth in RMS Mismatched Beams*

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Abstract

As shown in a separate paper [1], a charged-particle beam, whose rms size is not matched when injected into a transport channel or accelerator, has excess energy compared with that of a matched beam. If nonlinear space-charge forces are present and the mismatched beam transforms to a matched equilibrium state, rms-emittance growth will occur. The theory yields formulas for the possible rms-emittance growth, but not for the time it takes to achieve this growth. In this paper we present the results of systematic simulation studies for a mismatched 2-D round beam in an ideal transport channel with continuous linear focusing. Emittance growth rates obtained from the simulations for different amounts of mismatch and initial charge will be presented and the emittance growth will be compared with the theory.

I. INTRODUCTION

The theoretical model analyzed here is a generalization of the previous model [2-5] for transverse emittance growth of rms-matched particle beams with nonequilibrium distributions, injected into continuous linear focusing channels. That emittance growth was associated with the conversion of excess space-charge field energy to particle thermal energy, through the action of nonlinear space-charge forces. Now we test the more general assumption that, whenever a charged-particle beam has a total tranverse energy larger than that of an equivalent matched beam, the excess or free energy can be transformed to thermal energy, which results in emittance growth, provided nonlinear forces act on the beam. For the case of an initial rms-mismatched space-charge dominated beam, we assume that, as a result of the nonlinear space-charge forces, the beam will relax to a final stationary state that is rms-matched and has a uniform charge density (uniformity is a characteristic of stationary beams in linear focusing channels only in extreme space-charge limit). Transverse energy conservation in a continuous focusing channel then results in predictions for growth of the final, matched rms-beam size and growth of the final emittance.

II. THEORY

We summarize briefly the results of the theoretical model, which is described in detail in Ref.1. Emittance growth requires both free energy and nonlinearity. If there are no internal or external nonlinear forces, the beam envelope will oscillate about the value of the matched beam envelope, as predicted by the envelope equation, and the free energy, associated with the mismatched beam, will not be thermalized. However, if significant nonlinear external or space-charge forces are present (or if there is intrabeam scattering or scattering in a background gas), thermalization of the free energy will occur.

The extra free energy ΔE of a mismatched beam, injected into a continuous linear-focusing channel, can be calculated by comparing the total energy of the initial non-stationary beam E_o with the energy of the equivalent matched beam E_i so that $\Delta E = E_o - E_i$. The equivalent matched beam is defined here as an rms-matched beam with the same current and emittance and a uniform charge-density distribution.

By using transverse energy conservation and assuming that the final beam is stationary, we can obtain a relation for the final beam size a_f, as a function only of the initially known parameters:

$$\left(\frac{a_f}{a_i}\right)^2 - 1 - \chi \ln\left(\frac{a_f}{a_i}\right) = h, \qquad (1)$$

where a_i is the initial matched beam size, $\chi = 1 - \frac{k_i^2}{k_o^2}$, and h is a free-energy parameter. The quantity k_o is equal to $2\pi/\lambda_o$, where λ_o is the wavelength of the betatron oscillations without space-charge, and k_i is equal to $2\pi/\lambda_i$, where λ_i is the wavelength of the betatron oscillations with space-charge.

For a mismatched beam with a nonuniform distribution, h will be the sum of h_m, related to the mismatch, and h_s caused by the nonlinear component of the internal space-charge field, given by Ref.1.

$$h_m = \frac{1}{2}\frac{k_i^2}{k_o^2}\left(\frac{a_i^2}{a_o^2} - 1\right) - \frac{1}{2}\left(1 - \frac{a_o^2}{a_i^2}\right) + \left(1 - \frac{k_i^2}{k_o^2}\right)\ln\frac{a_i}{a_o}$$

$$h_s = \frac{1}{4}\left(1 - \frac{k_i^2}{k_o^2}\right)\frac{U}{w_o}, \qquad (2)$$

where a_o is the initial mismatched waist or crest radius, $w_o = I^2/(16\pi\epsilon_o\beta^2c^2)$ and U represents the difference between the initial internal energy of the non uniform beam and of the uniform beam.

The emittance growth is given by

$$\frac{\epsilon_f}{\epsilon_i} = \frac{a_f}{a_i}\left\{1 + \frac{k_o^2}{k_i^2}\left[\frac{a_f^2}{a_i^2} - 1\right]\right\}^{1/2} \qquad (3)$$

where using Eq.(1) we can relate the emittance growth to the free energy of the beam through the dimensionless parameter h.

Not all the free energy ΔE is avaliable for thermalization (mean kinetic energy increase) and emittance growth. Because the beam size increases, some of the free energy can be exchanged with potential energy associated with the external field and with field energy, as well as with kinetic energy. However, this beam-size effect is accounted for in the model.

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III. Numerical Simulations

A complete description of the program that we used can be found in ref.6. It simulates the transverse motion of N-interacting macroparticles for long distances in a continuous linear, external-focusing field. To calculate the space-charge fields, each particle is considered to be an infinitely long cylindrical sheet of charge. The space-charge electric field at any radius r is strictly radial and depends only on the amount of charge inside the radius r. Even at 1000 plasma wavelengths the emittance profiles still show small residual oscillations. For these studies we have identified the peak value of the oscillations at 1000 plasma wavelengths with the final emittance. We have considered two different initial distributions: the thermal or semi-Gaussian (uniform in space, Gaussian in divergence) and the full-Gaussian distributions.

Each simulation is characterized by two parameters, the initial space-charge tune depression k_i/k_o and the mismatch ratio defined as the ratio between the rms size of the initial mismatched and the equivalent matched beam. We will present the emittance growth and the rate of emittance growth as a function of tune depression for three mismatch ratios: 0.5, 1, and 1.5.

Figures 1, 2, and 3 show the theoretical curves and the numerical results of the emittance growth for the three mismatch ratios. The theoretical curves are obtained from numerical solution of Eqs.(1) to (3). Results are presented for both the thermal and the Gaussian distributions. When the beam is not mismatched, we find the same results as presented in Ref.3. In general the theoretical model slightly overestimates the numerical results for two reasons. First the final beam is assumed to have a uniform charge density, whereas in fact the final beam is not uniform and some free energy is converted to nonlinear field energy. The second reason is that, for some cases, especially for low-tune depression, the beam has not yet reached the final equilibrium state after 1000 plasma wavelengths i.e., not all the thermalization has occurred. We also notice that, for the same values of tune depression and mismatch ratio, the beam with a Gaussian distribution always has a larger emittance growth than the thermal beam. This can be explained by the extra amount of free energy in the Gaussian distribution caused by the initial nonuniform distribution.

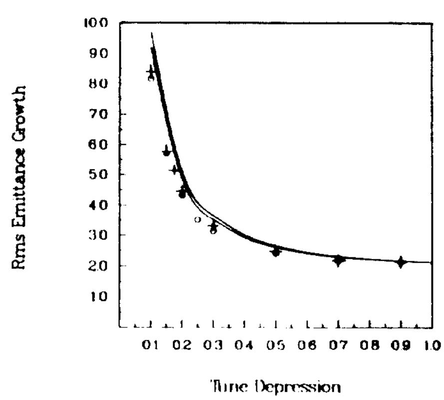

Figure 1. Theoretical curves for the emittance growth versus tune depression(upper and lower curves for Gaussian and thermal distributions respectively) and numerical data for thermal (O) and Gaussian (+) distributions for a mismatch ratio of 0.5.

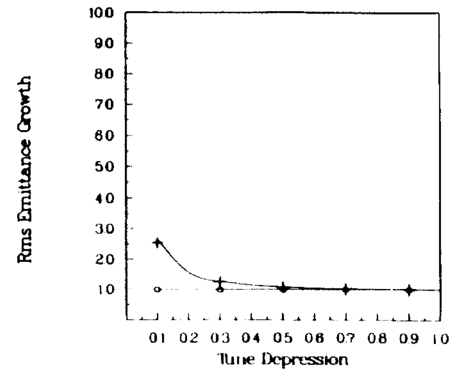

Figure 2. Theoretical curves for the emittance growth (upper and lower curves for Gaussian and thermal distributions respectively) and numerical data for thermal (O) and Gaussian (+) distributions with no mismatch versus tune depression.

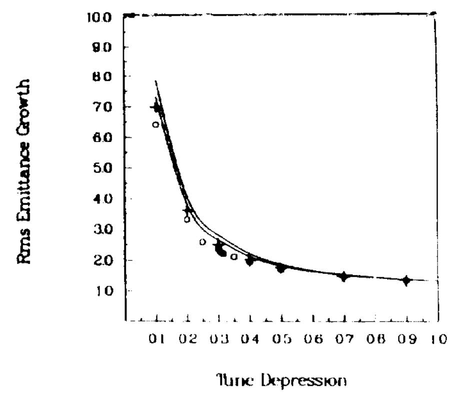

Figure 3. Theoretical curves for the emittance growth (upper and lower curves for Gaussian and thermal distributions respectively) and numerical data for thermal (O) and Gaussian (+) distributions for a mismatch ratio of 1.5 versus tune depression.

Figures 4 and 5 show typical plots of emittance growth versus axial distance normalized to the initial plasma period. We conclude that the growth rate of the emittance caused by an initial mismatch is not easy to characterize as a single numerical value. A general description of our initial results can be stated as follows. First, the beam undergoes a rapid charge-density redistribution, which occurs in approximately one-fourth of the plasma wavelength, during which a relativly small emittance growth is observed for nonuniform initial beams, see Fig.6. This effect is the same as was reported earlier for rms-matched beams [3]. For mismatched beams we observe additional emittance growth, which usually reaches an initial peak at few plasma wavelengths (typically around 5), for both the thermal and the Gaussian cases. For the higher tune depressions (Fig.4), the emittance at this initial peak is approximatly equal to the final value of the emittance, i.e., afterwards the beam emittance evolves with a pattern of damped oscillations to a final quasi-stationary state but with no further net growth. However as the tune ratio decreases (Fig.5), the initial peak contains a smaller fraction of the total emittance growth. The remaining growth continues at a very slow rate for these cases, and 90% of the final emittance value may not be reached for several hundred plasma wavelengths. The reduced overall growth rate for the lower tune depressions may be caused by a reduction of nonlinear space-charge forces at lower tune depressions. This happens in a linear focusing channel where the beam quickly relaxes to a more uniform density distribution when the tune depression is lower. The plots of rms beam size (not shown) and rms emittance versus axial distance show damped oscillations, which appear initially as beats, perhaps between the mismatch oscillation frequency and the beam plasma frequency.

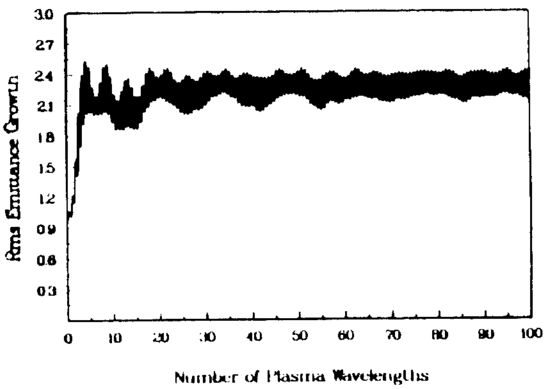

Figure 4. Emittance growth versus axial distance for a beam with initial thermal distribution, tune depression 0.5, and mismatch ratio 0.5.

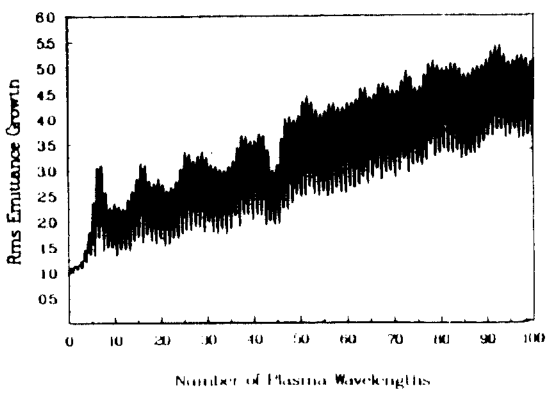

Figure 5. Emittance growth versus axial distance for a beam with initial thermal distribution, tune depression 0.15, and mismatch ratio 0.5.

IV. CONCLUSIONS

Our studies show that the mismatched beams evolve to a final quasi-stationary state with accompanying emittance growth, which can be large compared with the growth for matched beams. The values of emittance growth from the numerical simulations are also nearly equal to the predictions of the model. The good agreement of simulations with the model confirms that the emittance growth is associated with the conversion of free energy to thermal energy of the beam.

The final phase-space distributions (Fig.7) show that much of the emittance growth is a result of a large, well-populated halo surrounding the core of the beam. These studies suggest that beam mismatch may be the source of much of the halo observed in real beams.

Finally, we believe that the continuous linear focussing channel we have studied represents a smooth approximation idealization of a real quadrupole focusing channel. The next step is to test the predictions of this model against numerical simulation studies of beams in a quadrupole focusing channel.

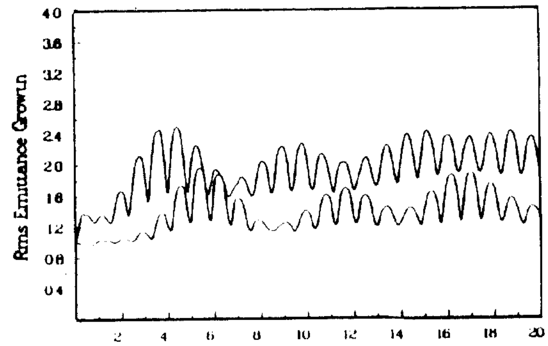

Figure 6. Emittance growth versus axial distance for tune depression 0.3 and mismatch ratio 1.5. The upper and lower curves correspond to the initial Gaussian and thermal distributions respectively.

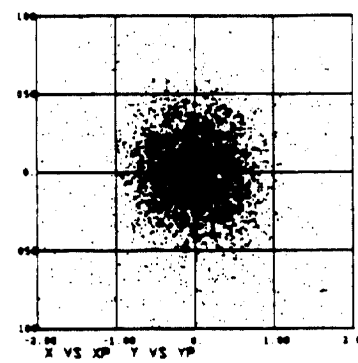

Figure 7. Final phase space distribution for an initial Gaussian beam with tune depression 0.3 and mismatch ratio 1.5.

V. ACKNOWLEDGMENT

We thank Ken Crandall for his help in using the computer code, which he wrote for general studies of space-charge induced emittance growth.

VI. REFERENCES

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