Experimental Studies of Emittance Growth Due to Initial Mismatch of a Space Charge Dominated Beam in a Solenoidal Focusing Channel*

D. Kehne, M. Reiser
Laboratory for Plasma Research
University of Maryland
College Park, Maryland 20742

H. Rudd
Rudd Consulting Services
6922 Nashville Rd.
Lanham, MD 20706

Abstract

Experimental studies of emittance growth resulting from beam mismatch have been performed at the University of Maryland. A 5-beamlet distribution of 44 mA and 5 kV passes through two solenoid matching lenses and into a 36 solenoid transport channel. Theory predicts substantial emittance growth due to mismatch. The 5-beamlet configuration is mismatched and the final emittance is measured. Experiment and simulation results suggest that a large halo is the source of the predicted emittance growth.

I. INTRODUCTION

High brightness beams envisioned for use in free electron lasers, colliders, and heavy ion inertial fusion sytems require low emittance in order to achieve a sufficiently small beam size in the interaction region. Past theoretical, simulation, and experimental studies [1-5] have shown that the free energy associated with nonuniform RMS-matched charge distributions is converted to random kinetic energy (emittance). RMS-mismatched and misaligned beams are also characterized by excess free energy. Recent theoretical results [6] offer a quantitative prediction of the emittance growth due not only to space charge homogenization but also to the damping of mismatch and misalignment oscillations as well.

II. BACKGROUND THEORY

Previous studies[1-5] have predicted and experimentally verified the emittance growth associated with the conversion of free energy in a nonuniformly distributed RMS-matched beam. This theory assumes that the beam average radius is constant. Recently developed theory[6] has incorporated the change in beam radius and also predicts the emittance growth due to the conversion of free energy in RMS-mismatched beams.

The theory predicts an emittance growth of

$$\frac{\varepsilon_f}{\varepsilon_i} = \frac{a_f}{a_i}\left[1 + \frac{\sigma_0^2}{\sigma_i^2}\left(\frac{a_f^2}{a_i^2} - 1\right)\right]^{1/2} \tag{1}$$

where σ_0 is the particle phase advance per period neglecting space charge and σ_i is the initial phase advance per period in the presence of space charge, a_i is the effective (2×RMS) radius of the equivalent initial stationary beam, and a_f is the effective radius of the equivalent final stationary beam. For period length S, the value of σ_i can be calculated from

$$\frac{\sigma_i^2}{S^2} = \frac{\sigma_0^2}{S^2} - \frac{K}{a_i^2} \tag{2}$$

where $K = 2(I/I_0)/(\beta\gamma)^3$ is the generalized perveance, I is the beam current, $I_0 = 1.7 \times 10^4$ A for electrons, $\beta = v/c$, v is the particle velocity, and γ is the relativistic energy factor. The ratio a_f/a_i can be calculated using the equation

$$\left(\frac{a_f}{a_i}\right)^2 - 1 - \left(1 - \frac{\sigma_i^2}{\sigma_0^2}\right)\ln\frac{a_f}{a_i} = h \quad . \tag{3}$$

For nonuniform charge distributions, h is defined as

$$h = h_s = \frac{1}{4}\left(1 - \frac{\sigma_i^2}{\sigma_0^2}\right)\frac{U}{w_o} \tag{4}$$

where U/w_o is a dimensionless parameter depending only the geometry of the nonuniform distribution.

For mismatched beams, $h = h_m$ is defined as

$$h_m = \frac{1}{2}\frac{\sigma_i^2}{\sigma_0^2}\left(\frac{a_i^2}{a_0^2} - 1\right) - \frac{1}{2}\left(1 - \frac{a_0^2}{a_i^2}\right) + \left(1 - \frac{\sigma_i^2}{\sigma_0^2}\right)\ln\frac{a_i}{a_0} \tag{5}$$

---

* Research supported by the U.S. Department of Energy

0-7803-0135-8/91$01.00 ©IEEE

where a_0 is the effective radius of the mismatched beam waist at the beginning of the channel.

In the presence of both effects, the associated free energy parameters h_s and h_m are added.

III. EXPERIMENTAL SETUP

The apparatus used to test this theory is the transport channel of the University of Maryland Electron Beam Transport Experiment[5,7]. The channel consists of 36 periodically spaced solenoids(period S = 13.6 cm). Two other solenoids are used to match the beam. As in previous work with multiple beams[5,7], the full 5 kV, 240 mA solid beam(2 μs pulse length, 60 Hz repetition rate) produced by the gun is apertured to form a 5-beamlet configuration of 44 mA.

The changing structure of the beam can be observed at any point in the channel or matching section using a movable phosphor screen. The beam emittance can be measured at the end of the channel using a slit/pinhole apparatus[5,7]. The analysis assumes the beam is axisymmetric which is valid if the 5-beamlet configuration has structurally merged.

As in past studies, the Particle-In-Cell code SHIFTXY has been used to simulate the beam. A thorough description of the code can be found elsewhere[7] and hence will be deferred here.

IV. RESULTS

Previous work with this 5-beamlet structure yielded the initial effective radius of 4.67 mm and 4×RMS (effective) emittance of 64.8 mm-mrad[7]. The phase advance per period neglecting space charge was calculated to be 77°. For the matched beam case, the theory predicts an emittance growth of 1.52 and a_f/a_i was calculated to be 1.05. Simulation also showed an emittance growth of about 1.5 and a_f/a_i of 1.05.

To maintain consistency with the theory, the ratio a_0/a_i is defined as the ratio of mismatched to matched beam radius one half period from the center of the first channel lens. This assumes that a beam waist is located at this point for each. The SHIFTXY code was used to find the matching lens settings that result in a mismatch of $a_0/a_i = 0.5$.

For this mismatch case , equations. (1) through (5) predict a total emittance growth of 3.8 and growth in radius of 1.30. Both of these numbers are supported by simulation[8]. The simulation results shown in Figure 1 and Figure 2 are the effective radius and effective emittance plotted as a function of period number. In Figure 1, the large mismatch oscillations near the beginning of the channel diminish essentially to matched oscillations by the end. Figure 2 reveals the total emittance growth occurring entirely in 15 periods, less than half the channel length.

Similar emittance results were expected experimentally at the end of the channel. Beam pictures taken along the channel verified that the simulation was an accurate representation of the actual experiment. As a reference, a schematic of the aperture plate is shown in Figure 3. Pictures, located 44.2, 98.6, and 524 cm from the aperture plate, are shown in Figure 4 with the corresponding simulation pictures. The structural agreement is extremely good. At z = 524 cm, the central portion of the beam has been blacked out to reveal the halo.

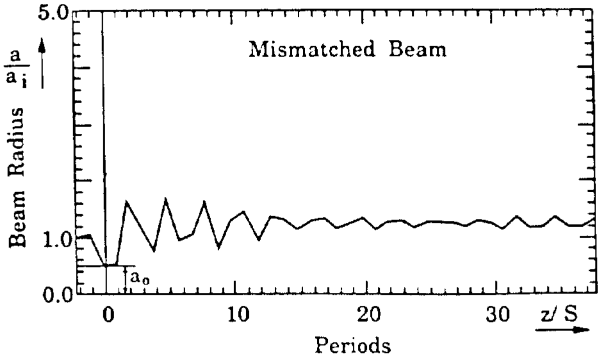


Figure 1. Ratio of the RMS radius to initial RMS radius of the beam plotted versus period number.

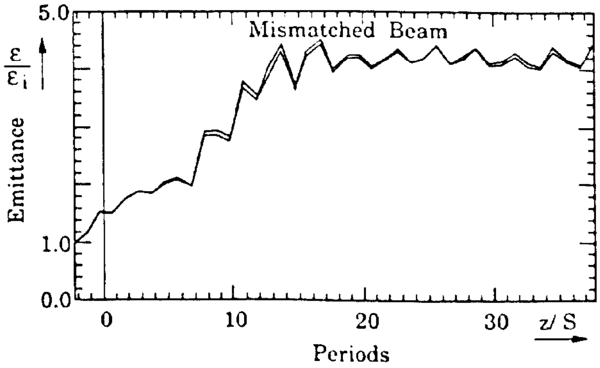
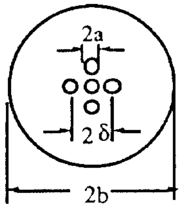


Figure 2. Emittance growth plotted versus period number.

When the matched and mismatched beam emittances were measured, values of 130 mm-mrad (beam diameter 10.5 mm) and 126 mm-mrad (beam diameter 13.5 mm) respectively resulted, though the outer halos were not included. The error of each measurement is about 20%. While the matched beam emittance is greater than predicted, the measured emittance the core of the mismatched case is less than half of the predicted total emittance.



b = pipe radius = 19.0 mm
(14 mm in matching section)

a = beamlet radius = 1.19 mm

δ = 3a = 3.57 mm

Figure 3. Schematic of the 5-beamlet aperture.

Figure 5 is the x-y and x-x' phase space plots of the simulated beam at z = 520 cm. Unfortunately, the intensity of the halo was too low to accurately include it in the experimental emittance measurement. A beam profile derived from the simulation of the matched and mismatched beams at z = 520 cm is shown in Figure 6. Note the different intensity layers of the two beams, especially the absence of a halo in the matched case. Simulation work is in progress to determine the emittance as a function of beam radius. This data should reveal the fraction of emittance growth due to the low-intensity halo found around the beam.

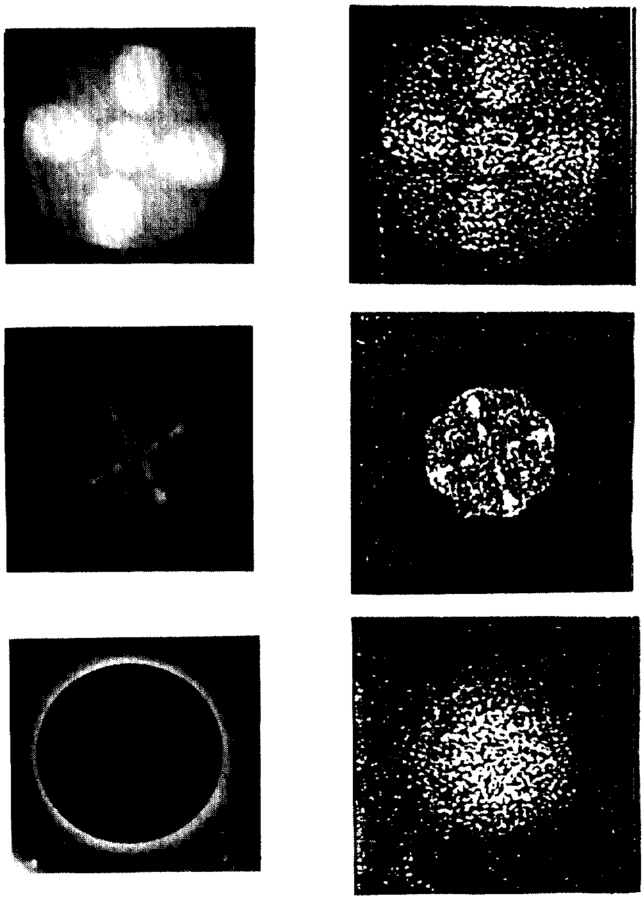


Figure 4. Comparison of Experimental pictures (left) and simulation real space plots at locations z = 44.2 cm (top), 98.6 cm, and z = 525 cm(bottom). The experimental picture at z = 525 cm is blacked so that the halo can be seen. The scale is 0.57 beam cm per paper cm.

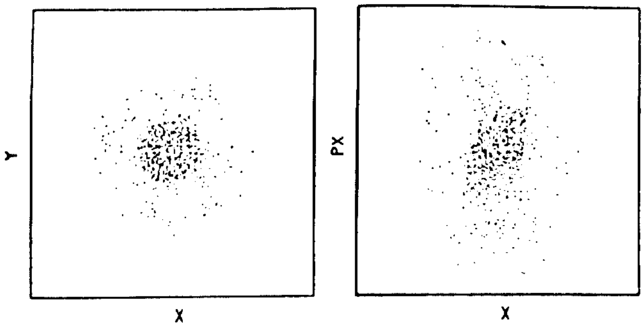


Figure 5. Simulation x-y and x-x' phase space plots at z = 520 cm showing halo.

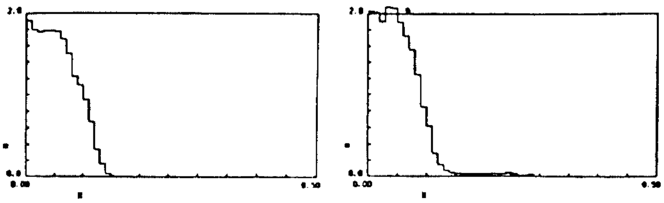


Figure 6. Simulation profiles of matched and mismatched beams at the end of the channel (z = 520 cm).

V. CONCLUSION

Theoretically predicted emittance growth due to the conversion of free energy in a mismatched beam has been partially confirmed. The theoretically predicted radius and emittance growth has been confirmed by simulation. Simulation and experiment show strong agreement in beam structure throughout the channel including a strong halo at the end. Measured emittances of the cores of the matched and mismatched beams are essentially the same indicating that the halo is the source of a large percentage of the predicted emittance growth in the mismatched case.

VI. REFERENCES

[1] J. Struckmeier, J. Klabunde, and M. Reiser, "On the Stability and Emittance Growth of Different Particle Phase-Space Distributions in a Long Magnetic Quadrupole Channel," Part. Accel., Vol. 14, pp. 227-260. 1984.

[2] T.P Wangler, K.R. Crandall, R.S. Mills, and M. Reiser, "Relation Between Field Energy and RMS Emittance in Intense Particle Beams," IEEE Trans. Nucl. Sci. NS-32, 2196, 1985.

[3] O.A. Anderson, "Internal Dynamics and Emittance Growth in Space-Charge Dominated Beams," Part. Accel. Vol. 21, p. 197, 1987.

[4] I. Hofmann and J. Struckmeier, "Generalized Three-Dimensional Equations for the Emittance and Field Energy of High-Current Beams in Periodic Focusing Structures," Part. Accel., Vol. 21, p.69, 1987.

[5] M. Reiser, C.R. Chang, D.Kehne, K. Low, T. Shea, H. Rudd, and I. Haber, "Emittance Growth and Image Formation in a Nonuniform Space-Charge Dominated Electron Beam," Physical Review Letters, Vol. 61, No. 26, pp. 2933-2936, 1988.

[6] M. Reiser, "Free Energy and Emittance Growth in Nonstationary Charged Particle Beams," CPB Tech. Rep. #90-034, Oct. 25, 1990, Univ. of Maryland; to be published; paper QGR5, this proceedings.

[7] I. Haber, D. Kehne, M. Reiser, and H. Rudd, "Experimental, Theoretical, and Numerical Investigation of the Homogenization of Density Nonuniformities in the Periodic Transport of a Space-Charge Dominated Beam," accepted for publication in Physical Review A.

[8] M. Reiser, "Studies of Beam Transport, Longitudinal Compression, and Resistive-Wall instability at the University of Maryland," to be published in Part. Accel.