Transverse Emittance of an Intense Electron Short Pulse Just Emitted by the Cathode of a RF-FEL Photoinjector : Influence of Electrodynamic Effects

J.-M. Dolique and M. Coacolo
Laboratoire de Physique des Plasmas. Université Grenoble I.
BP 53 X. 38041 Grenoble Cedex. France
and
CEA.Centre d'Etudes de Bruyères-le Châtel. BP 12. 91680 Bruyères-le-Châtel. France

Abstract

The emittance considered is that of the intense electron short pulse of a high-power RF-FEL, immediately after it has been emitted from the cathode of a photoinjector. In the absence of any azimuthal fluid motion, and for an axisymmetric pulse, the normalized rms transverse emittance for the whole pulse : $<\varepsilon_r>=(2/mc)(<r^2><p_r^2> - <rp_r>^2)^{1/2}$ is analytically calculated by averaging : over r, for a given z, which gives the transverse emittance in a particular slice $<\varepsilon_r>_z$, or over r and z, which gives $<\varepsilon_r>$. As a source of p_r, thermal effects are negligible as compared to the electromagnetic effects, which result from the RF field, and from the self-field. The latter one cannot be evaluated as a space-charge field relevant to a Poisson equation written in a beam frame. Near to the cathode the acceleration is so strong that the relativistic acceleration and retardation effects have to be taken into account. The dependence of $<\varepsilon_r>_z$ on z will be considered, as well as the dependence of $<\varepsilon_r>$ on various parameters : J, the emitted current density (which is assumed to be constant during photoemission), R the beam radius on the cathode, τ, the pulse duration, and E_0 the RF field on the cathode. Conditions for a minimum emittance will be looked for.

I. INTRODUCTION

Beam quality, as measured by some brilliance or emittance, is known as being a critical factor for free electron laser efficiency. The photoinjector is a potential source of high-current, low emittance, short bunch-length electron beams, particularly adapted for FEL. It consists of a laser-driven cathode set in an RF cavity followed by more RF cavities. Photoemitted electrons experience an extremely quick acceleration from thermal to multiple-MeV energy. In the emittance growth of a given beam slice, from cathode to photoinjector exit, initial thermal emittance (at the cathode) may be neglected before the one acquired under the influence of RF field or self-field. Moreover, for high current beam (say J>100 A/cm^2) self-field effects are dominant over RF-effects, in emittance growth.

Various theoretical analyses of transverse emittance growth in the photoinjector, under the influence of space-charge, have been given, which assume a uniform density and, principally, that all the electrons of the beam pulse have the same axial velocity [1]-[4]. Under these conditions it is possible, using the beam frame, to reduce the calculation of the electromagnetic field map to an electrostatic problem. Although it does not take into account any axial energy dispersion inside the pulse, this assumption may be a first approximation as long as the pulse is sufficiently far from the cathode. It cannot be kept for the just emitted beam pulse, inside which are found, at the head, electrons already accelerated by the RF field to a relativistic velocity, and at the back, next to the cathode, electrons with thermal velocities. Moreover, the density is far from being uniform.

Besides electron density non-uniformity, electrodynamic effects must be taken into account : a) the acceleration field is no longer negligeable before the velocity field, b) the relativistic retardation strongly limits that part of the beam of which the field is effectively experienced by a given electron. Lastly, a boundary condition is imposed by the presence at the pulse back of the equipotential cathode.

A theoretical treatment of this situation, based on the general Liénard-Wiechert expression for the electromagnetic field, is given in a companion paper [5]. In the present work, we apply the methods and results of [5] to the calculation of both slice and global rms transverse emittance, of a just emitted beam pulse, for various parameter values of practical interest.

II. THEORETICAL MODEL

A. Considered emittances

In the absence of azimuthal fluid motion, and for an axisymmetric unmagnetized beam, the normalized rms transverse emittance may be defined, in cylindrical coordinates (r,θ,z) by :

$$<\varepsilon_r>=(2/mc)(<r^2><p_r^2> - <rp_r>^2)^{1/2}$$

where the numerical factor 2 allows the identification of rms- and phase transverse emittances for a KV beam.

p_r is the radial mechanical (or canonical) momentum , m the rest mass.

For long beams, the average is usually a 2D one, taken in some particular beam slice z : $< >_z$:

$$<\varepsilon_r>_z = (2/mc)(<r^2>_z<p_r^2>_z - <rp_r>_z^2)^{1/2} \quad \text{(slice emittance)}$$

For short beam pulses, rather than this slice emittance, the global emittance is more usually considered, where the average < > is a 3D one, taken over the whole beam pulse.

Supposing the transverse and axial motions be decoupled, the growth of this global rms transverse emittance, would give a measure of the importance of non-linear effects, whether these effects are due to the external field or to the self-field.

But generally, for a short beam pulse, this assumption can at best be a rough approximation, so that $<\varepsilon_r>$ has essentially

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to be considered as a beam quality figure of merit, within easier theoretical or experimental reach than the ideal figure of merit, the distribution function $f(\mathbf{x},\mathbf{p}|t)$ would be.

B. Assumptions

The current density of the emitted electrons are supposed to be constant in time on $(0,\tau)$, and radially uniform on the cathode : $0 \le r \le R$.

The RF field of the extracting cavity is assumed to be constant, purely axial, and uniform, during photoemission ($0 \le t \le \tau$), and over the whole beam pulse ($\pi R^2 \sim 1$ cm^2, $L(\tau) \sim 1$ cm) : $\mathbf{E}_{RF} = -E_0 \mathbf{u}_z$. This assumption is appropriate for photoinjectors like that of Bruyères-le-Châtel [6], where the RF-field period and wavelength are large as compared to τ, and $R \sim L(\tau)$, respectively. A known time- and space-dependent $\mathbf{E}_{RF}$ could eventually be introduced with no consequences other than complication.

The only considered source of emittance are the self-field effects (an expression more appropriate than space-charge effects in the present conditions where electrodynamic phenomena play an important role). For high-current beams (say $J > 100$ A/cm^2), and during the studied stage, when electrons of the slices located close to the cathode are not yet relativistic, these effects would be dominant over RF-effects even in the presence of a slowly time- and space dependent RF field.

For a beam current density $|J|$ not in the vicinity of the self-field limit [5], the paraxiality condition $I/I_A = I/\beta\gamma I_0 \approx I$ (kA)$/17\beta\gamma \ll 1$, is satisfied over the whole beam pulse length, except for a narrow layer next to the cathode (Fig.1).

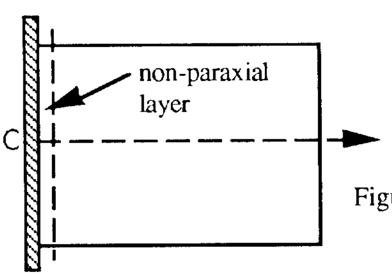


Figure 1

Except in this layer, all the electrons of a given slice have the same energy γ and the same $\beta_z \approx \beta \gg \beta_r$. The axial motion may be decoupled from the radial one.

Last assumption : the small angular divergence of the beam pulse is neglected on $(0,\tau)$, so that for each emitted electron $r(t) = r(t_0)$.

C. Procedure

Owing to the paraxiality, the axial motion to be studied is that of an electron slice. The axial motion of the slice emitted at time t_0 obeys

$$\frac{d}{dt}[\beta(t|t_0)\gamma(t|t_0)] = -\frac{e}{mc}\left\{E_0 + E_z[r=0, z(t|t_0), t]\right\},$$

equation in $z(t|t_0)$, where $c\beta(t|t_0) = [d/dt]z(t|t_0)$, and where $\mathbf{E}(r,z,t)$ is the electric field map at time t. As shown in [5], the latter cannot be calculated from a Poisson equation ; electrodynamic phenomena play an essential role, which make it necessary to use the general Liénard-Wiechert formalism. In order to solve the rather intricate resulting system of equations, an iteration procedure has been taken up [5]. At order 0, the electromagnetic field map is calculated at time t, from the trajectories (between $t=0$ and t) of electrons only accelerated by the RF field, according to the analytical expressions given in [5].Then, with $E_z = E_z^{(0)}$ introduced in the above equation, one finds an order 1 slice axial motion $z^{(1)}(t|t_0)$ which, in its turn, leads to an order 1 field map, ... A good convergence is obtained after only 3 iterations, which lead on the one hand to the self-consistent slice axial motion $z(t|t_0)$, and on the other hand to the self-consistent electromagnetic map $\mathbf{E}(r,z,t)$, $\mathbf{B}(r,z,t)$.

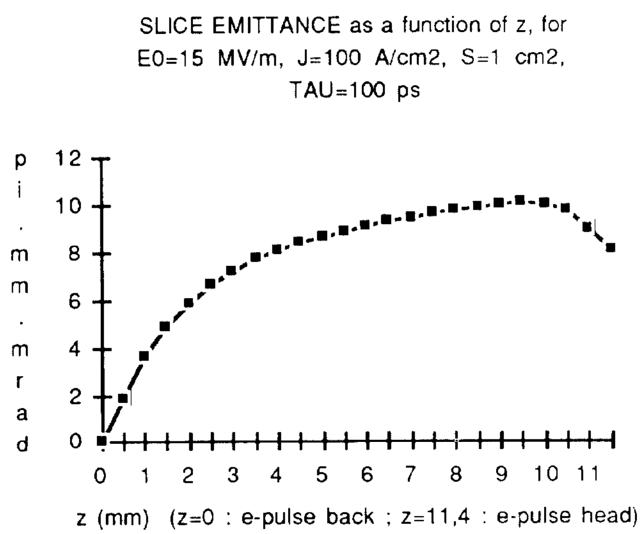
The evolution of the electron radial momentum p_r from the emission time t_0 (when $p_r = 0$) to τ, is then studied by

$$\frac{d}{dt}p_r(t|r,t_0) = -\left\{E_r[r, z(t|t_0), t] - \beta(t|t_0)B_\theta[r, z(t|t_0), t]\right\},$$

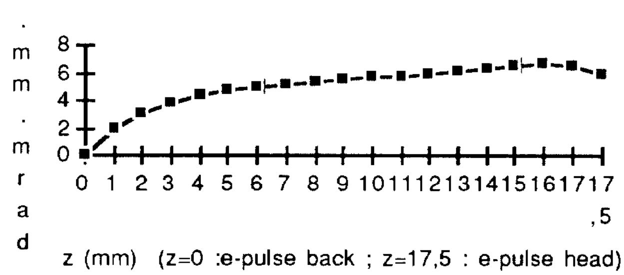
where $r = r(t_0)$, and where $z(t|t_0)$ is the known function previously determined ; $c\beta(t|t_0) = (d/dt)z(t|t_0)$. Thus, one finds $p_r(\tau|r,t_0)$, from which one can deduce, using the axial self-consistent slice motion law $z(t|t_0)$, the function $p_r(r,z,\tau)$ needed to evaluate the emittances.

III. SLICE EMITTANCES

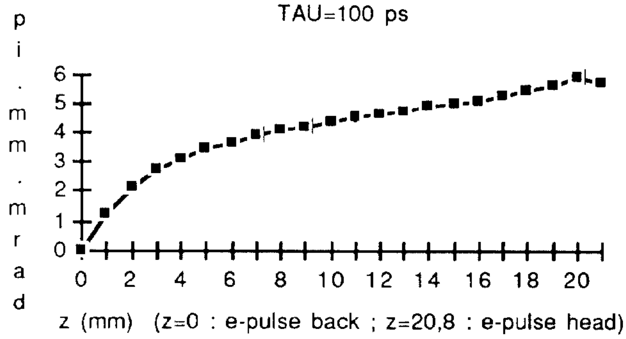
Figures 2 to 4 give some examples of slice emittances $<\varepsilon_r>_z$ as a function of z in the beam pulse. Except for E_0=45 MV/m, there is a maximum near to the beam head.



SLICE EMITTANCE as a function of z, for
E0=30 MV/m, J=100 A/cm2, S=1 cm2,
TAU=100 ps



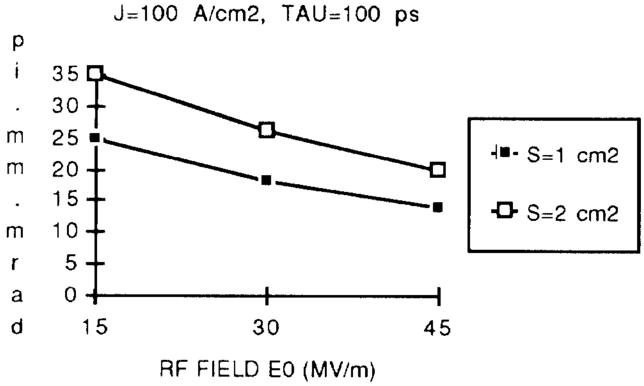
SLICE EMITTANCE as a function of z, for
E0=45 MV/m, J=100 A/cm2, S=1 cm2,
TAU=100 ps



IV. GLOBAL EMITTANCE : SAMPLE RESULTS AS A FUNCTION OF THE 4 PARAMETERS : J, R, E_0, τ

Figures 5 to 8 show, on some sample results, the influence of the 4 parameters J, R, E_0, τ.

GLOBAL EMITTANCE, as a function of E0, for
J=100 A/cm2, TAU=100 ps



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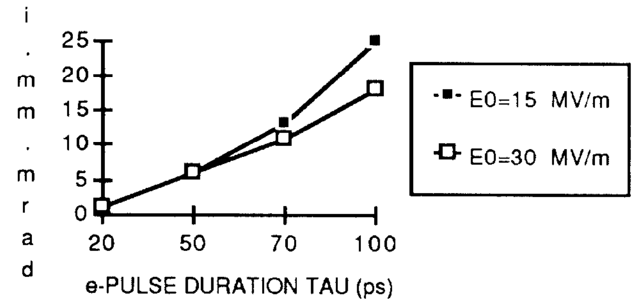
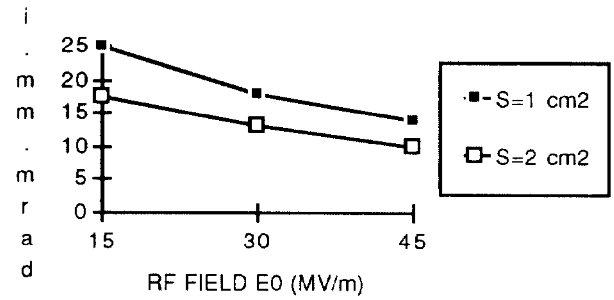
[6] R. Dei-Cas *et al.*, "Overview of the FEL activities at Bruyères-le-Châtel", *1990 Int. Free Electron Laser Conference*

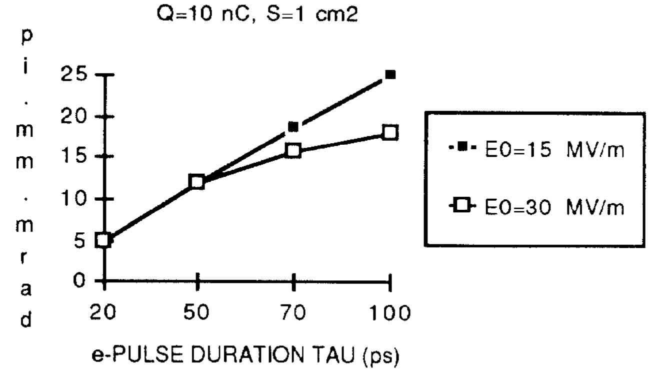
GLOBAL EMITTANCE, as a function of TAU, for
J=100 cm2, S=1 cm2



GLOBAL EMITTANCE as a function of E0, for
Q=10 nC, TAU=100 ps



GLOBAL EMITTANCE as a function of TAU, for
Q=10 nC, S=1 cm2



V. REFERENCES

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