Relativistic Acceleration and Retardation Effects on Photoemission of Intense Electron Short Pulses, in RF-FEL Photoinjectors

J.-M. Dolique and M. Coacolo
Laboratoire de Physique des Plasmas. Université Grenoble I.
BP 53 X. 38041 Grenoble Cedex. France
and
CEA.Centre d'Etudes de Bruyères-le Châtel. BP 12. 91680 Bruyères-le-Châtel. France

Abstract

In high-power free electron lasers, self-field effects in the electron beam are often the most important phenomenon on which the beam quality depends. These effects are generally conceived as space-charge effects, and described by a Poisson equation in a beam frame. In RF-FEL photoinjectors, the electrons of the intense short pulse produced by laser irradiation are submitted, just after their photoemission, to such a strong acceleration that relativistic acceleration and retardation effects have to be taken into account. The importance of these effects will be discussed, from the rigourous calculation of the Liénard-Wiechert velocity- and acceleration electric and magnetic fields, as a function of RF-electric field and beam parameters. The beam pulse is assumed to be axisymmetric, with a constant photoemitted current density. Consequences for the maximum current density that can be extracted will be considered (the "self-field limit", a name more appropriate than "space-charge limit" for the present conditions where electrodynamic phenomena play an important role).

I. INTRODUCTION

For the efficiency of electron-photon energy conversion in FEL, as well as in other coherent radiation sources such as gyrotrons, the beam quality, as measured by some brilliance or emittance, is a critical factor.

In the particular case of the short electron pulses of a RF FEL where the electron source is a photocathode, the beam emittances (transverse and longitudinal) along the line are essentially those acquired during the first stage of electron generation and acceleration, i.e. the photoinjector stage. Various phenomena can contribute to the emittances observed in the beam pulse at photoinjector exit. For powerful FEL, where the beam current exceeds say 100 A/cm^2, the space charge effects are dominant over the RF-field ones.

Theoretical calculations have been given of the transverse emittance growth inside the photoinjector for a short beam pulse either in ballistic drift or submitted to the RF accelerating field [1] -[4]. These calculations assume that the beam density is uniform, and principally that all electrons are moving with the same axial velocity $\beta_z c$, not only in a beam slice as the consequence of the well-founded hypothesis of paraxiality, but for the whole beam pulse.This assumption, the chief advantage of which is to allow an electrostatic calculation of the space-charge field map in the beam frame, may be regarded as a rather reasonable first approximation when the beam pulse is sufficiently removed from the cathode. It cannot be retained for the beam pulse which has just been emitted. On the one hand, at the beam head, about 1 cm away from the cathode (for a 100 ps pulse), there are electrons which are already accelerated by the RF field to a relativistic velocity. On the other hand, at the beam back, next to the cathode, one may find electrons with thermal velocities. Moreover, the density is far from being uniform.

In these conditions the electromagnetic field map inside the beam pulse strongly differs from the map which would be deduced from an electrostatic calculation made in the beam centroid frame. In particular : a) the acceleration field is no longer negligeable before the velocity field, b) the relativistic retardation effects have to be taken into account, c) the electromagnetic force suffered by an electron in the z-slice is no longer $-e\gamma(z)^{-2}E_r(r,z)$, but $-e[E_r(r,z) - \beta_z(z)cB_\theta(r,z)]$.

In Sec. II, a theoretical study of electron motion is presented, which takes these electrodynamic effects into account. In Sec. III, sample maps of the electromagnetic field inside the beam pulse are given and discussed. In Sec. IV, consequences on the self-field limitation for photoemission is examined.

In a companion paper [5], the transverse emittance of the just emitted electron beam pulse is calculated as a function of the parameters. It appears that, before any propagation, this transverse emittance already has a value of the order of 10 or 50 π mm.mrad (according to the pulse duration).

In fact, the main part of the emittance growth due to the "space-charge" field, more properly called the "self-field", takes place during photoemission.

II. THEORETICAL MODEL

A. *Electromagnetic field : general Liénard-Wiechert expression*

All the physical quantities will be calculated in the laboratory frame. The field (**E**,**B**) generated at time t, at point P, by an electron which, at time t, is at point M (t), is given by the general Liénard-Wiechert expression

$$\mathbf{E}(P,t) = \frac{-e}{4\pi\varepsilon_0}\left\{ \frac{\mathbf{n} - \mathbf{v}(t')/c}{\gamma(t')^2[1-\mathbf{n}.\mathbf{v}(t')/c]^3\; M(t')P^2} + \frac{\mathbf{n}\wedge\{[\mathbf{n}-\mathbf{v}(t')/c]\wedge\dot{\mathbf{v}}(t')/c\}}{c\; M(t')P\;[1-\mathbf{n}.\mathbf{v}(t')/c]^3} \right\}.$$

0-7803-0135-8/91$01.00 ©IEEE

$$\mathbf{B}_P(t) = \frac{1}{c}\,\mathbf{n}\wedge\,\mathbf{E}_P(t),$$

where :

$$\mathbf{n} = \frac{\mathbf{M}(t')\mathbf{P}}{M(t')P},$$

and where the retardated time t' is such that $M(t')P = c\,(t-t')$. The first term in $\mathbf{E}$ is the velocity field, which decreases as $1/r^2$, the second one the acceleration field -the radiation field far from the source- which decreases as $1/r$.

B. Electron motion : iteration procedure

Except for a narrow layer next to the cathode, the paraxial condition $I \ll I_A = 17\beta\gamma$(kA) is satisfied for $I \geq 100$ A, if the axial RF electric field on the cathode has an amplitude $E_0 > 10$ MV/m. In these conditions, for almost the whole beam pulse, axial and radial electron motion can be decoupled, and all the electrons of a given slice have the same γ. The correction due to the narrow non-paraxial zone will be discussed later.

Taking advantage of the decoupling, and considering an axisymmetric beam, where r,z are the cylindrical coordinates, the following iteration procedure has been used to calculate the electromagnetic field map inside the pulse, at different times $t \in (0,\tau)$, where τ is the pulse duration.

At order 0, the $(\mathbf{E},\mathbf{B})^{(0)}$ map is determined from the slice axial trajectories $z^{(0)}(t|t_0)$ in the axial RF field alone, where t_0 is the emission time for the considered slice. Then, order 1 slice trajectories $z^{(1)}(t|t_0)$ are calculated for electron slices submitted to both the RF field and the (0)-axial self-electric field on the beam axis $E_z^{(0)}(r=0,z,t|t_0)$. From $z^{(1)}(t\,|t_0)$, a new field map $(\mathbf{E},\mathbf{B})^{(1)} = [\mathbf{E}^{(1)}(r,z,t),\ \mathbf{B}^{(1)}(r,z,t)]$ is deduced,,... Three iterations have shown themselves sufficient.

C. Analytic treatment of the electrodynamic and retardation effects: the case of $E_z^{(0)}(r,z,t) = E_{z\beta}^{(0)}(r,z,t)$ (velocity field) $+ E_{z\dot{\beta}}^{(0)}(r,z,t)$ (acceleration field)

The beam pulse is assumed to be cylindrical, with radius R, carrying a current I, emitted by the cathode with a constant and radially uniform current density J. Taking into account the retardation effects -the detailed, rather intricate, discussion of which cannot take place here- and the boundary condition imposed on the cathode by equipotentiality, one finds for $\mathbf{E}^{(0)}(r,z,t)$, with the notations of Fig.1 :

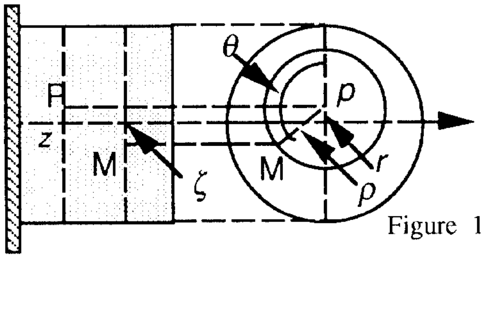


Figure 1

- $$E_{z\beta}(P,t) = \frac{-J}{4\pi\varepsilon_0 c}\int_0^{\zeta_{max}} \frac{1}{\beta(|\zeta|)\gamma(|\zeta|)^2}\cdot\Big\{\chi[\zeta,\beta(|\zeta|)|P,t] + \chi[-\zeta,-\beta(|\zeta|)|P,t]\Big\}\,d\zeta$$

- $$E_{z\dot{\beta}}(P,t\,|\zeta) = \frac{-J}{4\pi\varepsilon_0 c^2}\int_0^{\zeta_{max}} \frac{\dot{\beta}(\zeta)}{\beta(\zeta)}\cdot\Big\{\sigma[\zeta,\beta(|\zeta|)|P,t] + \sigma[-\zeta,-\beta(|\zeta|)|P,t]\Big\}\,d\zeta,$$

where :

$$\chi[\zeta,\beta(|\zeta|)|P,t] = \int_{\mathcal{D}(P,\zeta,t)} \frac{\zeta - z + \beta(|\zeta|)\sqrt{\rho^2+(\zeta-z)^2}}{\left[\sqrt{\rho^2+(\zeta-z)^2}+\beta(|\zeta|)\,(\zeta-z)\right]^3}\,\rho\,d\rho\,d\theta,$$

$$\sigma[\zeta,\beta(|\zeta|)|P,t] = \int_{\mathcal{D}(P,\zeta,t)} \frac{\rho^2}{\left[\sqrt{\rho^2+(\zeta-z)^2}+\beta(|\zeta|)(\zeta-z)\right]^3}\,\rho\,d\rho\,d\theta,$$

and : $$\beta(|\zeta|) = \frac{\sqrt{|\zeta|(|\zeta|+2\Lambda)}}{\Lambda+|\zeta|}.$$

$\Lambda = mc^2/eE_0$. $\mathcal{D}(P,\zeta,t) =$ disc $[p, \rho_{max}(\zeta)] \cap$ beam

$$\rho_{max} = \sqrt{\left[ct - \sqrt{\zeta(\zeta+2\Lambda)}\right]^2 - (\zeta-z)^2}.$$

and

$$\zeta_{max} = \frac{(ct+z)^2}{2(\Lambda+ct+z)}.$$

III. ELECTROMAGNETIC FIELD MAP : SAMPLE RESULTS

Table I shows the axial electric field at cathode center ($r=0, z=0$), when the whole electron beam pulse is extracted ($t=\tau$), for : $J = 100$ A/cm^2, $\pi R^2 = 1$ cm^2, $E_0 = 30$ MV/m,

τ=100 ps. $E_z(0,0,\tau) = E_{z\beta}(0,0,\tau) + E_{z\dot\beta}(0,0,\tau)$ is compared on the one hand to the 0-order value $E_z^{(0)}(0,0,\tau)$ and, on the other hand, to the values which would be deduced from an electrostatic calculation for either a constant uniform emitted current density, or a uniform density.

TABLE I

E_0	$E_{z\beta}$	$E_{z\dot\beta}$	E_z	$E_z^{(0)}$	$E_{z\ stat\ J}$	$E_{zstat\ \rho}$
			(MV/m)			
15	5.14	0.56	5.70	5. 72	6.10	4.30
30	3.60	0.74	4.34	4.35	4.68	3.08

E_0 : RF electric field ; E_z : axial electric self-field at cathode center ($E_{z\beta}$: its velovity part, $E_{z\dot\beta}$: its acceleration part) ; $E_z^{(0)}$ zero-order E_z (axial electric self-field generated by beam electrons which are supposed to be accelerated by the RF field only). $E_{z\ stat\ J}$: axial electric self-field at cathode center deduced from a Poisson equation for a constant J photoemission. $E_{zstat\ \rho}$: axial electric self-field at cathode center deduced from a Poisson equation for a constant ρ photoemission.

IV. PHOTOEMISSION SELF-FIELD LIMIT

Fig. 2 and 3 show the maximum extracted current $|J_{max}|$ as a function of the pulse duration τ, for πR^2=1 cm^2, and for E_0=15 MV/m or 30 MV/m.

Figure 2

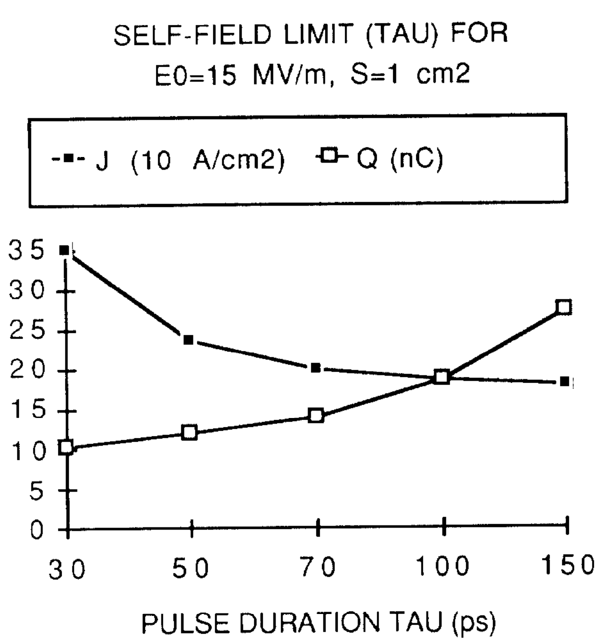
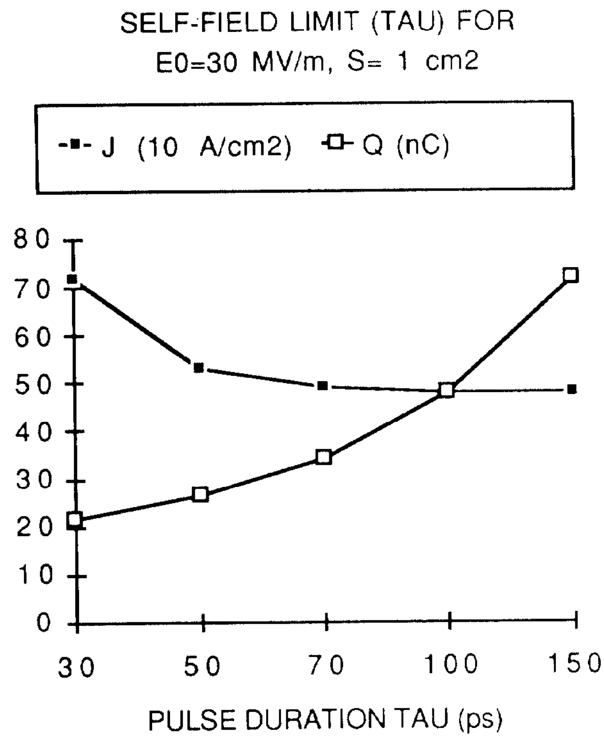


Figure 3



V. REFERENCES

[1] M.E. Jones and B.E. Carlsten, "Space-charged-induced emittance growth in the transport of high-brightness electron beams", in *1987 IEEE Particle Accelerator Conference Proceedings*, Washigton,DC, March 1987, pp.1319-1321.

[2] B.E. Carlsten and R.L. Sheffield, "Photoelectric injector design consideration", in *1988 Linear Accelerator Conference Proceedings*, Williamsburg,VA, October 1988

[3] K.J. Kim, "RF and space-charge effects in laser-driven RF electron guns", Nuclear Instr. Meth., A 275, pp. 201-218 (1989).

[4] B.E. Carlsten, "Photoelectric injector design code", in *1989 IEEE Particle Accelerator Conference,* pp. 313-315

[5] J.-M. Dolique and M. Coacolo, "Transverse emittance of an intense electron short pulse, just emitted by the cathode of a RF-FEL photoinjector : influence of electrodynamic effects", *this Conference*