

## Observation and Analysis of Time-Dependent Closed Orbit Motion in the LAMPF Proton Storage Ring

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### Abstract

When the stored beam is artificially offset in a section of the LAMPF Proton Storage Ring by changing selected ring dipole strengths, there is evidence for a small time dependence of the offset during the course of beam injection. A complete discussion of the time dependence of orbit offsets should take into account at least the following possibilities: 1) correlations between the injection timing pattern and ring dipole field ripple, 2) correlations between the injection timing pattern and changes of beam position monitor characteristics, and 3) growth of space-charge effects as the number of stored protons increases. Since there is no *a priori* reason to expect the correlations mentioned, we have analyzed the observed time dependence of the beam offset in terms of space-charge effects only, although the other possible causes cannot be ruled out. The buildup of circulating charge during proton injection leads to a shift of the betatron tune of individual protons because of space-charge forces; this shift can cause a change of the individual proton closed-orbit positions, and consequently a change in the position of the beam as a whole. At the end of a PSR injection cycle there are approximately  $2.5 \times 10^{13}$  protons stored in the ring. The observed time dependence of the beam offset indicates a horizontal-plane tune shift of  $-0.03 \pm 0.02$ ; this is consistent with a theoretical estimate of a maximum expected space-charge tune shift of  $-0.09$  when  $2.5 \times 10^{13}$  protons are stored in the ring.

### I. INTRODUCTION

In a storage ring the average position of the stored beam at a given location depends on the betatron tune of the individual protons averaged over the whole ensemble of protons circulating in the ring. If, for any reason, the average tune changes in time, e.g., because of space-charge forces, the average orbit position is affected. We made use of this fact to look for evidence of space-charge horizontal-plane tune shifts in the LAMPF Proton Storage Ring (PSR) at LANL. We observed the horizontal beam position at a fixed location in the ring at three different times during proton injection as the amount of circulating charge, and the resulting space-charge effects, increased. These observations were made both before and after the beam was offset by changing selected ring dipole strengths (referred to from here on as dipole errors). The time dependence of the offset provides a measure of space-charge tune shift.

### II. THEORY

#### A. Closed-orbit Distortion

It has been shown that the change of an individual particle's closed-orbit position, caused by insertion of dipole errors in a

storage ring, depends both on the errors and on the betatron tune of the particle [1,2]. Therefore, if dipole errors are introduced purposely in order to offset the closed orbit away from its initial position, the magnitude of the offset at a given location in the ring provides a measure of the betatron tune of the particle.

If dipole errors are represented as angular kicks,  $\phi(k)$ , at locations,  $k$ , then the offset,  $\Delta u(s)$ , of the horizontal position of the closed orbit of a given proton at location,  $s$ , can be written as

$$\Delta u(s) = \sum_k \frac{1}{2} \phi(k) \sqrt{\beta(s) \beta(k)} \frac{\cos(\pi Q - \mu(s,k))}{\sin \pi Q} \quad (1)$$

where  $\beta(s)$  and  $\beta(k)$  are the betatron functions at the location of interest and at the location of the  $k$ th kick respectively,  $Q$  is the betatron tune of the proton, and  $\mu(s,k)$  is the phase advance measured from the  $k$ th kick to the location,  $s$  [2].

#### B. Space-charge Tune Shift

Near the end of a PSR injection cycle, the circulating charge is large enough to exert significant space-charge forces on protons; the forces are defocusing and reduce the tune of individual protons. The net defocusing effect, as manifested in relatively large tune shifts, is largest for those protons undergoing small-amplitude betatron oscillations about the center of charge of the beam; protons following trajectories with large amplitude oscillations experience relatively smaller space-charge tune shifts. This amplitude dependence of the tune shift can be explained in a qualitative way by using a harmonic-oscillator-like picture. In this model the ring quadrupoles provide a restoring force (with an effective spring constant) to proton motion transverse to its average orbit. The oscillation frequency or tune depends on the net effect of all the quadrupoles, increasing with quadrupole strength. The effect of the repulsive space-charge forces is to cancel some of the primary attractive forces exerted by the quadrupoles; the result is a weakening of the effective spring constant and a decrease in the oscillation frequency or betatron tune. This explanation can be made a little more quantitative by considering a model in which the space charge is distributed with cylindrical symmetry about the average orbit, the total charge within a radius  $r$  being given by  $q(r)$ . The repulsive force on a proton at  $r$  is then proportional to  $q(r)/r$ . If the force is written in the same form as the force defining a harmonic oscillator, i.e., as proportional to the displacement  $r$ , the space-charge force is expressed as

$$f(r) = \frac{a q(r)}{r^2} \times r \quad (2)$$

where  $\alpha \times q(r)/r^2$  looks like a spring constant.

At small  $r$  and for realistic charge distributions,  $q(r)$  is very nearly proportional to  $r^2$  so that the spring constant describing the space-charge force is approximately constant independent of  $r$ . This constant subtracts from the constant representing the effects of the quadrupoles. So, for those protons with small betatron-oscillation amplitudes, the result is a shift of the oscillation frequency or betatron tune downward. For protons undergoing large-amplitude oscillations, the average  $q(r)$  over the full range of  $r$  spanned by the proton motion tends to a constant as  $r_{\max}$  increases so that the effective spring constant expressed above tends toward smaller values when averaged over the full range of  $r$ . In this case there is relatively little space-charge tune shift.

### C. Beam Position Offset

Equation (1) shows that the magnitude of a proton closed-orbit offset, introduced by the insertion of dipole errors, depends on the space-charge tune shift because of the offset dependence on  $Q$ . Since each proton has a different betatron oscillation amplitude, its tune shift, and therefore its closed-orbit offset, will be unique. Any measurement of beam position offset provides a measure of the average closed-orbit offsets for all the protons each having a different betatron oscillation amplitude.

## III. MEASUREMENTS

### A. Description of the Proton Storage Ring

The PSR, sketched in Figure 1, is filled with protons by first stripping  $H^-$  ions to  $H^0$  with a 1.8-Tesla dipole magnet in the ring injection line and then stripping the  $H^0$  to  $H^+$  with a 200- $\mu\text{g}/\text{cm}^2$  carbon foil in the ring itself.

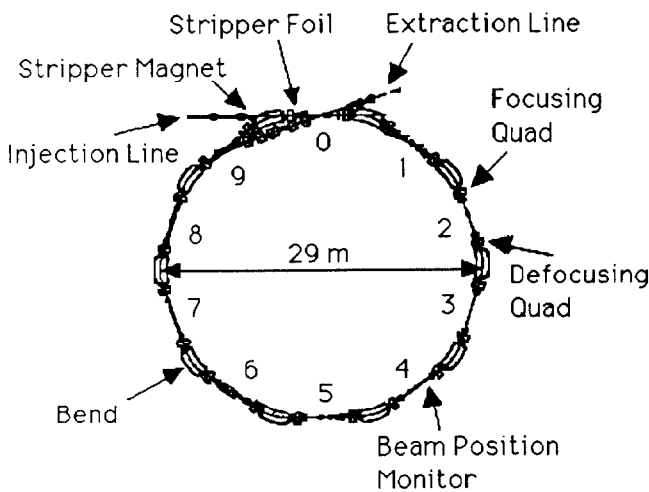


Figure 1. Layout of the LANL Proton Storage Ring

After the ring is filled by proton injection, the stored beam is extracted and transported to the neutron production target at the Los Alamos Neutron Scattering Center (LANSCE). The injection/extraction cycle occurs twenty times per second. Near the end of an injection interval there are approximately

$2.5 \times 10^{13}$  protons circulating in the ring, enough to give rise to significant space-charge forces.

### B. Offset and Measurement of Beam Position

In section II it was pointed out that the magnitude of the beam offset caused by insertion of dipole errors depends on the space charge tune shift because of the tune dependence of the offset shown in equation (1). We used this orbit offset behavior as a tool to look for evidence of a space-charge tune shift as the amount of beam circulating in the ring increases. For the measurements described, the nominal unshifted horizontal-plane betatron tune was 3.16.

Two sets of measurements were done. During the first set there were no dipole errors introduced into the ring artificially. The beam position was measured at the beam position monitor shown in section 4 of the ring (Figure 1). The position was measured at three different times during an injection interval as the amount of stored beam, and therefore the space-charge effects, built up. The measurements were done at 200, 400, and 580  $\mu\text{s}$  after the start of injection. At 200  $\mu\text{s}$  the amount of stored beam is small, but at 580  $\mu\text{s}$  the effects of space-charge forces were expected to be detectable.

During the second set of the same three measurements, dipole errors had been inserted into the main ring benders located at the start of sections 1–6 by shunting part of the current in these magnets. The dipole strengths were adjusted to result in a nominal 5-mm outward (or positive) orbit shift at the section-4 beam position monitor. Figure 2 shows the relative sizes of the angular kicks introduced by the dipole errors while Figure 3 shows the theoretical (assuming the nominal tune of 3.16) closed-orbit offsets to be expected near the upstream end of the ring quadrupoles.

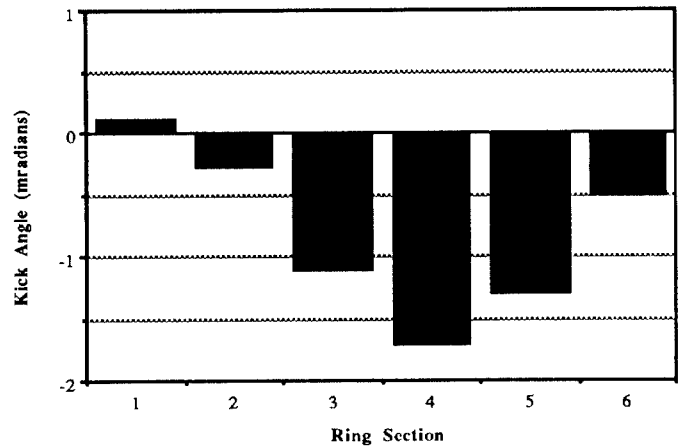


Figure 2. Kick Angles Introduced at Ring Bending Magnets at the Start of Ring Sections 1 – 6.

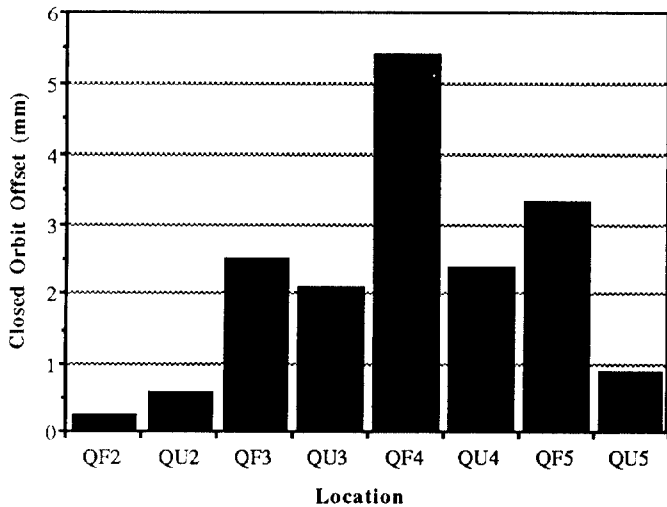


Figure 3. Closed-orbit Offsets Expected Near the Upstream End of the Ring Quadrupoles After Dipole Changes were Inserted.

QFn = focusing quadrupole near beginning of section n.  
 QU<sub>n</sub> = defocusing quadrupole near end of section n.

#### IV. RESULTS

The time dependence of the dipole-error-induced orbit offset (at the section 4 beam position monitor) is plotted in Figure 4. Measurements of the beam position were done by photographing an oscilloscope trace of the beam-position-monitor signal, and the error bars in Figure 4 represent errors in estimating the signal strength from these photographs.

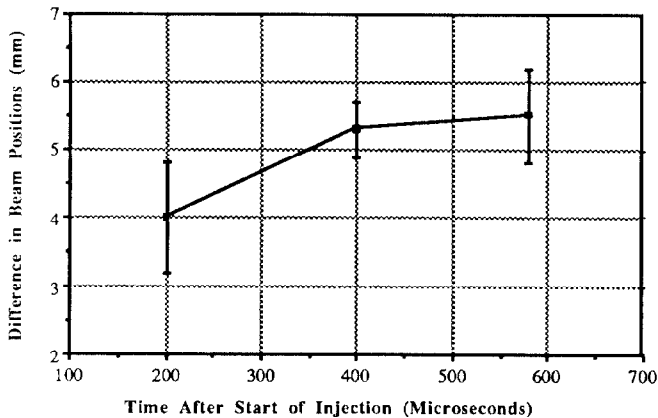


Figure 4. Plot of Difference Between Orbit Positions Before and after Insertion of Dipole Strength Changes

The change of the orbit offset, as time progresses from 200 to 580  $\mu$ s after start of injection, is  $+1.5 \pm 1.0$  mm. The change is interpreted here as evidence for a space-charge tune shift caused by the increase in the number of circulating protons from near zero to approximately  $2.5 \times 10^{13}$ . By substituting a range of values of Q into equation (1), along

with the magnitude of the beam angular kicks corresponding to the inserted dipole errors, a change of Q can be found which results in a change of the calculated value of  $\Delta u(s)$  that is equal to the measured orbit offset change of  $+1.5 \pm 1.0$  mm; the change of Q consistent with the offset change is found to be  $-0.03 \pm 0.02$  and is interpreted as a space-charge tune shift. Recall that this represents a kind of average tune shift over all protons since the tune of an individual proton depends on the amplitude of its betatron oscillations.

Using Guignard's [3] expressions for calculating tune shifts due to direct space-charge forces, we estimate that the maximum expected tune shift for protons in the PSR, when it is operating with a nominal horizontal betatron tune of 3.16 and with  $2.5 \times 10^{13}$  stored protons, is approximately  $-0.06$ . This is somewhat larger than our experimentally estimated value of  $-0.03 \pm 0.02$ , but it is not surprising since the orbit position measurements average the closed-orbit offsets for all the protons in the beam; some protons have small-amplitude betatron oscillations and relatively large tune shifts, while others have larger oscillations and smaller tune shifts.

#### V. REFERENCES

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- [3] G. Guignard, Selection of Formulae Concerning Proton Storage Rings, CERN 77-10, June 1977, pp. 43-44.