Separation Criterion and Scaling Law for Long-Range Beam-Beam Interactions in the Tevatron

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Introduction

Future collider runs at Fermilab will be fundamentally different from the first (86-87) and the second (88-89) runs. Starting with the third (1992) run, closed orbits of protons and antiprotons will be helically separated everywhere in the Tevatron except at interaction regions B0 and D0.

The helical separation scheme makes it possible to improve the initial luminosity greatly but also brings the problem of long-range beam-beam interactions. What is the contribution of long-range interactions to beam lifetime, to emittance growth, to background noise in the detectors? How much separation will be sufficient? What are the scaling laws relevant to long-range interactions? In this paper we address some of these questions. A more detailed discussion can be found in Ref.[2].

Separation Parameter

In any analysis it is important to find the natural parameter relevant to the particular problem being studied. Here we introduce a new separation parameter, S, which will reduce the parameter space by 2 and facilitate the formulation of scaling laws.

Fig.(1) illustrates the difference between long-range and head-on beam-beam interactions. The distance between the centers is d and the conventional (dimensionless) separation parameter is given by $D \equiv d/\sigma_{\text{source}}$. Here "source" refers to the bunch with higher intensity; in the case of the Fermilab Collider it is the proton bunch. The "probe" refers to the weaker bunch (antiproton). This terminology is also suggestive of the weak-strong model where the source (strong) bunch creates the field and the probe (weak) bunch samples it. The source bunch distribution is not changed by the probe beam. Fig.(2) shows the beam-beam kick. Note that for $-1 < x/\sigma < 1$ the beam-beam kick is fairly linear. It is strongly nonlinear between $1 < x/\sigma < 2$ and $-2 < x/\sigma < -1$ and also exhibits nonlinearity for $|x/\sigma| > 2$ as the strength of the kick decreases asymtotically ($\sim 1/r$).

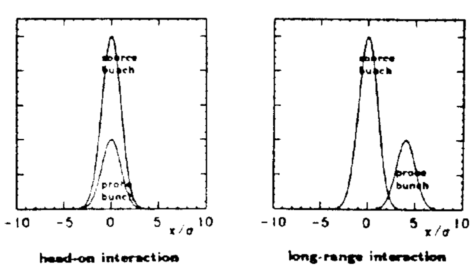

Figure 1: Transverse profiles for source and probe bunches.

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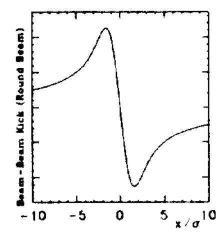

Figure 2: The kick experienced by an oppositely charged probe particle as it passes by a round gaussian source bunch.

We need a separation parameter which is also a function of the probe particle amplitude. We introduce

$$S \equiv \sqrt{D^2 - 2DA + A^2} = | D - A | \tag{1}$$

$$D \equiv \sqrt{(d_x/\sigma_x)^2 + (d_y/\sigma_y)^2} \tag{2}$$

$$A \equiv \sqrt{(a_x/\sigma_x)^2 + (a_y/\sigma_y)^2} \tag{3}$$

Here, a_x and a_y are the probe particle amplitudes, d_x and d_y are the projections of d on horizontal and vertical planes respectively. σ_x and σ_y are the r.m.s. (source) beam widths.

The parameter S should be interpreted as the "effective amplitude" since the probe particle oscillates around a closed orbit which is separated from the source beam by D, the difference $|D - A|$ gives the "effective" amplitude relative to the source bunch. The instantaneous amplitude relative to the source bunch can be greater or smaller than S depending on the separation. We take the absolute value of the difference because a negative amplitude would not be physicaly meaningful.

If there are many beam-crossing points then we simply take the average of S over the beam-crossing points. Henceforth we will use S and $< S >$ interchangably.

Measuring the Nonlinearity

The parameter S can be computed for a given configuration. One can also compute the average separation over turns. We used the beam-beam simulation code HOBBI[1] to plot $< S >_{\text{turns}}$ versus S for a mesh of a_x/σ_x, a_y/σ_y amplitudes. We call this average quantity

$$\chi = < S >_{\text{turns}} \tag{4}$$

we also plotted

$$\eta \equiv (\chi - S)/S \tag{5}$$

versus S for a mesh of amplitudes. η is a measure of nonlinearity since χ would be equal to S in a linear system. Fig.(3) demonstrates the power of χ-S and η-S diagrams. As we predicted in the previous section, the region

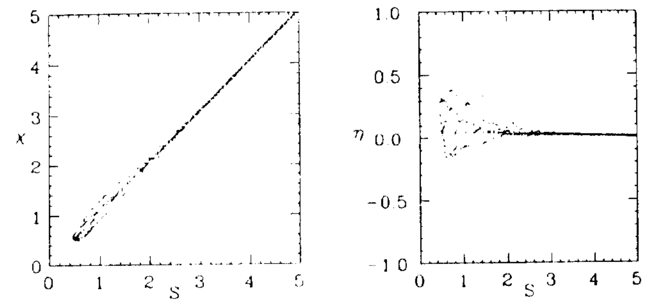

Figure 3: Diagrams showing the effect of nonlinearity for $1 < S < 2$. In a perfectly linear system these plots would be straight lines. Normalized deviation of S from its initial value as the particle circulates around the ring is caused by nonlinearity due to beam-beam interactions. The diagram has been generated from a mesh of amplitudes.

of $1 < S < 2$ exhibits a high degree of nonlinearity. If there were no beam-beam interactions, the χ-S curve would have been a 45° straight line and the η-S curve would have been identically zero for all S. Fig.(4) shows that the nonzero value of η is caused by long-range interactions only. Head-on interactions contribute very little.

Another observation from Fig.(4) is that the η value for small amplitude (probe) particles is not zero when long-range interactions are turned on. This means that the small amplitude particles are suffering from nonlinearities.

Scaling Law for Long-Range Interactions

The "η-measure" developed in the previous section can be used to quantify the nonlinear effects due to long-range interactions. In this section we investigate the dependence of η (for a given S) on bunch intensity, emittance and the number of long-range interactions. Here, we will concentrate on the small amplitude particles since they are the ones that contribute most to the luminosity. We computed $\eta(S \approx D)$ (small amplitude particles) for the 34x34 configuration which would be used in a Tevatron collider run with the Main Injector. In this configuration 34 proton bunches collide on 34 antiproton bunches. Bunch loading involves the injection of 3 batches (12 coalesced bunches) into the Tevatron establishing a 3-fold symetry, then knocking out 2 bunches to create abort gaps.

First, we varied the protons per bunch, N_p, while keeping other parameters constant. We found that $| \eta(S \approx D) |$ increases linearly with N_p (Fig.(5)).

We also generated similar plots by keeping all beam parameters but proton emittance constant and plotted $\eta(S = 3)$ versus emittance in Fig.(6). The error bars in this graph indicate the thickness of the η-S plots at $S = 3$. The conclusion from Fig.(6) is that the η for a given S does not depend on the emittance to first order. This conclusion seems counter-intuitive but closer thought reveals that the parameter S was indeed the right choice because it successfully reduces the dimension of the parameter-space by 2. In other words, one no longer deals with probe particle amplitude A, separation D or emittance ε but just one parameter S. The size of error bars, however, increase with emittance, indicating a second or higher order effect.

Finally, we studied how the number of long-range interactions affect the strength of nonlinearity. As expected it

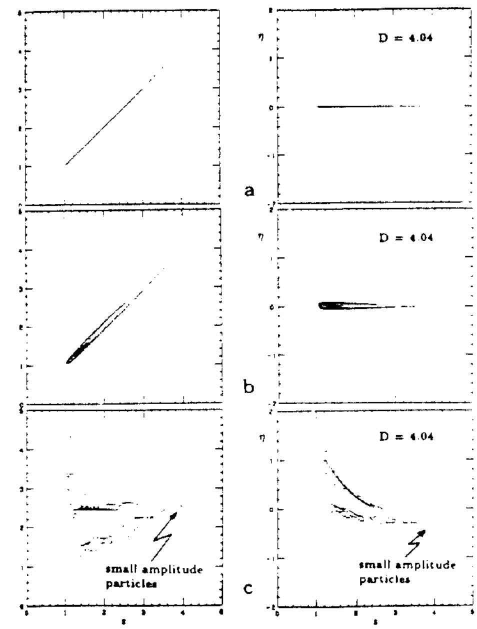

Figure 4: $\chi - S$ diagrams (left) and $\eta - S$ diagrams (right) for the Collider Run with the Main Injector. a) no beam-beam interactions b) only head-on interactions c) 2 head-on, 66 long-range interactions. Note that the nonzero value of $| \eta |$ is caused mostly by long-range interactions. Head-on interactions contribute very little.

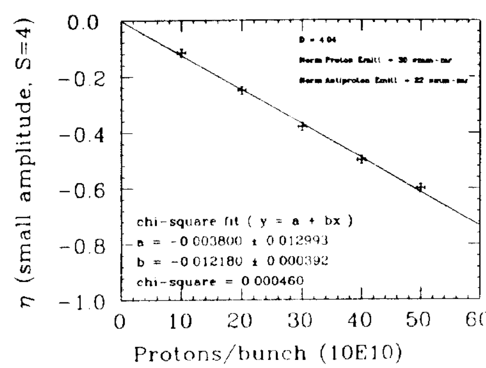

Figure 5: The strength of nonlinearity increases linearly with bunch intensity.

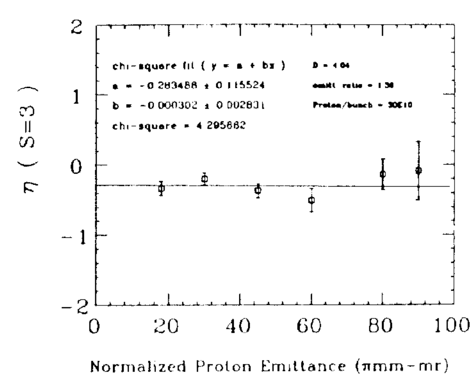

Figure 6: The strength of nonlinearity does not depend on the emittance to first order. The size of error bars, however, increase with emittance, indicating a second or higher order effect.

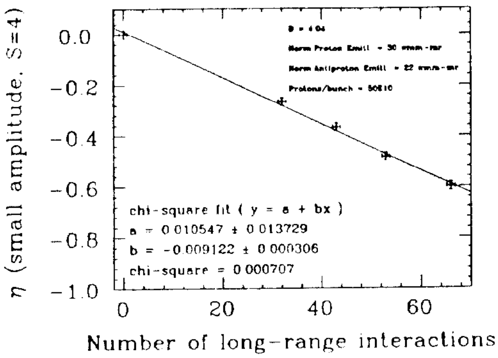

Figure 7: The strength of nonlinearity increases linearly with the number of long-range interactions.

is a linear relationship as shown in Fig.(7).

The conclusion from Fig.(5), Fig.(6) and Fig.(7) is that the strength of nonlinearity scales as

$$| \eta | \propto N_p N_{lr} \qquad (6)$$

where N_{lr} is the number of long-range interactions. Head-on interactions do not contribute to $\eta(S \approx D)$ (for small amplitude particles).

Tolerable Nonlinearity Criterion

In order to determine the proportionality constant in Eq.(6) one needs to know the value of η at which the nonlinearity is so strong that it causes beam loss. One could investigate this limit by studying the long term behaviour of particles in simulation. This approach would be more relevant for large amplitude particles and is currently being pursued. Since the scaling law, Eq.(6), was deduced for small amplitude particles we should find the η-limit when $S \approx D$. It is very fortunate that in the case of small amplitude particles, the η-limit can be established directly from operational experience, by relating the "scaling law" to the beam-beam tune shift limit.

The tune shift for small amplitude antiprotons from a single head-on beam-beam interaction is given by

$$\xi = \frac{N_p r_p}{\pi(\varepsilon_p \beta\gamma)} \qquad (7)$$

for a round beam. ε_p is the proton emittance, N_p is the proton bunch intensity. r_p is the classical proton radius and β, γ are the relativistic factors. At the beam energy of 1 TeV the numerical value is

$$\xi = 0.00733 \frac{N_p\ [10^{10}]}{\varepsilon_p\ [\pi\mathrm{mm}-\mathrm{mr}]} \qquad (8)$$

The operational experience at the CERN $Sp\bar{p}S$ and the Tevatron Collider has shown that the the beam-beam tune shift/spread limit is

$$\text{Tuneshift/spread} < 0.024 \qquad (9)$$

This can be rewritten as

$$0.00733 \frac{N_p\ [10^{10}]}{\varepsilon_p\ [\pi\mathrm{mm}-\mathrm{mr}]} N_{ho} R \ < \ 0.024 \qquad (10)$$

where N_{ho} is the number of head-on interactions, $R \equiv (\Delta\nu_{\mathrm{total}}/\Delta\nu_{\mathrm{head-on\ only}})$ is the ratio that shows the tune shift contribution from long-range interactions. R can be found from tuneshift-footprint diagrams[2].

In the Tevatron Collider the number of head-on collisions will always be 2 (CDF and B0 detectors). Using $R = 1.23$ (obtained from tuneshift-footprints[2]), and the nominal parameter $\varepsilon_p = 30$ πmm-mr, we come up with an upper limit on proton bunch intensity

$$N_p < 40 \times 10^{10} \qquad (11)$$

It is found graphically from Fig.(5) that $| \eta | \cong 0.5$ when $N_p = 40 \times 10^{10}$. Therefore the beam-beam η-limit is

$$| \eta(S \approx D) | < 0.5 \qquad (12)$$

We shall call Eq.(12) the "tolerable nonlinearity criterion". This can also be viewed as the connection between the head-on and the long-range beam-beam interactions. In other words, a beam stability criterion based on the head-on beam-beam experience is related to another criterion which applies to long-range interactions.

Eq.(12) also serves as the "separation criterion". In Ref.[2] this criterion is translated into "separation rules" for each step in the colliding beams sequence.

Configuration Limit

In Fig.(5), Fig.(7), the tolerable nonlinearity limit is reached when $N_p = 42 \times 10^{10}$ while $N_{lr} = 66$ or when $N_p = 50 \times 10^{10}$ while $N_{lr} = 56$. The $N_p N_{lr}$ product is 2.772×10^{13} and 2.800×10^{13} respectively which suggests that there is a proportionality constant K

$$K = 1.8 \times 10^{-14} \qquad (13)$$

which can be used to write the "configuration limit"

$$K N_p N_{lr} < 0.5 \qquad (14)$$

Here, it is important to point out that K is not a universal constant. It is related to the average separation D. The larger the D the smaller the K. The form of the relationship between K and D will be pursued in a later publication.

In the 1992 Collider Run, $N_p = 10 \times 10^{10}$, $N_{lr} = 10$, thus the product $K N_p N_{lr} = 0.018$ will be much smaller than 0.5 . In the Collider Run with the Main Injector, with $N_p = 30 \times 10^{10}$, $N_{lr} = 66$ the product $K N_p N_{lr} = 0.3564$ will still be less than 0.5, so the bunch intensity can be increased up to $N_p = 40 \times 10^{10}$ while keeping $K N_p N_{lr} \leq 0.5$

References

[1] Developed by the author. It was based on the code HELICAL written by Leo Michelotti.

[2] S.Saritepe, "Future Collider Runs in the Tevatron: Beam-Beam Simulation Results", FERMILAB FN-563 (1991)