Simulation of High Energy Beam Focusing by a Plasma*

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Abstract

We study the focusing of a high energy electron beam and a positron beam by means of a plasma. The simulation is done using the code CONDOR. We present the results of simulation and simple model calculations, and demonstrate the significant focusing of the beams attained.

I. Introduction

The study of focusing of high energy beams of electrons and positrons has been of interest for several years[1−4]. The mechanism offers a way to attain higher luminosity, without great technical complications, by reducing the spot size. This can be compared with the involved optical schemes necessary to attain small spot sizes in linear colliders. Of course the presence of a gas target will present problems of significant background for physics experiments as well as strong aberrations, present in the focusing mechanism, especially for positron beams.

In this paper we present model calculations for electron beams and compare with the results of a particle-in-cell simulation. We assume the beam has $3*10^{10}$ electrons(positrons),emittance $4*10^{-8}$ cm-rad and a beta function at the initial waist of 5 mm. During the simulation the beam used was round with a length of 138 microns. This study is further motivated by a recent report by one of the authors[5]. We study the focusing of the beam inside the plasma for both electron and positron beams.

II. Electron Focusing

The model for the focusing is based on the assumption that the front of the beam, representing a very small percentage of the total number of particles ejects the plasma electrons due to the strong fields of the beam transversely and as a result the rest of the beam is focused by the transverse electric fields of the ions left behind. We review the optics of electron focusing.

If K is the focusing strength and β the beta function the evolution of the beta function is given by

$$\beta''' + 4K\beta' + 2K'\beta = 0 \qquad (1)$$

or

$$\frac{d}{ds}(\beta'' + 4K\beta) = 2K'\beta \qquad (2)$$

The focusing strength K for a uniform density column of ions is given by

$$\frac{2\pi r_e n_0}{\gamma} \qquad (3)$$

where n_0 is the plasma density, γ the relativistic factor for the beam and r_e is the classical electron radius. Outside the longitudinal boundaries of the plasma we are in a drift space where $K = 0$. Integrating equation (2) through the boundary of the plasma one obtains

$$(\beta'' + 4K\beta)_{inside} - (\beta'' + 4K\beta)_{outside} = 2K\beta_0 \qquad (4)$$

here K is the focusing strength of the plasma lens and β_0 is the beta function at the entrance to the plasma. Outside the plasma lens $K = 0$ and

$$\beta = \beta_0^*(1 + \frac{s^2}{\beta_0^{*2}}) \qquad (5)$$

where s is measured from the waist in drift space in the direction of beam motion. Therefore $\beta'' = 2/\beta_0^*$ and

$$\beta_0 = \beta(s_0) = \beta_0^*(1 + \frac{s_0^2}{\beta_0^{*2}}).$$

The equation becomes

$$\beta'' + 4K\beta = 2K\beta_0 + \frac{2}{\beta_0^*}.$$

The solution[3] is

$$\beta = (\frac{\beta_0}{2} + \frac{1}{2K\beta_0^*}) + (\frac{\beta_0}{2} - \frac{1}{2K\beta_0^*})\cos(2\sqrt{K}(s - s_0)) + \frac{s_0}{\sqrt{K}\beta_0^*}\sin(2\sqrt{K}(s - s_0)) \qquad (6)$$

If the entrance to the lens is at $s_0 = 0$, the waist in drift space then the beta function oscillates as a cosine between $\beta_0 (= \beta_0^*)$ and $1/K\beta_0^*$. For focusing we clearly require

$$\frac{1}{K\beta_0^*} < \beta_0^* \Rightarrow K > \frac{1}{\beta_0^{*2}}.$$

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The focal length[5] is

$$s^* = \frac{\pi}{2\sqrt{K}}. \tag{7}$$

For the beam of interest one finds substituting the numbers for a plasma half as dense as the beam, n_0 equal to $0.865 * 10^{18}/cm^3$ and beam energy 0.45 GeV ,the final beam size to be 0.14 microns from the initial size of 2.828 microns with a focal length .381 mm. We do not expext significant emittance degradation, due to non-linearities in the focusing arising from the ion motion during the passage of the beam. The ion motion is much slower than the electron motion. The simulation was done using the code CONDOR developed at Lawrence Livermore Laboratory and SLAC. The beam size can be estimated best from the plots of the contours for the magnetic field (which is dominantly from the beam current) and we find the final size is the size of the transverse cells. Smaller sizes can't be resolved without finer transverse resolution. The focal length for the front portion of the beam which does not see the effects of ion motion is indeed what is expected from the model. For the rear of the beam the ion motion is such as to increase the focusing of the beam and this is observed in the reduction of the focal length as the rear is focused at later times.(See Figs. 1-4).

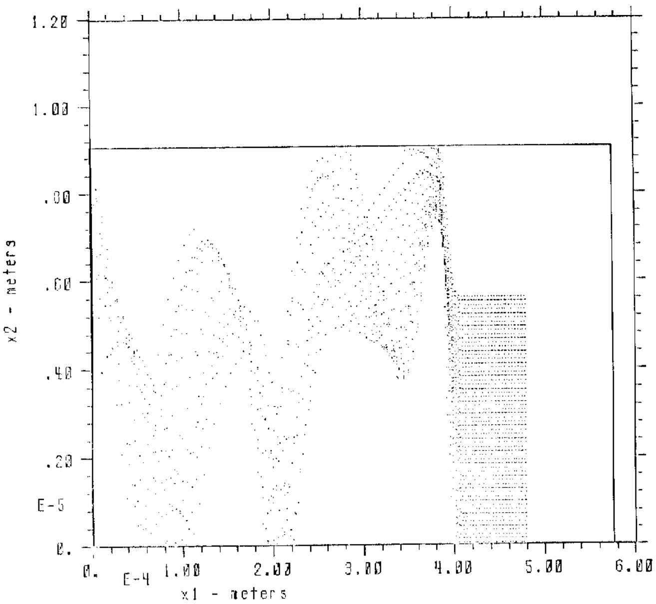

Fig 1.This shows the plasma electron motion. The snapshot is at the time the front of the beam is focusing down to its waist.Note the electrons are always outside the initial beam radius (2.828 microns) and move rapidly outside the beam.

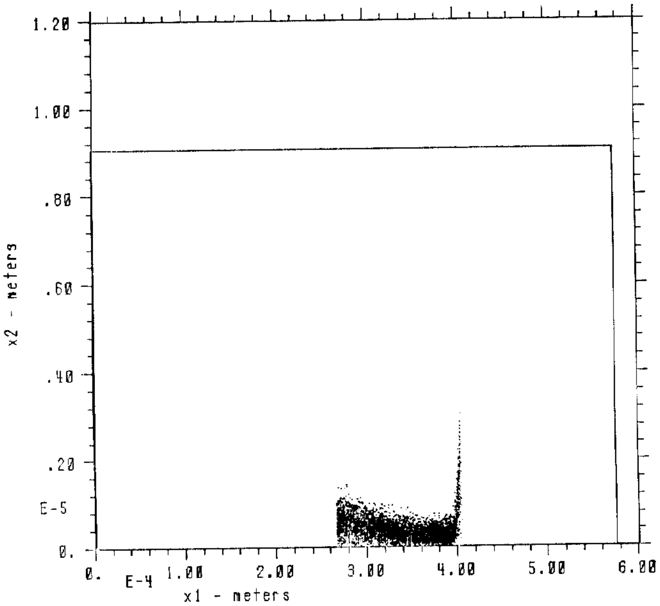

Fig 2.The electron beam focuses down to the waist at a distance about 380 microns from the plasma entrance as expected with motionless ions.

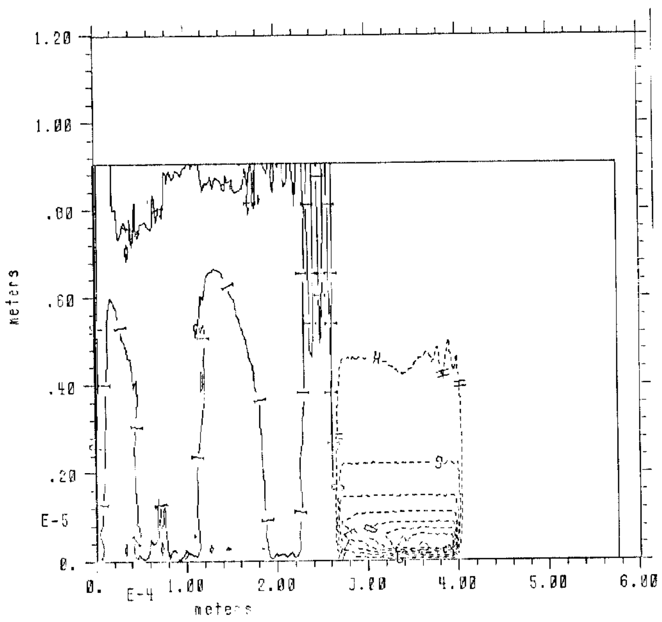

Fig 3.The contours of the magnetic field show that the waist of the beam is about the size of the transverse cell. The transverse resolution is not enough to resolve smaller beam sizes, but the emittance is preserved and the focal length is close to the predicted value. Thus we may say that the focusing is as expected.

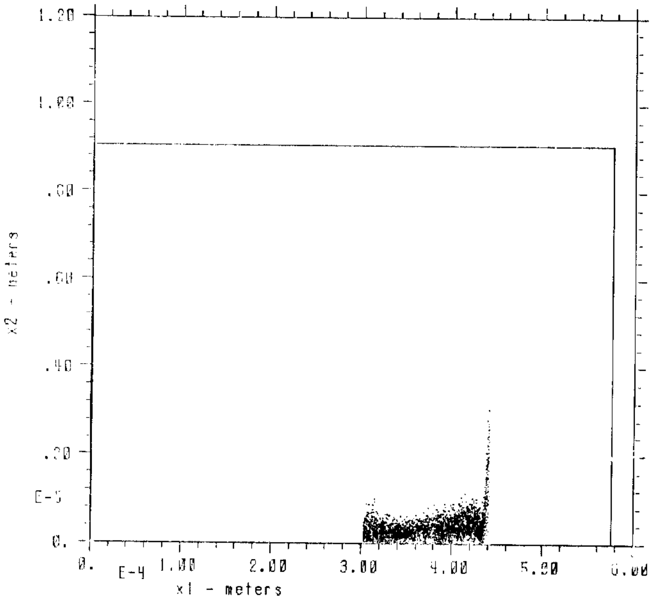

Fig 4. The portrait of the back of the electron beam focusing down to its waist at a different focal length from the front of the beam.

III. Positron Focusing

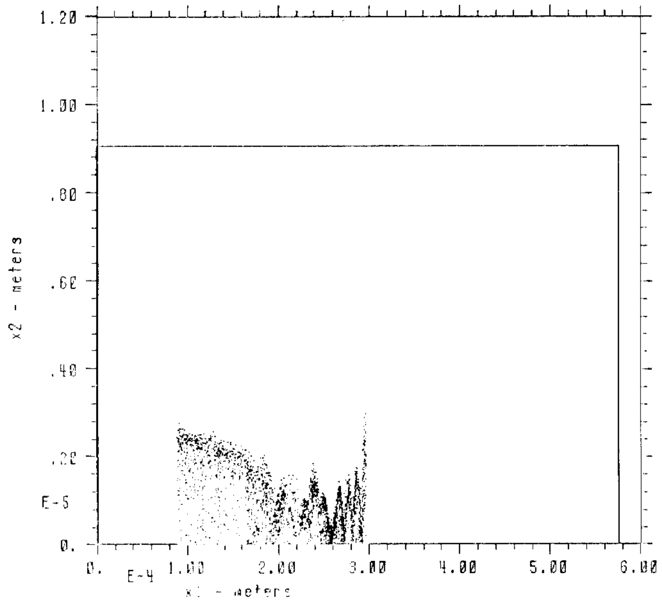

Fig 5. The focusing down of the positrons near the front of the beam. As can be seen from the plasma electron profile, most of the electrons are inside the initial beam radius,and since there are twice as many plasma electrons as positrons per unit length we would naively expect the focal length with linear focusing to be about 270 microns. With stronger non-linear focusing it would be smaller.

The plasma electrons oscillate in the fields of the beam and the ions. The motion is mainly transverse with a smaller longitudinal component due to the beam magnetic field.As a result the focusing seen by a beam positron at a distance behind the head of the beam depends on the distance. Some positrons are focused by a dense electron core and others by a diffuse electron profile. As a result the focused positron beam has different waists at different positions and a snapshot would reveal a wavy profile.The positron beam had a length of 207 microns.

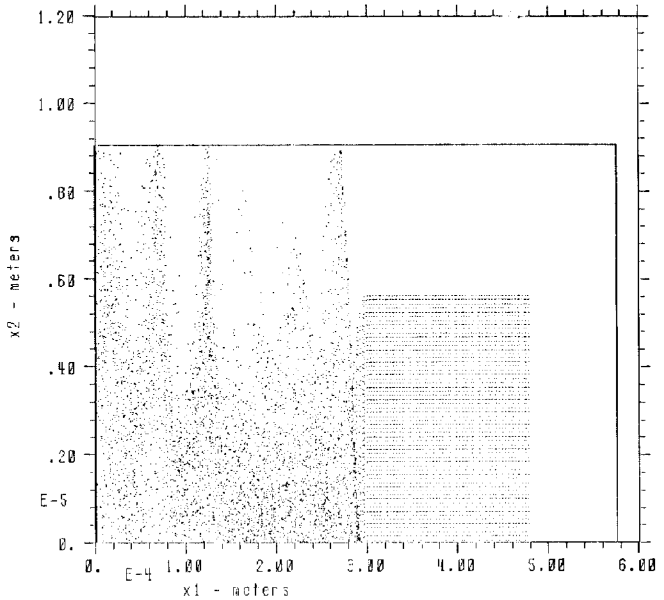

Fig 6. Plasma electron motion. The tranverse oscillations can be clearly seen.

IV. Conclusions

We have presented a preliminary analysis of the focusing of beams by plasmas. More realistic beam distributions , namely Gaussian or similiar, have to be studied through simulation. Further work needs to be done to understand in terms of tractable models, the positron focusing - the nonlinearities thereof and the growth in emittance which is observed in the simulation. A more detailed report of these studies will be published elsewhere in the near future. The authors would like to thank D. B. Cline,W. Gabella, K. Ko, K. Eppley and C. Ng for useful discussions.

V. References

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