GEOMETRICAL (LIENARD-WICHERT) APPROACH IN ACCELERATOR PHYSICS

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A geometrical (Lienard-Wichert) approach is used to define the field of a relativistic beam (both in vacuum approximation and taking into account reflections from vacuum chamber walls). The approach allows to take into account the curvature of the trajectory and the discreteness of the beam current.

Such an approach to the problems of accelerator physics was first used by I.Tamm [1] in 1947-1948. Recently the importance of the curvature effects is admitted in accelerator physics (see, e.g. [2-6]). However, these works are based on the continuous approximation of the current and in reality the whole component of the field related to the radiation is fallen out. This component essentially distorts the Coulomb term at distances as small as interparticle distances within the beam. Adequate to the physical condition is the conception of represnting the field as a sum of microscopic fields of individual particles averaged over the specific physical effect. The continuity of the current over the total field is not satisfied in modern electron accelerators [7,8].

In this report the results of the previous works [7-11] based on the remarkable spatial pattern of the Lienard-Wichert field of a particle are presented (the influence of these fields on dynamics is estimated). The reflections of the beam self-field from conducting surfaces of the accelerator are also considered. The incidence of the hard part of synchrotron radiation on the exterior wall of the vacuum chamber results in a localized "spot" of induced charge, which moves along that wall faster-than-ligth. The field and the force lines resulting from motion of this "spot" are determined.

1. Beam self-field

The bunch of electrons can be considered as a continuous one if [7,8]

$$\mu_e (R\sigma_x)^{3/2}\gamma^{-4} > 1 , \qquad (1)$$

where μ_e is the bunch density, R is radius of curvature of the bunch orbit, σ_x is the transverse size of the bunch in the orbit plane, γ is the Lorentz-factor of particles. Derivation of (1) is based on the analysis of the beam self-field as a superposition of Lienard-Wichert fields of particles [9-11]. The fact, that the hard part of synchrotron radiation of a particle is localized in a extended narrow γ-region that stretches from the particle (see formula (4)) is taken into account. Out of this region the field of the particle is substantially asymmetric with respect to the forward and backward directions :

$$\frac{R^2}{e}\vec{E}=\begin{cases} -\dfrac{2\hat{x}}{(3s/R)^{5/3}}+\dfrac{2\hat{s}}{(3s/R)^{4/3}}+\dfrac{2(z/R)\hat{z}}{(3s/R)^{7/3}}, \\ \dfrac{R}{2s^3}[(x^2-z^2-s^2)\hat{x}+2sx\hat{s}+2zx\hat{z}], \end{cases} \qquad (2a)$$

$$\frac{R^2}{e}\vec{H}=\begin{cases} \dfrac{2(z/R)\hat{x}}{(3s/R)^{7/3}}-\dfrac{(z/R)\hat{s}}{3(s/R)^2}+\dfrac{2\hat{z}}{(3s/R)^{5/3}}, \\ \dfrac{R}{2s^3}[2xz\hat{x}+2sz\hat{s}+(z^2-s^2-x^2)\hat{z}], \end{cases} \qquad (2b)$$

where s is the viewpoint coordinate relative to the particle along the trajectory, x and z are transverse deviations of this point, ($\hat{x}$, $\hat{s}$, $\hat{z}$) is the corresponding system of unit vectors, upper and lower formulae satisfy the conditions $s>0$ and $s<0$. The formulae (2) deviate in $1 \gg x/R \gg \gamma^{-2}$ approximation (the viewpoint is out of the Coulomb region) ,and also when $1 \gg s/R \gg (x/R)^{3/2}$, $1 \gg z/R \gg \gamma^{-1}(2x/R)^{1/2}$. One can "draw" the field of a particle by the lines of the field, the covariant techniques of which are developed in [12-16]. Illustrations of the field of a particle of Lorentz-factor γ=1.5 and γ=100 are presented in fig.1.

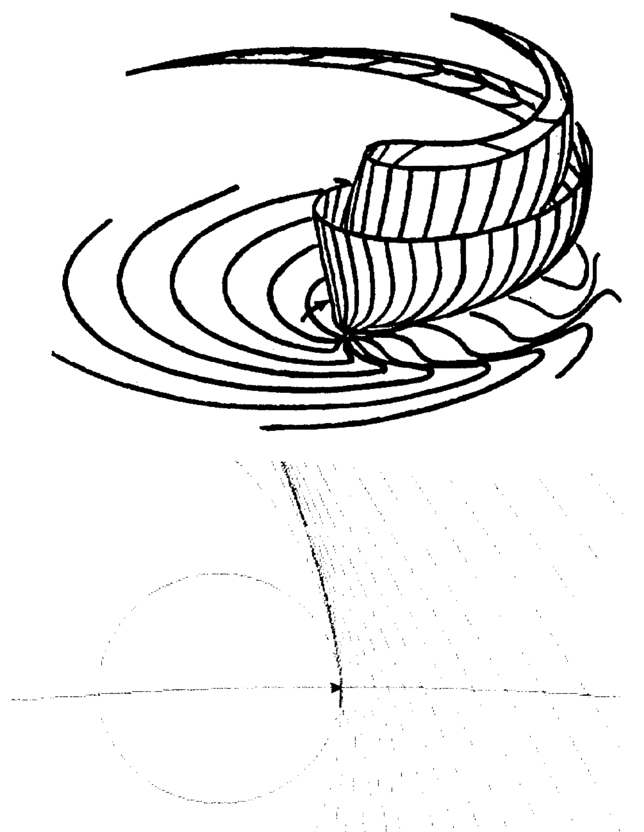

Fig.1. The field of the charge near the γ-region represented with the help of force lines, when γ=1.5 (A), γ=100 (B). The whole system of lines in the space can be represented as a set of line systems on surfases one enclosed in another. Two of these surfaces are shown in fig.1A (the trajectory is marked by an arrow). The force lines on the orbit plane are shown in fig.1B. When increasing γ, the size of the γ-region $R\gamma^{-3}\to 0$, however the structure of the field qualitively remains the same.

Following [11], one can obtain a general formula for the Lorentz force by which a one-dimensional bunch of uniform linear charge density λ acts on the unit charge moving at the velocity $\vec{\beta}c$ (all particles of the bunch move along the curve $\vec{r}_o(s)$ at the same rate βc):

$$\vec{F}=\lambda\left\{\beta\frac{\vec{n}-\vec{\beta}-[\vec{\beta}\times[\vec{n}\times\vec{\beta}]]}{l(1-\vec{n}\vec{\beta})}\Bigg|_{u_1}^{u_2}+\int_{u_1}^{u_2}du\frac{\vec{n}-[\vec{\beta}\times[\vec{n}\times\vec{\beta}]]}{l^2}\right\} \quad (3)$$

where $l=|\vec{r}-\vec{r}_o(u)|$ is the distance between the retarded position $\vec{r}_o(u)$ and the viewpoint $\vec{r}$, $\vec{n}=(\vec{r}-\vec{r}_o(u))/l$, the values of $\vec{\beta}$ are taken at the corresponding retarded positions. The first radiation term of (3) is caused by the bunch edges and the second term is caused by continuous charges and currents. Using (3) one can estimate the required quantity of terms in expansion of the force $\vec{F}$ over the small deviations of the viewpoint from the bunch center and corresponding influence of these terms on the beam dynamics (unwieldy expansions in [3] lead to inaccurate results).

The condition (1) is not satisfied for the present cyclic electron accelerators (for PETRA the left side of (1)$\simeq 3\times 10^{-3}$). Therefore the self-field of the beam consists of a smooth component (the sum of fields (2)) and in average non-intersecting γ-regions (stochastic component, i.e. spatial localization of the hard part of synchrotron radiation).

The main result of smooth component action is the shift of the betatron oscillation frequency ν_x [8,10] by the value $\Delta\nu_x \simeq \Lambda\mu_e R\sigma_z r_o/(2\nu_x\gamma)$, where $r_o=e^2/mc^2$, $\Lambda\simeq 10$, σ_z is the vertical size of the beam. There is no γ^{-2} factor in comparison with the value of the frequency shift counted in the rectilinear approximation of the tracks (see, e.g. [17]). This fact is due to the absence of compensation of electric and magnetic forces of two-particle Lorentz forces, which is restored at the distances $\lesssim R\gamma^{-2}$ (this quantity is less than average distance between the beam particles).

The influence of the stochastic component of the field is similar to the quantum swinging of betatron oscillations caused by self radiation [11]. In fact, the question is about mutual scattering of the particles by synchrotron radiation.

2. Reflections of synchrotron radiation from the vacuum chamber walls

Accelerator chamber deforms the vacuum field of the beam. However, at small distances from the particle, the vacuum component predominates over the reflected fields. For a particle, the space in the curvilinear chambers of cyclic accelerators is divided into a lighted part and a shadow region. The lighted region has an abrupt front at a distance of $\simeq(2d)^{3/2}R^{-1/2}/3$ from the particle and extends $\simeq 6.7\times(Rd)^{1/2}$, where 2d is transverse size of the vacuum chamber in the orbit plane. The first value $\simeq 0.03$ cm for $R=10^3$ cm, d=5 cm, and $\simeq 0.1$ cm for $R=10^4$ cm. In both cases it is far greater than the distance between particles. Therefore, when studying the influence of the nearest particles on the electron dynamics, one may neglect the reflected fields.

Let's consider the hard part of the field of a particle that circulates inside a azimuthal symmetric vacuum chamber. Since the hard part of synchrotron radiation is formed within the angle $<\gamma^{-1}$ to the direction of the particles motion, one may consider reflections from the exterior walls of the chamber only. Along this wall, perturbations of the charge density run at the rate $\beta c(1+d/R)$. For the β, d and R under consideration this value is greater than the speed of light. The maximum of the perturbation is caused by the field of γ-region. The electric field polarization is almost radial (the radiation incidence angle is $\simeq (2d/R)^{1/2}$). The field in the γ-region is determined by the formulae [7,18,19]:

$$\vec{E}=\frac{4e\gamma^4}{lR}\,\frac{(1+\alpha^2-\eta^2)\hat{x}_s+2\alpha\eta\hat{z}_s}{(1+\alpha^2+\eta^2)^3},$$
$$\vec{H}=\frac{4e\gamma^4}{lR}\,\frac{(1+\alpha^2-\eta^2)\hat{z}_s-2\alpha\eta\hat{x}_s}{(1+\alpha^2+\eta^2)^3}, \quad (4)$$

where $\hat{x}_s$, $\hat{z}_s$ are the unit vectors at the retarded point, l is the distance between the given point and the point at the center of the cross-section of γ-region, $\gamma^{-1}\alpha l$ is the vertical deviation of the viewpoint in that cross-section, the parameter η is related to the horizontal deviation $\gamma^{-3}\chi R$ of the viewpoint from the central point by relation: $\eta=(((1+\alpha^2)^3+9\chi^2)^{1/2}+3\chi)^{1/3}-(((1+\alpha^2)^3+9\chi^2)^{1/2}-3\chi)^{1/3}$. It is evident, that the transverse sizes of the γ-region on the vacuum chamber are $\simeq\gamma^{-3}R$ in the orbit plane (the characteristic wavelength of synchrotron radiation) and $\simeq\gamma^{-1}(2Rd)^{1/2}$ in the vertical direction.

If the walls of the chamber were ideal conductors the induced charge q in this "spot" would be $q\simeq eE(R\gamma^{-3})(\gamma^{-1}(2rd)^{1/2})\simeq e$. It is obvious, that in the range of wavelengths characteristic to the synchrotron radiation (10^{-8} cm and less) it is necessary to take into account the finiteness of the conductivity of the chamber walls. Following [20] let's carry out some simple estimates. Let's consider the classical motion of the nonrelativistic electron of the medium in the field of the γ-region (in this range of wavelengths one may consider the electrons of the medium as free ones and the interaction with them – noncoherent). The radiation friction force to the Lorentz force within the γ-region ratio is $\simeq r_o\gamma^3/R \ll 1$ for characteristic values of R & γ. Neglecting the friction force, for the transferred pulse Δp and shift Δx of the electron during the time $T\simeq R/(c\gamma^3)$ of passage of the synchrotron radiation pulse, we have

$$\Delta p\simeq 4\sqrt{2}mc(r_o/R)(d/R)^{-1/2}\gamma,$$
$$\Delta x\simeq r_o(R/d)^{1/2}\gamma^{-2} \quad (5)$$

(the quantity $\Delta p\to 0$ when $T\to\infty$). When deriving (5), we took into account the radiation term only. Note that the shift of the electron due to the Coulomb term during the same time $\Delta x_c \simeq r_0(R/d)^2\gamma^{-5} \ll \Delta x$, if synchrotron radiation has been formed ($d \gg R\gamma^{-2}$).

The "instantaneous" polarization (5) of the conducting medium electrons of charge density n gives rise to a field of $\simeq en\Delta x$ on the surface of the conductor. This field is $\simeq nr_0 \times R^2\gamma^{-6}$ from the intensity of the synchrotron radiation field. For $n=5.3\times10^{24}$ cm^{-3} (copper), $R=10^4$ cm, $\gamma=10^4$ this ratio is $\simeq 1.5\times10^{-4}$, i.e. the real value of the charge q induced within the synchrotron radiation "spot" on the chamber wall is $q \simeq -1.5\times10^{-4}e$.

3. Field of the faster-than-light circulating charge

The perturbation of the charge density on the wall of the vacuum chamber can be represented as a charge that moves faster-than-light along a circle of radius R+d (similar to [21]). The Machus surface [22] is the caustic of all wave fronts emitted from the trajectory and separetes the beat wave of the field. When approaching to the point A on the Machus surface from its inner side the field increases as [23]

$$E \simeq q\varepsilon^{-3/2}(2l(1+lB\Gamma\cos\psi/(R+d)))^{-1/2}, \qquad (6)$$

where $B=\beta(1+d/R)$, $\Gamma=(B^2-1)^{-1/2}$, l is the radius of the wave front tangent to the surface at the point A, $\varepsilon>0$ is the distance of the viewpoint from the Machus surface. The "edge of return", that correspods to the condition $R+d+lB\Gamma\cos\psi=0$ is separated out on the Machus surface. The intersection of the surface with the orbit plane forms a characteristic "beak" ($\psi=\pi$, i.e. $l=(R+d)/B\Gamma$). near which the field is proportional to $\varepsilon^{-5/3}$. In the direction tangent to the "beak" $E \propto \varepsilon^{-2}$ ($\varepsilon>0$).

One may realize the geometry of the field by the force lines presented in fig. 2.

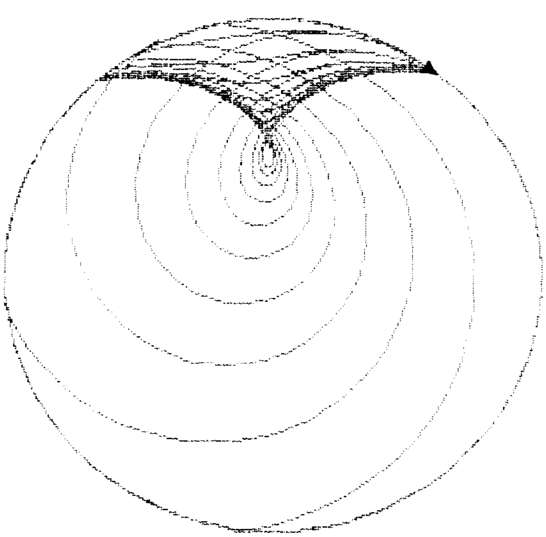

Fig.2. The field lines of the faster-than-light moving charge along the exterior wall of the chamber. Two sets of intersecting lines are seen (here may exist more than one solutions of the retardation equation). The field is largest near the characteristic "beak" - the intersection of the "edge of return" on the Machus surface by the orbit plane. It is essential, that the "beak" falls at the distance $\simeq d^{3/2}R^{-1/2}$ backwards from the charge, which induces charge perturbation on the chamber wall and on the same track with on accuracy of $R\gamma^{-2}$.

Here the boundary condition is not satisfied, since we are interested in the field near the "beak" only. One is to take into account that this pattern is spread, because the surface charge q is not a point one.

The estimates given above show that the effects associated with the trajectory curvature are important both for the vacuum field pattern and the influence of the chamber on the field of the beam. The field asymmetry found causes perturbations of the beam dynamics. Most of these effects are estimated in the first approximation, but one may see that the geometrical (Lienard-Wichert) approach is useful when it is based on the spatial pattern of the field and not on the "almost infinite" expansion ($\simeq\gamma^3$ terms) in the main harmonics.

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