Lattice Function Perturbations Caused by the Beam-Beam Interaction

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Abstract

At small betatron amplitude, the beam-beam force acts as a focusing magnetic quadrupole in both planes. This effective focusing at the beam crossing points of a collider can cause a change in the effective beta functions and phase advances of the unperturbed lattice. [1] These changes can, in turn, lead to real changes in the luminosity by altering the β^* and dispersion at the interaction region (IR). An error in the calculated luminosity can also be induced because the beam emittances as measured by flying wire profile monitors depend on the beta functions and dispersions at the wire locations. A model of these effects is presented and a comparison is made between the the model and the data from the last Tevatron collider run.

Introduction

Over the course of the 1989-1990 Fermilab Collider run, beam parameters from machine diagnostic devices and the counting rates from the CDF detector were recorded at regular intervals. The luminosity can be determined from these data in two separate ways. From the CDF experiment, the luminosity is the counting rate divided by the effective cross-section into the solid angle seen by the detector. The effective cross-section can be measured directly or inferred from other experiments.

The luminosity can also be determined from the measurements of beam parameters, using calculated values for the beta functions and dispersion of the machine. The bunch intensities and lengths are measured with a digitized sampling scope and the transverse profiles are monitored with flying wire scanners.

Comparisons of these two methods of luminosity determination display remarkable agreement at low luminosity. However, as the luminosity increases, the two measures no longer agree. At a luminosity of 2 E 30 $cm^{-2}sec^{-1}$ there is about an 8% difference. [2]

Before these comparisons were made, perturbations in the lattice functions were noted while studying the effects of dispersion at the crossing points on the linear beam-beam tune shift parameter. [3] Namely, the same forces which produce the tune shift also cause the betatron amplitude function to change throughout the lattice.

This work is a first attempt at a quantitative explanation of the discrepancy between the luminosity as determined by CDF and that from beam measurements. The model is a simple one in which only the linear part of the beam-beam interaction is used to predict a change in the beta function of the machine.

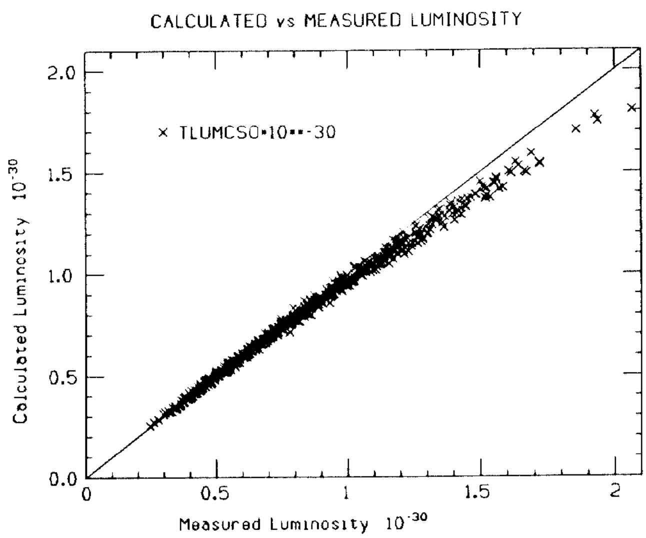

Figure 1: Plot of calculated verses measured luminosities from Collider Run II data. Ref. 1

Beam-Beam Tune Shift

The beam-beam tune shift parameter, ξ, is the maximum tune change that a test particle at small amplitude would experience in passing through a bunch of N particles with transverse normalized emittance, ϵ_N. It is given by,

$$\xi_{x,y} = \frac{N r_p(1+\beta^2)}{4\pi\beta\gamma(\sigma_x+\sigma_y)}\frac{\beta_{x,y}}{\sigma_{x,y}} \quad \textit{per crossing} \qquad (1)$$

where N is the bunch intensity, r_p is the classical proton radius, $\beta = v/c$ and for the Tevatron equals 1, $\beta_{x,y}$ is the beta function at the crossing, γ is the energy normalization, and $\sigma_{x,y}$ is the "strong" beam size at the crossing. The expression for the beam size is

$$\sigma^2_{x,y} = \frac{\epsilon_N\beta_{x,y}}{6\pi(\gamma\beta)} + \eta^2(\frac{\sigma_p}{p})^2 \qquad (2)$$

where $\sigma_{x,y}$ is the standard deviation of the transverse beam profile distribution, ϵ_N is the normalized emittance, β is the Courant-Snyder amplitude function, $\gamma\beta$ is a kinematical factor for normalizing the emittance, the 6 in 6π gives a 95% estimate

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emittance, η is the dispersion function, and σ_p/p is the standard deviation of the momentum distribution.

In a linear model, the total tune shift is just the sum of the individual beam-beam crossings in one revolution.

The linear beam-beam tune shift parameter is a measure of the strength of the interaction, but it is the tune spread, due to the different amplitudes of the particles passing through the bunch, which increases the area occupied by the beams in tune space (ν_x, ν_y). A simple tune shift can be compensated by correction quadrupole adjustments. We will speak of a change or shift of a beta function, but there is no recognized analogous parameter for the spread in lattice functions. For the moment we will calculate the changes in the beta functions due to linear the beam-beam interaction and note that including the non-linear part of the forces will only dilute the effects.

Effects on Luminosity

The interesting terms in the expression for the luminosity are those describing the of the brightness of the beams, given by

$$L \propto \frac{N\overline{N}}{(\sigma_x^2+\overline{\sigma_x^2})^{1/2}(\sigma_y^2+\overline{\sigma_y^2})^{1/2}} \qquad (3)$$

where the N and $\overline{N}$ are the proton and pbar bunch intensities, and $\sigma_{x,y}$ and $\overline{\sigma_{x,y}}$ are the usual measure of the transverse beam size for the proton and pbar beams at the collision point. The beam size (eq. 2), for a given ϵ_N and σ_p/p, is a function of the lattice and is usually considered to remain constant for all bunch intensities. Therefore, the luminosity function should be linear with increasing bunch intensity. If the beam size at the IR changes with bunch intensity, the actual luminosity (i.e. counting rate) would not remain linear.

The expected luminosity at the IR may be calculated by measuring σ's elsewhere in the ring, determining the emittance via eq. 2, and using this invariant emittance to determine the σ's at the IR. [2] If the lattice functions do not remain constant with intensity, the calculation of the expected luminosity as a function of intensity will be in error. Using the above method based upon the linear lattice to calculate an expected luminosity, an error in the calculated luminosity will be proportional to an error in the lattice functions at the wires and inversely proportional to an error in the lattice functions at the IR. The magnitude and sign of the error are dependent on the number of perturbations, the magnitude, and location of each perturbation.

Figure 1 clearly shows the calculated luminosity (using the unperturbed lattice functions) underestimate the measured luminosity (from B0 counting rates) at luminosities above 1 E 30 $cm^{-2}sec^{-1}$. This discrepancy suggests that the lattice functions at the wires have increased from those of the linear lattice while the lattice functions at the interaction have decreased due to the beam-beam force.

The Model and Calculation

If we equate the tune shift due to a thin quad,

$$\delta\nu = \frac{1}{4\pi}\beta\frac{1}{f} \qquad (4)$$

where β is the beta function at the crossing, with equation 1, a first order approximation to the focal length of the beam-beam lens is found to be

$$\frac{1}{f} = \frac{2N r_p}{\gamma\sigma_{x,y}(\sigma_x+\sigma_y)} \quad \textit{per crossing.} \qquad (5)$$

The last Collider run had 12 head-on collisions for 6 proton and 6 pbar bunches. We use two lattice codes, SYNCH and TEVLAT [5] to predict the resultant tune shift and distortion to the beta functions. The distortion to the dispersion function was noted but not included in these calculations. The 12 crossing locations when the beams are in collision at B0 were calculated and inserted into the lattice codes.

Based upon an analysis of the beam characteristics during the last run, [4] we selected the average proton emittance of 20 π-mm-mr in both planes and a range of bunch intensities from 1 E 09 to 1 E 11 for a comparison between the linear tune shift predicted by eq. 1 to that predicted by the lattice codes.

The focal lengths were calculated from eq. 5 at the locations of the 12 crossings using the $\sigma_{x,y}$ from eq. 2 in the unperturbed lattice for a number of bunch intensities and the single value of emittance. Matrices representing quadrupole lenses focusing in both planes were added at these locations to the lattice codes data files describing the 1/2 meter β^* low beta lattice used during the last Collider run.

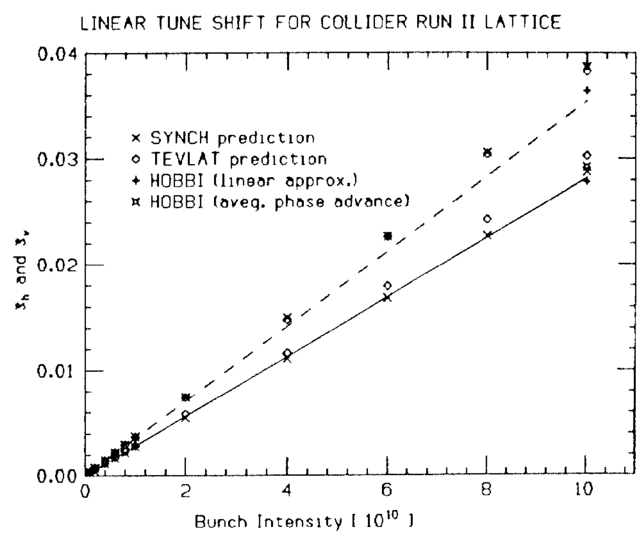

Figure 2: Linear beam-beam tune shift calculated from eq. 1 compared with SYNCH, TEVLAT, and HOBBI calculations.

Figure 2 shows the horizontal (solid) and vertical (dashed) linear tune shift (as a sum of the tune shift of the individual crossings) as a function of bunch intensity for a constant emittance.

As the bunch intensity was linearly increased in the lattice codes, the difference between the predicted tunes with and without the linear lenses are calculated and shown in figure 2. As the bunch intensity increases beyond about 2 E10 the lattice code determinations of the tune shift diverge from the linear approximation. A third code, HOBBI [6] was used to predict the expected tune shifts for the largest bunch intensity and the results are also shown in figure 2.

For each intensity, the lattice functions determined by the lattice codes were recorded. The ratio of the beta functions β in the lattice with the bean-beam lens to those in the unperturbed lattice β_0 are plotted in Figure 3.

The solid lines represent the calculations by SYNCH. The change of the beta function at the three locations predicted by TEVLAT are plotted as data points. The results appear to be linear below a bunch intensity of about 2 E 10. Both lattice codes predict the beta functions increase at each of the flying wire locations and decrease at the IR. This would increase the actual luminosity at the IR while reducing the predicted value of the luminosity based upon the linear lattice.

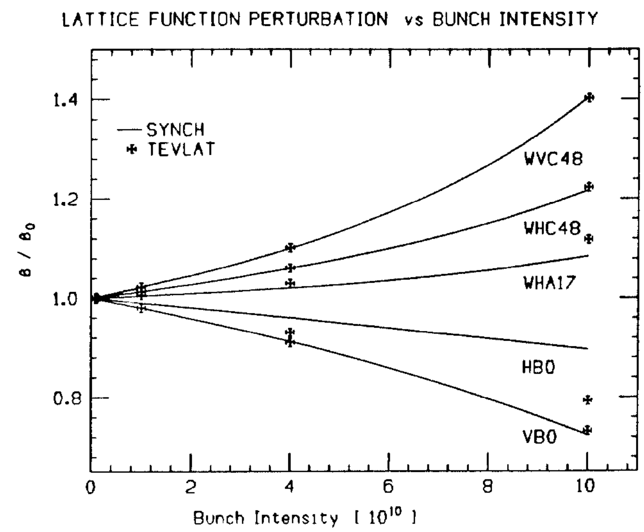

Figure 3: Lattice code calculation of beta function perturbations due to the beam-beam lenses.

Comparison with Data

We selected a random store from the last Collider run, shot 1824, to recalculate the luminosity using the perturbed beta functions due to the beam-beam tune shift. The average intensity, the calculated horizontal and vertical emittance (based on the linear lattice) of the protons and pbars for each data point during the store were used to determine the strength of the beam-beam lens at each crossing. This was done for both protons and pbars since the lattice functions for the protons will be distorted due to the pbars, and visa versa. The lattice code TEVLAT was used to calculate the perturbed lattice functions since it represents out best estimate of the actual lattice by incorporating measured high order multipoles and excludes missing or shorted tune quads. The luminosity was recalculated using these perturbed lattice functions (for both protons and pbars) at the wires and IR using the same calculation used for the unperturbed lattice functions. [2]

Figure 3 shows the measured luminosity from B0, the calculated luminosity using unperturbed lattice functions, and the calculated luminosity using the perturbed lattice functions by the beam-beam lenses. These results show the calculated luminosity, based upon the perturbed beta functions over estimate the measured luminosity. However, the effect of the spread in lattice functions due to large amplitude particles and the perturbed dispersion function were neglected in this model. Inclusion of these will tend to reduce the effect and improve the agreement with the measured luminosity.

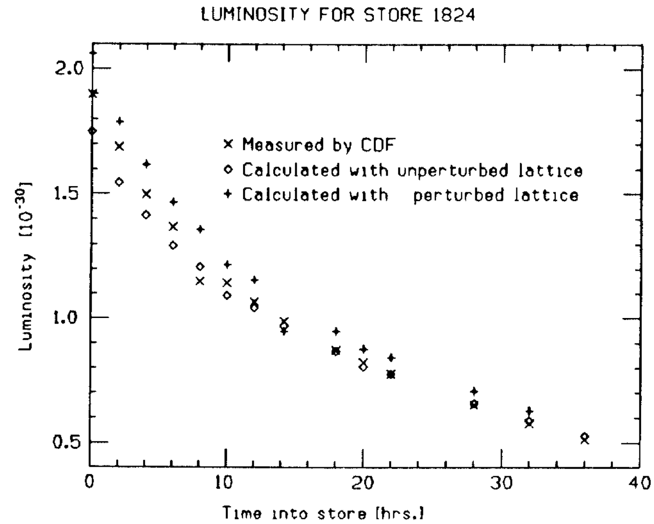

Figure 4: Plot of measured verses calculated luminosity for store 1824 as a function of time into the store.

Conclusions

A model of the beam-beam interaction has been used to explain the discrepancy between the luminosity as derived from experimental counting rates versus the luminosity derived from beam measurements. The physical picture is that the beam-beam interaction distorts the lattice functions of the machine. These distortions change the real luminosity by altering the beam size at the IR. The calculated luminosity is also affected in that the lattice distortions cause beam parameters to be incorrectly determined.

Pbar-p colliders tend to operate with a maximum tune spread, consistent with the free space between resonances on the tune diagram, to obtain the highest possible luminosity. In the SPS and the Tevatron the numbers of bunches have changed and the use of electrostatic separators has changed the number of beam crossing points. Nevertheless, the luminosity is normally limited by the total tune spread. Consequently, the beta function perturbations due to the beam-beam force should always be calculated for any proposed collider lattice..

References

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