CIRCUIT DESCRIPTION OF PULSED POWER SYSTEMS

M.J. Rhee and T.A. Fine
Laboratory for Plasma Research and Electrical Engineering Department
University of Maryland, College Park, MD 20742

Abstract

We summarize the results of a comprehensive analysis of basic pulsed power systems. Two capacitive energy storage systems, lumped capacitor bank and LC generator, are described. Also described are counterparts in transmission line system, a voltage charged transmission line and Blumlein line. The inductive energy storage systems, which are duals of the capacitive energy storage systems, are described by the dual solutions obtained for the capacitive systems.

Introduction

Over the past few decades, there has been rapid development in electrical pulsed power systems. Although a variety of systems have been developed, they may be conveniently classified into a few basic categories. Two different means of energy storage are utilized in electrical pulsed power systems: capacitive energy storage in which the energy is stored in the form of electric field energy, and inductive energy storage in which the energy is stored in the form of magnetic field energy. Relatively slow systems may be constructed with lumped components and described conveniently by lumped circuit theory, whereas fast systems may be constructed with transmission lines amd described by transmission line theory. The circuits of inductive storage systems are the duals[1] of the corresponding capacitive storage systems. In this paper, we summarized the results of a comprehensive analysis of basic pulsed power systems. Two capacitive energy storage systems, lumped capacitor bank and LC generator, are described in terms of circuit theory; their counterparts in transmission line systems, voltage charged transmission line and Blumlein line, are described by transmission line theory. The inductive energy storage systems are duals of the above mentioned capacitive energy storage systems, and found from the solutions obtained from the capacitive systems. Such relationships are depicted in Fig. 1. We propose new inductive energy storage systems which are the duals of the LC generator and Blumlein line and have similar features to those of corresponding capacitive systems.

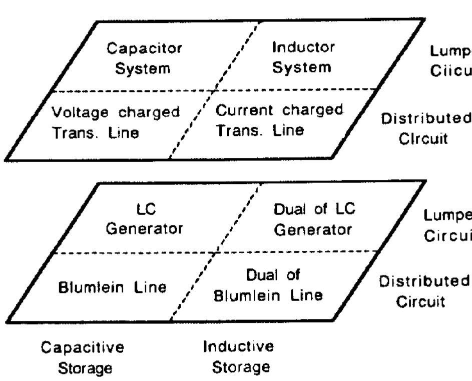

Fig. 1. Classification of pulsed power systems.

Capacitive Energy Storage Systems

Lumped Capacitor System

The lumped capacitor system has been widely used as a simple system for relatively slow pulse applications. Typical examples include the capacitor bank system used in many pulsed plasma devices. and the Marx generator, a simple high voltage pulse step-up device. Such systems may be schematically represented by a series RLC circuit as shown in Fig. 2. By closing the switch S at t = 0, the energy stored initially in the capacitor $U = \frac{1}{2}CV_0^2$ is released to the resistive load. The governing equation is a second order linear differential equation:

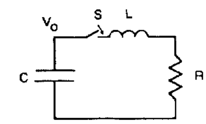

Fig. 2. Schematic of lumped capacitor system.

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0. \tag{1}$$

There are three solutions of output waveforms with α = R/2L, and $w_0 = (LC)^{1/2}$:

a) For $\alpha > w_0$ (overdamped)

$$V(t) = \alpha V_0/(\alpha^2 - w_0^2)^{-1/2}e^{-\alpha t}\sinh(\alpha^2 - w_0^2)^{1/2}t, \tag{2}$$

$$i(t) = V(t)/R \tag{3}$$

b) For $\alpha = w_0$ (critical damping)

$$V(t) = 2\alpha V_0 te^{-\alpha t},\ i(t) = V(t)/R. \tag{4}$$

c) For $\alpha < w_0$ (underdamped), with $w_d = (w_0^2 - \alpha^2)^{1/2}$

$$V(t) = 2\alpha V_0/w_d e^{-\alpha t}\sin w_d t,\ i(t) = V(t)/R. \tag{5}$$

These solutions, with typical circuit parameters, are plotted in Fig. 3. Due to the presence of series inductance L, the output waveforms have a finite risetime resulting in a reduction of peak amplitude.

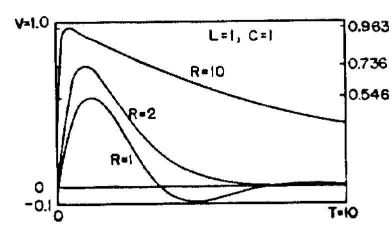

Fig. 3. Typical output voltage waveforms of RLC circuit showing overdamped (R = 10Ω), critical damping (R = 2Ω), and underdamped (R = 1Ω) cases.

The peak amplitude V_p and the risetime t_p (at which the amplitude peaks) for the above three cases are listed in Table I.

Table I. Risetime t_p and peak amplitude V_p of output voltage of LCR circuit, where $\alpha = R/2L$, $w_0 = (LC)^{-1/2}, w = (\alpha^2 - w_0^2)^{1/2}, w_d = (w_0^2 - \alpha^2)^{1/2}$.

Damping condition	Risetime t_p	Peak amplitude v_p
Overdamped $\alpha > w_0$	$\frac{1}{w}\tanh^{-1}\frac{w}{\alpha}$	$\frac{2\alpha V_0}{w_0}\left(\frac{\alpha-w}{\alpha+w}\right)\frac{\alpha}{2w}$
Critical $\alpha = w_0$	$\frac{1}{\alpha}$	$\frac{2V_0}{e}$
Underdamped $\alpha < w_0$	$\frac{1}{w_d}\tan^{-1}\frac{w_d}{\alpha}$	$\frac{2\alpha V_0}{w_d}\left(\frac{\alpha-w_d}{\alpha+w_d}\right)\frac{\alpha}{2w_d}$

CH2669-0/89/0000-1954$01.00©1989 IEEE

Voltage Charged Transmission Line

In lieu of the capacitor and inductor in the RLC circuit, a transmission line is employed as a pulse forming network. The transmission line is initially charged to V_0 and switched to the load resistance, R, which is matched to the characteristic impedance of the transmission line Z_0. The output is a square wave of voltage $V_0/2$, current $V_0/2Z_0$, and pulse duration $2\tau = 2\ell/v$, where ℓ is the length of the transmission line and v is the propagation velocity of wave in the transmission line. If the load resistance is mismatched, the pulse is reflected backwards and forwards in the transmission line giving successive pulses of the same duration 2τ with gradually decreasing amplitude. The decay rate and polarity depend upon the ratio R/Z_0 displaying a resemblance to those of RLC circuits; three typical cases are shown in Fig. 4.

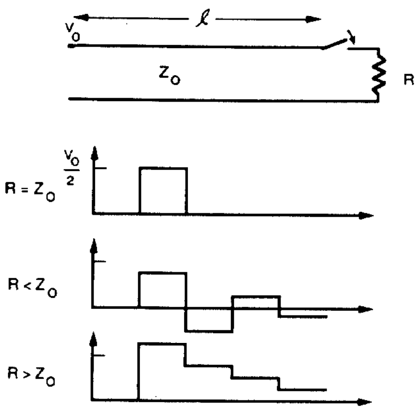

Fig. 4. Schematic of voltage charged transmission line and typical output waveforms.

LC Generator

The LC generator as shown schematically in Fig. 5, is not as popular as the capacitor bank system in practical application, but is an important basic circuit. The main advantage of this system over the simple capacitor system is that with a given charging voltage, the output voltage is nearly twice that obtained from the simple capacitor system. The circuit is represented by a third order differential equation,

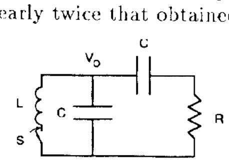

Fig. 5. Schematic of LC generator.

$$RC\frac{d^3i}{dt^3} + 2\frac{d^2i}{dt^2} + RL\frac{di}{dt} + \frac{i}{LC} = 0. \tag{6}$$

With appropriate initial conditions, $i(0) = 0, di/dt(0) = 0$, and $d^2i/dt^2(0) = V_0/(RLC)$, the solution is found to be

$$V(t) = \frac{V_0\omega_0^2}{(\alpha-\beta)^2+\omega^2}\left[e^{-\beta t}\left(\cos\omega t + \frac{\alpha-\beta}{\omega}\sin\omega t\right) - e^{-\alpha t}\right] \tag{7}$$

$$i(t) = V(t)/R \tag{8}$$

where $r = -1/(6RLC^2) - 8\,(27R^3C^3)$, $q = 1/(3LC) - 4/(9R^2C^2)$, $d = q^3 + r^2$, $\alpha = [r + d^{1/2}]^{1/3} - [r - d^{1/2}]^{1/3} + 2/(3RC)$, $\beta = 1/2[r + d^{1/2}]^{1/3} - 1/2[r - d^{1/2}]^{1/3} - 2/(3RC)$, and $\omega = 3^{1/2}/2\left\{[r + d^{1/2}]^{1/3} - [r - d^{1/2}]^{1/3}\right\}$. The behavior of this solution depends largely upon the parameters, R, L, and C; three typical cases are plotted in Fig. 6.

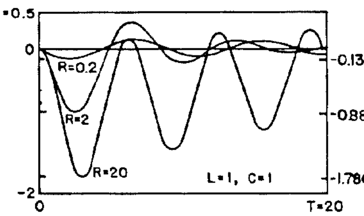

Fig. 6. Typical output waveforms of LC generator.

Blumlein Line

The Blumlein line is the counterpart of the LC generator in transmission line systems. Two identical transmission lines are series connected with a resistor which is twice the characteristic impedance of the transmission line to be matched. Both lines are charged initially to V_0 and one end of the line is shorted by the switch S at t = 0, resulting in a pulse negative V_0 which propagate along the line towards the load resistor. In a time $\tau = \ell/v$ this pulse arrives at the midpoint; at this point the impedance seen by the pulse is the sum of $R = 2Z_0$ and Z_0. It is straightforward to show that a $\frac{1}{2}V_0$ pulse is reflected back and a pulse V_0 appears across the load resistor R, and a pulse $-\frac{1}{2}V_0$ is transmitted to the other section of the line. Both reflected and transmitted pulses are reflected again by the shorted and open ends of the lines respectively, and return to the midpoint in time 2τ, canceling the initial charging voltage along the entire transmission line. Thus the output voltage across the load resistance is V_0 with a pulse duration of 2τ. If the load resistance is mismatched, as in the case of the single transmission line system, pulses are continuously reflected in the lines, resulting in a train of successive pulses of decreasing amplitude across the resistor, as shown in Fig. 7.

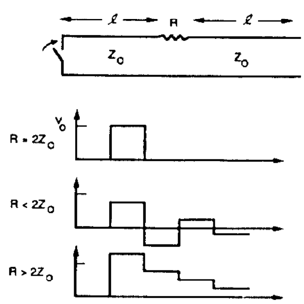

Fig. 7. Schematic of Blumlein line and typical output waveforms.

Inductive Energy Storage System

It is apparent, from the circuit point of view, that the dual circuits of all of the capacitive energy storage systems are the inductive energy storage systems. Therefore the circuits described in this section are the duals of previously described capacitive systems. Of particular interest are the dual circuits of the LC generator and Blumlein line, which are not well know but have features just as interesting as their counterparts.

Lumped Inductor System

A typical inductive energy storage system employing a lumped inductor, as shown in Fig. 8, is the exact dual of the RLC capacitive system circuit shown in Fig. 2. The most successful application of such a system is found in the induction coil system of the automobile engine. The inductor is initially current charged to I_0 to store an inductive energy of $\frac{1}{2}LI_0^2$ in the inductor. Then the current is interrupted at t = 0 by the opening switch S, resulting in a sudden release of the stored inductive energy into the resistive load. The governing equation and solutions are the duals of Eqs. (1), (2), (3), and (4).

$$c\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{v}{c}=0 \tag{9}$$

a) For $\alpha > \omega_0$ (overdamped)

$$i(t)=\alpha I_0(\alpha^2-\omega_0^2)^{-1/2}e^{-\alpha t}\sinh(\alpha^2-\omega_0^2)^{1/2}t \tag{10}$$

$$v(t)=Ri(t). \tag{11}$$

b) For $\alpha = \omega_0$ (critical damping)

$$i(t)=2\alpha I_0 te^{-\alpha t}, \tag{12}$$

$$v(t)=Ri(t). \tag{13}$$

c) For $\alpha < \omega_0$ (underdamping)

$$i(t)=\frac{2\alpha V_0}{\omega_d}e^{-\alpha t}\sin\omega_d t \tag{14}$$

$$v(t)=Ri(t), \tag{15}$$

where $\alpha = L/2R, \omega_0 = (LC)^{-1/2}$, and $\omega_d = (\omega_0^2-\alpha^2)^{1/2}$. Furthermore, the duals of other results such as the peak amplitude, the risetime, and the plots of output waveforms for the RLC circuits equally apply.

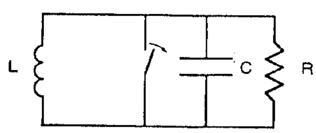

Fig. 8. Schematic of lumped inductor system.

Current Charged Transmission Line

As a transmission line counterpart of the lumped inductor system or the dual system of the voltage charged transmission line, the current charged transmission line system,[2] shown in Fig. 9, is a logical choice for the production of high power square pulses. The resulting output waveforms are the duals of the voltage charged transmission line system.

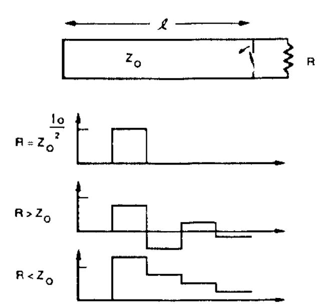

Fig. 9. Schematic of current charged transmission line and typical waveforms.

Dual of LC Generator

The dual circuit of the LC generator is shown in Fig. 10. Similar (dual) advantages of this system over the single inductor system are found, just as in the capacitive counterpart system. The output current is twice that of the single inductor system, and the switch

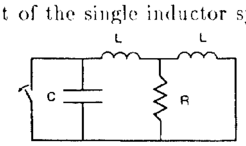

Fig. 10. Schematic of dual of LC generator and typical output waveforms.

is conveniently located away from the load. Again, the governing equation and solutions are the duals of those of the LC generator.

$$\frac{L}{R}\frac{d^3v}{dt^3}+2\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0 \tag{16}$$

$$i(t)=\frac{I_0\omega_0^2}{(\alpha-\beta)^2+\omega^2}\left[e^{-\beta t}(\cos\omega t+\frac{\alpha-\beta}{\omega}\sin\omega t)-e^{-\alpha t}\right] \tag{17}$$

$$v(t)=Ri(t) \tag{18}$$

where $r = -R/(6CL^2) - 8R^3/(27L^3)$, $q = 1/(3LC) - 4R^2/(9L^2)$, $d = q^3+r^2$, $\alpha = [r+d^{1/2}]^{1/3}-[r-d^{1/2}]^{1/3}+2R/L$, $\beta = \frac{1}{2}[r-d^{1/2}]^{1/3}-\frac{1}{2}[r+d^{1/2}]^{1/3}+2R/L$, and $\omega = 3^{1/2}/2\ [r-d^{1/2}]^{1/3}+[r+d^{1/2}]^{1/3}$.

Dual of Blumlein Line

The dual circuit of the Blumlein line system, as shown in Fig. 11, is another system of particular interest. Note that the value of matched resistive load is a half of the characteristic impedance of the transmission lines, $R = Z_0/2$. The output waveforms are the duals of those of the Blumlein system, and have similar features, such

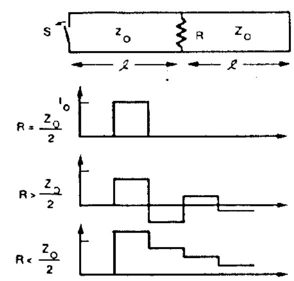

Fig. 11. Schematic of dual of Blumlein line. and typical output waveforms.

as output current equal to the charging current I_0 (which is twice that of the single current charged line system), and switch location conveniently placed at one end of the transmission line.

Conclusions

We have summarized the results of a comprehensive analysis of basic pulsed power systems: two lumped capacitive energy storage systems, their transmission line counterparts, two lumped inductive energy storage systems which are the duals of the lumped capacitive systems, and two transmission line inductive energy storage system which are the duals of the capacitive transmission line systems. We proposed two new inductive energy systems, the duals of the LC generator and Blumlein system, which have inherently interesting features.

Acknowledgements

The authors wish to recognize the contributions of C.C. Kung in helping with the circuit analysis. This work was supported by the Air Force Office of Scientific Research and the U.S. Department of Energy.

References

1. See e.g., W.H. Hayt, Jr., and J.E. Kemmery, Engineering Circuit Analysis, 3rd ed. (McGraw-Hill, New York, 1978), p. 158.
2. M.J. Rhee and R.F. Schneider, IEEE Trans. Nucl. Sci. NS-30, 3192 (1983).