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#### BEAM LOADING AND CAVITY COMPENSATION FOR THE GROUND TEST ACCELERATOR\*

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#### Summary

The Ground Test Accelerator (GTA) will be a heavily beam-loaded H<sup>-</sup> linac with tight tolerances on accelerating field parameters. The methods used in modeling the effects of beam loading in this machine are described. The response of the cavity to both beam and radio-frequency (RF) drive stimulus is derived, including the effects of cavity detuning. This derivation is not restricted to a small-signal approximation. An analytical method for synthesizing a predistortion network that decouples the amplitude and phase responses of the cavity is also outlined. Simulation of performance, including beam loading, is achieved through use of a control system analysis software package. A straightforward method is presented for extrapolating this work to model large coupled structures with closely spaced parasitic modes. Results to date have enabled the RF control system designs for GTA to be optimized and have given insight into their operation.

#### Introduction

Because of the stringent performance and operational requirements of the GTA RF system, a thorough effort has been undertaken to model and simulate the performance of the cavity field control loops. The use of cryogenically cooled cavities to accelerate a pulsed beam of up to 200 mA leads to peak beam-loading factors of up to 80%. The relatively large cavity detuning required for this level of beam loading exacerbates the amplitude modulation to phase modulation (AM/PM) coupling inherent in the accelerating cavity, leading to stricter control-loop performance requirements. An accurate model of the dynamic performance of the cavity is essential to the success of this undertaking.

Models of the response of accelerating cavities to drive and beam stimuli have been presented in the literature.<sup>1,2</sup> These models, because they were derived using amplitude and phase analysis, are quite useful but generally restricted to the small-signal regime. When the superposition of responses resulting from RF drive and beam current is expressed in terms of amplitude and phase, strong nonlinear and transcendental couplings appear in the analysis, limiting practical applications to small purturbations.<sup>3</sup> In the method described here, a complex envelope model of the in-phase and quadrature (I/Q) responses of the cavity is derived. Thus the stimulus, behavior, and resultant response of the accelerating cavity are expressed using a set of linear and fully orthogonal signals. No restrictions other than linearity and time-invariance are placed on this I/Q model; therefore, effects such as large-signal perturbations, cavity detuning, and finite resonator Q can be conveniently and accurately included. The extension of this technique to include structures with closely spaced resonant modes, such as large coupled-cavity linacs, is uncomplicated.

#### **Cavity Model Development**

To reduce the computational effort involved in simulation to a tractable level, a baseband model of the RF accelerating cavity has been developed using the complex envelope concept.<sup>4,5</sup> This method is entirely equivalent to direct simulation at RF frequencies while avoiding the high associated sampling rate. In essence, the analysis is performed in a reference plane that is rotating at the RF frequency; therefore, only the complex baseband modulation envelope remains. As mentioned above, the I/Q form of the complex envelope is used in this derivation. An electrical equivalent circuit of an RF accelerating cavity system near resonance is shown in Fig. 1. Starting



Fig. 1. Equivalent circuit of an accelerating cavity system.

from the fundamental differential equation describing the voltage response of the cavity circuit to a current impulse, i.e., the impulse impedance response, the time-domain operational model of the complex envelope, which is shown in Fig. 2, can be deduced, where

$$z_{c}(t) = \left(\frac{2R}{r}\right) \left(e^{\frac{-t}{v}}\right) \left[\cos \Delta \omega t - \left(\frac{1}{2Q}\right) \sin \Delta \omega t\right]$$
(1)

and

2

$$s_{s}(t) = \left(\frac{2R}{t}\right) \left(e^{\frac{-t}{v}}\right) \left[\sin \Delta \omega t + \left(\frac{1}{2Q}\right) \cos \Delta \omega t\right]$$

R = cavity shunt resistance (ohms).

r = cavity damping time constant (sec).

 $\Delta \omega = \text{cavity detuning frequency (rad/sec).}$ 

Q = cavity resonator loaded quality factor.



 $i_i(t)$  = in-phase component of cavity current  $i_q(t)$  = quadrature component of cavity current  $v_i(t)$  = in-phase component of cavity voltage  $v_q(t)$  = quadrature component of cavity voltage

Fig. 2. Operational model of cavity behavior in the time-domain.

This is the envelope observed in a reference frame rotating at the stimulus frequency. An equivalent model in the LaPlace transform domain can be generated, where

$$\begin{aligned} Z_{c}(s) &= \left(\frac{2R}{\tau}\right) \left(\frac{s+a_{1c}}{s^{2}+b_{1}s+b_{2}}\right) \\ Z_{s}(s) &= \left(\frac{R}{Q\tau}\right) \left(\frac{s+a_{1s}}{s^{2}+b_{1}s+b_{2}}\right) \end{aligned} \tag{2}$$

and

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with

$$a_{1c} = \left(\frac{1}{r} - \frac{\Delta\omega}{2Q}\right)$$
,  $b_1 = \left(\frac{2}{r}\right)$ ,  
 $a_{1s} = \left(\frac{1}{r} + 2Q\Delta\omega\right)$ , and  $b_2 = \left(\frac{1}{r^2} + \Delta\omega^2\right)$ .

Note that in both cases, the describing equations of the model are linear and simple in nature. No restrictions have been placed on the properties of the input current signals other than the existence of their LaPlace transform. The effects of cavity detuning and finite resonator Q have been included directly.





The responses of this cavity model to various perturbations are demonstrated in Figs. 3 and 4. In this simulation, the following assumptions have been made:  $Q = 1.0 \cdot 10^4$  and  $\tau = 7.5 \,\mu s$ . In Fig. 3, the cavity is driven on resonance and a 180° phase step is applied. The field collapses momentarily in the process of reversing sign, as expected. In Fig. 4, the cavity is driven at a frequency detuned from resonance by  $\Delta \omega = 3.9 \cdot 10^5 \, rad/s$ . Figure 4a shows the response to a unit step in input current, while Fig. 4b depicts the effect of a 30° phase perturbation on a cavity that is in the steady state. Note that this simulation faithfully describes the behavior of a detuned cavity with these stimuli.

## Augmentation of Cavity Model

The cavity model described above can easily be augmented to include beam loading, multiple resonances, and dynamic reflected voltage effects. An RF cavity, in normal operation, is a linear time-invarient device. Thus, the effects of beam loading on the field in the cavity can be expressed through linear superposition of the field induced by the RF drive stimulus and the field induced by the beam current passing through the cavity. For a relativistic beam, the response dynamics of the cavity are not modified by the presence of beam current. A beam-dependent susceptance should be added to the model to account for the effect of cavity voltage on beam velocity for nonrelativistic beams.<sup>1,6</sup>

The expression of the cavity stimulus and response in terms of linear orthogonal signals also permits the calculation of the forward and reflected waves within the RF drive line. Once again, linear superposition is used to determine the total traveling waves in each direction on the drive line. Both the beam and the drive amplifier contribute components to these waves. An operational model of a



Fig. 4. Responses of cavity with (a) unit amplitude step applied at 0 s and (b)  $30^{\circ}$  phase step applied at 60  $\mu$ s.

beam-loaded cavity, including the dynamic reflection calculations, is shown in Fig. 5. The cavity field and the total traveling waves that appear during a typical pulse are shown in Fig. 6. The beam current, cavity detuning, and coupling coefficients have been adjusted so that the load appears to be matched when beam current arrives at 10 µs. In reality, however, these parameter choices simply force the vector sum of the partial waves, induced by each source, traveling away from the cavity to equal zero. With respect to dynamic perturbations from this static condition, the drive amplifier still sees a detuned and overcoupled load at all times. This effect occurs in physical accelerating hardware systems as well.

Up to this point, it has been assumed that the cavity being modeled can be adequately represented by a single resonance. This may be an invalid assumption, however, if either the RF drive or the beam contain appreciable signal energy in the vicinity of nearby parasitic modes. Incorporation of this effect into the cavity model is straightforward. Because the modal fields are linear, a realistic electrical model of a cavity with excitable parasitic modes is shown in Fig. 7a. Each mode can be characterized by a unique resonant frequency and quality factor. The operational equivalent of this circuit appears in Fig. 7b, wherein linear superposition was again used to determine the total field in the cavity.



 $v_r^l = \text{total reflected voltage}$ 

Fig. 5. Operational model of cavity with beam loading and reflection calculations.

## **Decoupling Network Synthesis**

A resonant circuit, such as a cavity, exhibits crosscoupling between responses to amplitude and phase (or I/Q) modulation. This is especially evident when the cavity is driven slightly off-resonance, i.e., under detuned conditions. Substantial difficulty can arise when attempting to regulate the fields in a cavity of this sort, as this cross-coupling causes intercoupling between the feedback control loops. This cross-coupling effect can be eliminated, as shown in Fig. 8, wherein the decoupling network is used to predistort the I and Q drive signals so as to remove their interaction from the overall system. Here, the cross-coupled network could represent a simple cavity or a complex combination of a cavity and RF amplifiers, transmission lines, etc. The transfer function of these concatenated networks is

$$\begin{bmatrix} v_i \\ v_q \end{bmatrix} = \begin{bmatrix} c_{11} - c_{21} \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} d_{11} - d_{21} \\ d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} s_i \\ s_q \end{bmatrix}$$

The resultant compound transfer matrix is thus orthogonalized if

and

$$d_{11} = d_{22}$$

$$d_{12} = -\left(\frac{c_{12}}{c_{22}}\right); \quad d_{21} = -\left(\frac{c_{21}}{c_{11}}\right) .$$

= 1

Therefore, for a cavity, these predistortion functions are simple and realizable. Standard network synthesis methods can be used to design circuits of this type into the baseband, or video, section of an I/Q control module. Note that the full benefit of this simple decoupling scheme can be realized only if I and Q control loops are used to regulate the cavity field.







Fig. 7. Model of multimode cavity with (a) circuit representation and (b) operational simulation.

#### **RF System Simulation**

Numerical simulation of full RF control-loop behavior has been achieved through use of the MATRIXX<sup>®</sup> analysis package. The appropriate models, both linear and nonlinear, of each element were built in block-diagram form



## Fig. 8. Basic approach for adding decoupling predistortion to a cross-coupled network.

in this software, and all elements were subsequently connected in the same manner. While space does not allow a complete description here of the simulated loop, a block diagram containing the essential components is shown in Fig. 9. The I/Q control loops are placed around the cavity to



# Fig. 9. Typical RF system block diagram.

regulate the accelerating field. Feedforward methods are used to ameliorate the perturbing effects of beam loading. Also, psuedoderivative feedback is incorporated by sensing the cavity drive signal.

Detuning of the cavity leads to a strong cross-coupling between the I and Q control loops, with resulting degradations in stability and performance.<sup>2,3</sup> To minimize this effect, the I and Q control signals are predistorted as described in the previous section in order to decouple the control loops. This approach effectively orthogonalizes the two feedback control loops.

The cavity amplitude and phase responses, along with their respective loop errors, during a typical acceleration pulse are shown in Fig. 10. The cavity begins charging at time t = 0, while beam current enters the cavity at  $10 \text{ }\mu\text{s}$ . A beam rise time of 10 us is assumed. With this nonzero beam current rise time, the loop errors remain small throughout the pulse.

The cavity model in Fig. 5 lends itself to direct implementation in hardware. Because the model operates at baseband frequencies, the authors will synthesize a simulation of this type using either analog computational components or digital signal processing devices. This circuitry will be extremely useful for testing and integrating the hardware for various control applications.

## Conclusions

Reasonable models have been developed for all components in the GTA RF control loops. Included in these



Fig. 10. Cavity responses and controlloop errors during a typical pulse.

models are beam loading and all significant time responses, nonlinearities, and coupling phenomena. A thorough numerical simulation of loop performance has been achieved using these models. Used as a development tool, this approach has been beneficial for loop design and optimization, as well as providing insight into the workings of the RF system. Further work remains to incorporate the physical dynamics of particle beams into these models.

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