SPACE-CHARGE EFFETS IN THE FERMILAB MAIN RING AT 8 GEV

S.R. Mane
Fermi National Accelerator Laboratory,*P.O. Box 500, Batavia, IL 60510

Abstract

I use computer tracking to investigate the effects of space-charge on particle motion in the Fermilab Main Ring at $p = 8$ GeV/c. The results are found to agree with the Laslett tuneshift formula. Simple model cases are also studied to speed up the tracking. The effects of synchrotron oscillations, via tune modulation and dispersion, are included.

Introduction

The effects of space-charge in the Fermilab Main Ring, at $p = 8$ GeV/c, have been studied using numerical particle tracking with the program TEVLAT [1]. TEVLAT is a kick code for tracking a single test particle around the ring in the presence of arbitrary closed orbit distortions and transverse multipole fields. The effects of RF (synchrotron oscillations) are also included, but not the longitudinal motion. The program was modified so as to include a space-charge kick after every nonlinear multipole. The space-charge force was derived from a Gaussian beam (in all planes), and a single test particle was tracked. The Laslett tuneshift formula was reproduced by the tracking, which is a necessary check on the accuracy of the tracking.

Later on, a multiparticle tracking program was written, to track a distribution of particles, for a simpler model lattice, to study the problem from a different angle. In particular, the longitudinal degree of particle motion was included.

Accelerator Model

The Main Ring has a mean radius of $R = 1000$ m, with six superperiods (neglecting two vertical overpasses). The nominal horizontal and vertical tunes are $Q_x = 19.42$ and $Q_y = 19.38$ respectively, and the beta functions vary between 30 to 100 m. The average beta function is taken to be $\bar{\beta}_x = \bar{\beta}_y = 50$ m below. The harmonic number is $h = 1113$, and the total number of particles in the ring is taken to be $N = 2 \times 10^{13}$ in 1113 bunches. A bunching factor of $B = 0.1$ is assumed, which gives an r.m.s. bunch length (one standard deviation) of $\sigma_z = 0.225$ m, which is much larger than the transverse sizes σ_x and σ_y, as will now be shown.

The beam sizes are calculated as follows. The value used for the normalized transverse emittances is $\epsilon_{Nx} = \epsilon_{Ny} = \epsilon_N = 10\pi$ mm-mrad, where the figures refer to 95% of the beam. This is related to the r.m.s. unnormalized beam emittances via $\epsilon_{x,y} = \epsilon_{Nx,Ny}/(6\pi\beta\gamma)$. The average r.m.s. beam sizes around the ring are given by

$$\sigma_x = \left[\bar{\beta}_x\epsilon_x + \overline{(\eta\,\Delta p/p)^2}\right]^{1/2}, \qquad \sigma_y = [\bar{\beta}_y\epsilon_y]^{1/2}, \tag{1}$$

where overhead bars denote average values around the ring. All the results presented here are at $p = 8$ GeV/c momentum. Here η denotes the horizontal dispersion and $\Delta p/p$ denotes the momentum spread. The values used below are $\overline{\eta^2} = 9$ m^2 and $\Delta p/p = 10^{-3}$. Then, using the values quoted above for $\bar{\beta}_x$, etc., $\sigma_x = 4.22$ mm and $\sigma_y = 2.97$ mm, which are seen to be much less than σ_z.

A Main Ring lattice with linear tunes of $Q_x = 19.42$ and $Q_y = 19.38$ was used [1]. The lattice included two vertical overpasses, harmonic and chromaticity correction sextupoles, and systematic sextupole and decupole moments in the dipoles. It had no octupoles and no random fields.

Graphs were made of the orbital tune vs. amplitude, and phase-space plots of the trajectories at $\theta = 0$. Figs. 1 and 2 show the fractional horizontal and vertical tunes, respectively, as a function of starting amplitude for particles launched with $x \neq 0$, $y = 0$ (Fig. 1), and $y \neq 0$, $x = 0$ (Fig. 2). In both cases $x' = y' = 0$ initially. The crosses (circles) refer to data calculated with (without) space-charge. For starting amplitudes $y > 12$ mm, the particles were lost from the ring. The small-amplitude tune shifts are in agreement with the Laslett formula. The horizontal tune decreases with amplitude at large amplitudes because of the effect of the magnet nonlinearities.

Figs. 3 and 4 are plots of the trajectories in horizontal phase-space at $\theta = 0$, without and with space-charge, respectively. In both cases the starting condition was $x \neq 0$, $x' = y = y' = 0$. We see that there is some difference in the phase-space structure, because of the space-charge tune shift. Five resonance islands appear when the fractional part of the tune crosses the value of 0.4 in Fig. 4. We see that the space-charge force makes little difference to the oscillation amplitudes (action variables), even though it does produce resonance islands that would otherwise be absent.

Multiparticle Tracking

The above results were for one test particle, and did not include synchrotron oscillations. It is of interest to track a distribution, and also to include synchrotron oscillations. A multiparticle tracking program has been written for this purpose, and applied to a simple accelerator model. A sample result is shown in Fig. 5, for 100 particles, tracked for 10000 turns. It is seen that the emittance grows slightly with time. The model treated only one degree of freedom, with atune of 19.420. The initial emittance was 10π mm-mrad (95% normalized), with a Gaussian beam. There was a sextupole of $B''L/(2B\rho) = .01$ m^{-2}. An RF voltage of 0.65 MV was used, giving a synchrotron period of approximately 96.9 turns. The r.m.s. bunch length was 2.0 nsec (multiplied by c).

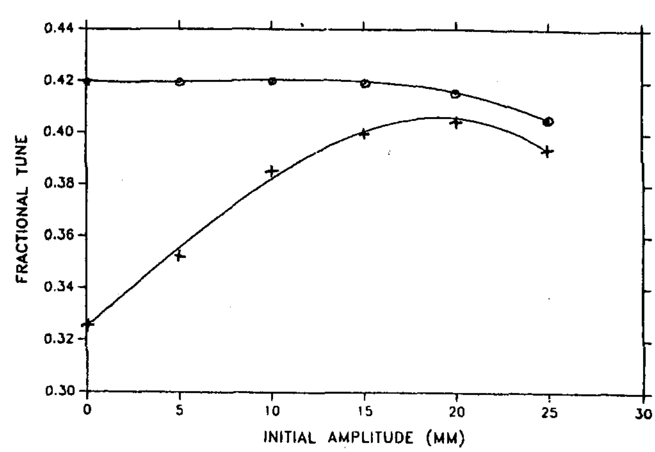

Fig. 1 Fractional horizontal tune vs. amplitude.

*Operated by the Universities Research Association Inc., under contract with the U. S. Department of Energy

CH2669-0/89/0000-1804$01.00©1989 IEEE

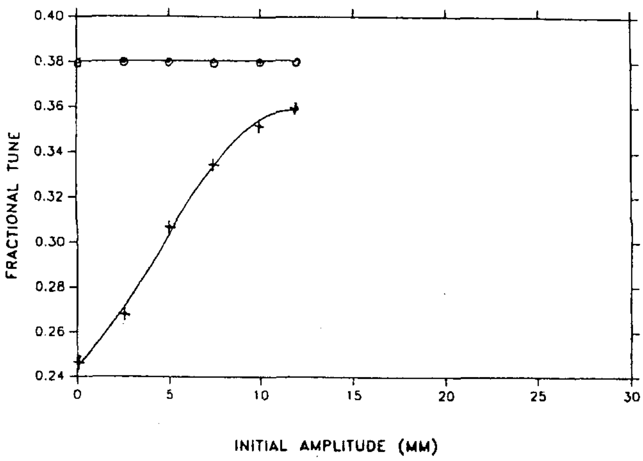

Fig. 2 Fractional vertical tune vs. amplitude.

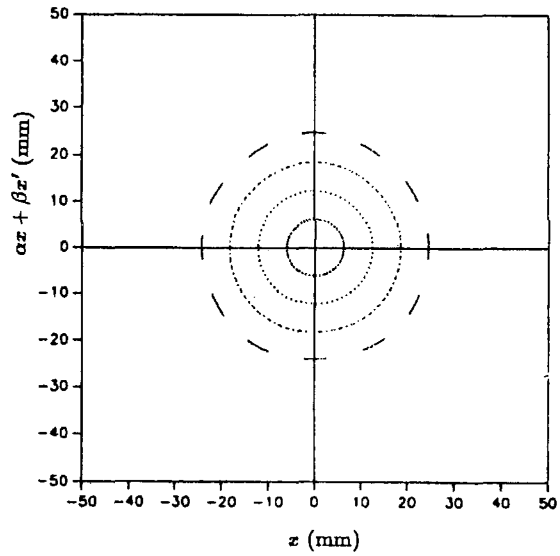

Fig. 3 Horizontal phase-space trajectories without space-charge.

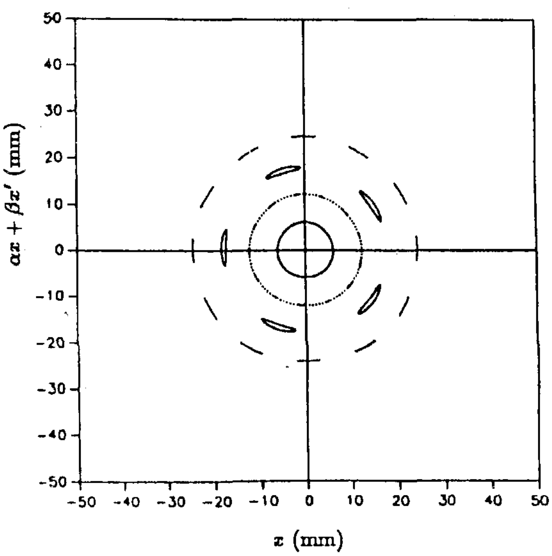

Fig. 4 Horizontal phase-space trajectories with space-charge.

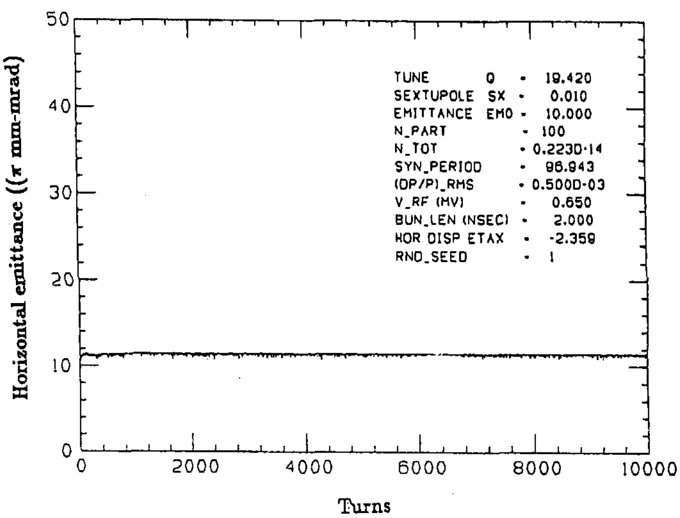

Fig. 5 Horizontal emittance vs. turn number.

Conclusions

The present results do not show any large change in beam behavior caused by space charge. There is distinct evidence for tuneshifts and consequent bounded resonance islands. Multiparticle tracking may yield more detailed results.

Acknowledgements

I thank C. Ankenbrandt, D. Edwards, R. Gerig, G. Jackson and S. Stahl. This work was supported by the Universities Research Association Inc., under Contract DE-AC02-76CH03000 from the Department of Energy.

References

[1] The version of TEVLAT and the Main Ring lattice used were obtained from R. Gerig.

[2] Many of the ideas here follow the work of M. Furman, SSC-115 (1987), and presented at the IEEE Particle Accelerator Conference, Washington D.C. (1987).