STUDIES AND CALCULATIONS OF TRANSVERSE EMITTANCE GROWTH IN HIGH-ENERGY PROTON STORAGE RINGS

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Abstract

In the operation of proton-antiproton colliders, an important goal is to maximize the integrated luminosity. During such operations in the Fermilab Tevatron, the transverse beam emittances were observed to grow unexpectedly quickly, thus causing a serious reduction of the luminosity. We have studied this phenomenon experimentally and theoretically. A formula for the emittance growth rate, due to random dipole kicks, is derived. In the experiment, RF phase noise of known amplitude was deliberately injected into the Tevatron to kick the beam randomly, via dispersion at the RF cavities. Theory and experiment are found to agree reasonably well. We also briefly discuss the problem of quadrupole kicks.

Introduction

When high energy storage rings are used to collide beams of particles and antiparticles for high energy physics experiments, it is important to obtain as high an integrated luminosity as possible. Reduction of integrated luminosity can arise from several factors, in particular from growth of the transverse beam sizes (transverse emittances). An example of this phenomenon was recently observed in the Tevatron [1,2] at Fermilab. During colliding beam operations in the Tevatron, it was noted that the luminosity decayed at an unexpectedly fast rate. Investigations showed that the horizontal and vertical beam emittances were growing linearly as a function of time, and this growth was the dominant cause of the poor luminosity lifetime [3]. Since it was observed that the beam was undergoing externally driven betatron oscillations [4], a search for accelerator components which were capable of driving the beam transversely and causing emittance growth was initiated.

We have therefore investigated the problem of transverse emittance growth in high energy storage rings caused by random dipole noise kicks to the beam. A similar investigation is reported in Ref. [5]. A theoretical formula for the emittance growth rate is derived, and compared against experimental measurements. In order to obtain quantitative results, noise of known amplitude and power spectrum was deliberately injected into the Tevatron, to kick the beam randomly. The Tevatron RF system was chosen for this purpose because the relationship between the injected noise and the kick to the beam is quantitatively understood. The theoretical formula itself is applicable to arbitrary noise sources, provided they satisfy certain criteria, to be specified below.

In the experiment, phase noise was introduced into the Tevatron RF system. Independent of previous work [6] which investigated the effect of RF noise on longitudinal beam dynamics, it was suspected that phase noise with the appropriate Fourier spectrum could induce horizontal emittance growth due to the existence of horizontal dispersion at the RF cavities. The measured dependence of horizontal emittance growth on phase noise amplitude is compared against the theoretically derived response. The theoretical derivation is presented below, followed by a description of the experiment. The full details of this work can be found in Ref. [7].

*Operated by the Universities Research Association under contract with the U. S. Department of Energy

Theory

First we shall describe the notation to be used below. The orbit of a particle is described by the functions [8]

$$\begin{pmatrix} x \\ p \end{pmatrix} \equiv \begin{pmatrix} x \\ \alpha x + \beta x' \end{pmatrix} = \sqrt{2I\beta} \begin{pmatrix} \sin(\Psi + \psi - \omega_\beta t) \\ \cos(\Psi + \psi - \omega_\beta t) \end{pmatrix} . \qquad (1)$$

Here I and Ψ are the action and angle variables, respectively, ω_β is the linear dynamical oscillation frequency (linear tune times revolution frequency) and ψ is the Floquet phase. We make the approximation that the phase-space trajectories in $\{x, p\}$ space are circles with action-dependent tunes. The unnormalized emittance is given by $\epsilon = \langle I \rangle$, assuming $\langle x \rangle = \langle p \rangle = 0$. The angular brackets denote an average over the beam at fixed t. We always define the emittance to be averaged over the beam. The emittance growth rate is $r = d\epsilon/dt$. Note that to calculate r it is not necessary that $\langle x \rangle = 0$; it is sufficient if $\langle x \rangle$ and $\langle p \rangle$ are bounded, because then $d\langle x \rangle/dt$ and $d\langle p \rangle/dt$ average to zero.

Suppose there is a random horizontal dipole kick at location t, so that

$$\begin{aligned} x(t+\delta t) - x(t) &= 0 \\ p(t+\delta t) - p(t) &= N\delta t . \end{aligned} \qquad (2)$$

It is sufficient to consider a kick which changes only p and not x. We shall add in the contribution of a kick which changes x below. We shall linearize the response of the beam with respect to the kicks, i.e. we calculate the changes to x and p to linear order in N only, and so the emittance growth rate will be of $O(N^2)$. Next, we calculate the rate of emittance growth. For this we need to study $x^2 + p^2$. Now

$$[x^2 + p^2]_{t+\delta t} = [x^2 + (p + N\delta t)^2]_t \simeq [x^2 + p^2]_t + 2pN\delta t \qquad (3)$$

and

$$\frac{p}{\sqrt{\beta}} = \int_{-\infty}^{t} \frac{N(t')}{\sqrt{\beta(t')}} \cos[\Phi_0(t) - \Phi_0(t')]\, dt' . \qquad (4)$$

Combining these results with the fact that $\delta(x^2 + p^2) = 2\beta\,\delta I$,

$$\delta I = \frac{pN\delta t}{\beta} = \frac{N\delta t}{\sqrt{\beta}} \int_{-\infty}^{t} \frac{N(t')}{\sqrt{\beta(t')}} \cos[\Phi(t) - \Phi(t')]\, dt' . \qquad (5)$$

Averaging over the beam,

$$\frac{d\epsilon}{dt} = \frac{d\langle I \rangle}{dt} = \frac{N}{\sqrt{\beta}} \int_{-\infty}^{t} D(t,t') \frac{N(t')}{\sqrt{\beta(t')}} \cos[\Phi_0(t) - \Phi_0(t')]\, dt' . \qquad (6)$$

We want the time average of the growth rate. The function ϵ itself grows indefinitely, and does not have a finite time average. The function $D(t,t')$ is a decoherence factor, and Φ_0 is the linear dynamical value of Φ, i.e. the value without tunespread. An expression for the decoherence factor, for tunespread due to an octupole moment, is given in Ref. [9]. A simpler model, which is motivated by radiation damping in synchrotron radiation theory, is to put [10]

$$\begin{aligned} D(t,t') &= e^{-\alpha_d(t-t')} && t > t' \\ &= 0 && t < t' . \end{aligned} \qquad (7)$$

Since we do not know the detailed decoherence mechanism in general, the choice of model is somewhat arbitrary, and so we shall use Eq. (7). It must be understood that this is a phenomenological step.

The mathematical manipulations are described in Ref. [7]. The time-averaged growth rate is

$$r \simeq |\widetilde{M}(-\omega_\beta)|^2 \frac{\Delta\omega}{8\pi} . \tag{8}$$

Here $\widetilde{M}(\omega)$ is the Fourier transform

$$\widetilde{M}(\omega) = \int dt \, \frac{N e^{i(\psi - \omega_\beta t - \omega t)}}{\sqrt{\beta}} . \tag{9}$$

Measuring frequencies in Hz, and adding the effect of a noise source which kicks x also, and introducing Fourier transforms $\widetilde{M}_1$ and $\widetilde{M}_2$ for kicks to x and p, respectively, the above result becomes

$$r = \frac{1}{4}\left(|\widetilde{M}_1(f_b)|^2 + |\widetilde{M}_2(f_b)|^2\right)\Delta f . \tag{10}$$

Here f_b is the betatron frequency in Hz, which separate both the upper and lower betatron sidebands from the revolution harmonics. We see that the integrand above is a Lorentzian with a maximum at $\omega = -\omega_\beta$ and a width α_d, and so the emittance growth is driven by harmonics of the noise in a range $\pm\alpha_d$ centered on the betatron frequency plus multiples of the revolution frequency.

Measurements

Each accelerator revolution the beam sampled a random phase shift introduced into the RF voltage waveform. Mathematically, this voltage is equal to $V(t) = V_0 \cos[\omega_0 t + \phi_0 + \phi_n(t)]$. The phase waveform was a random noise signal generated by a Hewlett-Packard 3561A Dynamic Signal Analyzer in the frequency band surrounding the lowest betatron sideband frequency (near 20 kHz). The RF voltage was measured with a RF cavity gap monitor. Based upon such measurements, the measured phase noise band amplitudes, as a ratio of the fundamental RF voltage amplitude, were measured three times (see Ref. [7]). The results are listed in Table 1.

Table 1. Ratio of phase noise band amplitudes to fundamental RF voltage amplitude.

Measurement	Relative Amplitude (db)	R.M.S. Phase Noise in 100 Hz (mrad)
1	-62 ± 1	1.1 ± 0.1
2	-68 ± 1	0.56 ± 0.06
3	< -90	< 0.04

The horizontal emittance of the beam is measured using devices called flying wires [11,12]. A 1 mil diameter carbon wire, oriented vertically, passes through the beam horizontally at a velocity of approximately 3 m/sec. As the wire traverses the proton beam, protons collide with wire atoms producing particle showers detected by scintillator/phototube monitors. The flying wire system is set up such that the phototube output voltage is proportional to the local proton density at the wire. By digitizing the wire position and phototube voltage on a turn by turn basis, one can map out the beam's horizontal density distribution. The horizontal emittance is calculated using r.m.s. beam widths at two flying wire monitors. These wires are placed at points in the Tevatron where the dispersion is very different, so that the contributions of horizontal emittance and momentum spread to the total beam size can be separated. By flying the wires periodically over the span of an hour or more, and doing a linear least square fit of emittance vs. time, the emittance growth rate of the beam is determined. Figure 1 is an example of horizontal emittance as a function of time, with the result of such a fit superimposed.

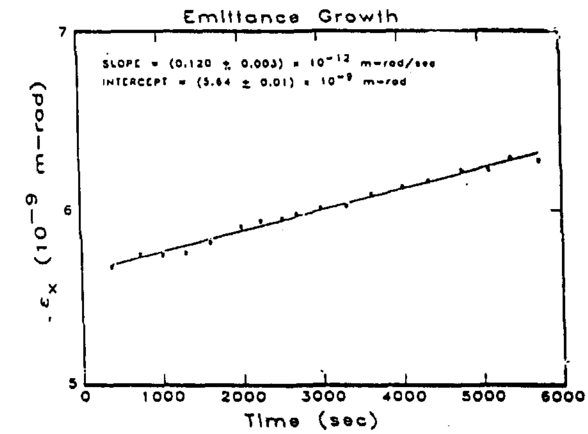

Fig. 1. Horizontal emittance as a function of time in the Tevatron.

Using the results of such fits, the horizontal emittance growth rates were measured three times, corresponding to the three phase noise amplitudes tabulated above (Figure 2). The growth data are listed in Table 2.

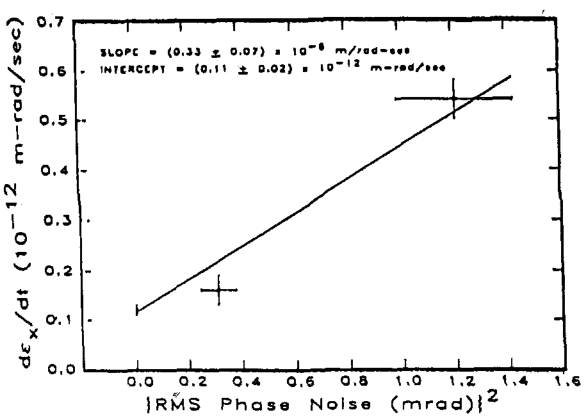

Fig. 2. Fit of horizontal emittance as a function of phase noise amplitude in 100 Hz binwidth.

Table 2. Emittance growth rate measurements corresponding to phase noise measurements in Table 1.

Measurement	Growth Rate (10^{-12} m-rad/sec)
1	0.54 ± 0.04
2	0.16 ± 0.03
3	0.12 ± 0.01

Analysis

In order to compare the above experimental data to theoretical predictions, the relationship between betatron position changes and RF phase modulation must be specified. Since the energy change u of a synchronous proton traversing an RF voltage V_0 with a phase error ϕ_n is $u = eV_0 \sin\phi_n$ and since ϕ_n is much less than unity, $u \simeq eV_0\phi_n$. If the horizontal position of this synchronous particle is described by $x = \sqrt{2I\beta}\cos(\omega_\beta t)$, then we can deduce the expressions for $\widetilde{M}_1$ and $\widetilde{M}_2$. This is done in Ref. [7], and the result is

$$r = \frac{f^2 H}{4}\left(\frac{eV_0}{E_0}\right)^2 |\tilde{\phi}(f_b)|^2 \Delta f , \tag{11}$$

where $\tilde{\phi}$ is the Fourier transform of ϕ and [13]

$$H = \{\eta^2 + [\beta\eta' - \frac{1}{2}\beta'\eta]^2\}/\beta . \tag{12}$$

Now the experimental noise harmonic $\tilde{\phi}_{sa}(\omega)$, as recorded by the spectrum analyzer, is actually the integral of $\tilde{\phi}(\omega)$ over the frequency bins Δf at the frequencies ω and $-\omega$, i.e.

$$|\tilde{\phi}_{sa}(\omega)|^2 = \left[|\tilde{\phi}(\omega)|^2 + |\tilde{\phi}(-\omega)|^2\right](\Delta f)^2 = 2|\tilde{\phi}(\omega)|^2(\Delta f)^2 \ , \qquad (13)$$

because $\tilde{\phi}(-\omega) = \tilde{\phi}^*(\omega)$ for a real function of time $\phi_n(t)$. Hence $\tilde{\phi}_{sa}$ has the same dimension as $\phi_n(t)$. Thus the time-averaged emittance growth rate is

$$r = \frac{f^2 H}{8\Delta f}\left(\frac{eV_0}{E_0}\right)^2 |\tilde{\phi}_{sa}(f_b)|^2 \ . \qquad (14)$$

The Tevatron conditions at the time of this experiment are lised in Table 3.

Table 3. Tevatron parameters used in experiment.

Parameter	Value	Unit
f	47.713	kHz
Δf	100	Hz
E_0	900	GeV
eV_0	1.16	MeV/turn
H_{RF}	0.090 ± 0.020	m

Substituting these values into eq. (14) yields the theoretical prediction

$$r = (4.3 \pm 0.9) \times 10^{-7}\,|\tilde{\phi}_{sa}(f_b)|^2 \quad \text{(m-rad/sec)} \ . \qquad (15)$$

Experimentally, one obtains the result (Figure 2 and Table 2),

$$r = (3.3 \pm 0.7) \times 10^{-7}\,|\tilde{\phi}_{sa}(f_b)|^2 \quad \text{(m-rad/sec)} \ . \qquad (16)$$

The agreement is within the errors quoted.

Quadrupole Kicks

It is reported in Ref. [2] that ripple in the quadrupole power supplies was also observed to drive emittance growth in the Tevatron. One can describe the effect of ripple on the betatron motion by a differential equation of the form

$$\ddot{x} + \omega_\beta^2(1 + a\cos(\omega_r t))\,x = 0 \ , \qquad (17)$$

where a is a dimensionless constant, ω_β is the betatron angular frequency, and ω_r is the ripple angular frequency. The principal ripple frequency which drives emittance growth is twice the betatron frequency, plus multiples of the revolution frequency $\omega_r = 2\omega_\beta + n\omega_{rev}$. For a model with only one kick in the ring, in the limit of small kicks and very low betatron tune, with the ripple frequency close to twice the betatron frequency, the emittance growth rate is approximately

$$\frac{\Delta\langle I\rangle}{\langle I\rangle} \simeq \frac{\beta^2}{8}k_{rms}^2\frac{f}{\Delta f} \ , \qquad (18)$$

where k_{rms} is the r.m.s. value of the fluctuations in the quadrupole gradient, β is the beta function at the quadrupole, f is the revolution frequency, and Δf is the binsize of the frequency intervals of the measuring apparatus. A similar formula, treating white noise kicks, is reported in Ref. [14]. Work on this subject is still ongoing, and further results will be reported elsewhere.

Conclusions

The theory surrounding stimulated transverse emittance growth of proton beams has been presented. Given a random dipole kick each turn, quantitative predictions for the r.m.s. beam centroid betatron oscillation amplitude and average emittance growth rate are made. Because the particles undergo deterministic betatron oscillations between kicks, the effects of successive kicks are correlated, hence only the noise harmonics at the betatron frequency plus multiples of the revolution frequency contribute to the emittance growth.

An experiment was performed at the Fermilab Tevatron, where the effect of RF phase noise on transverse emittance growth was measured. The RF system was chosen because it was quantitatively understood, thus enabling an absolute calibration between the emittance growth rate and the injected phase noise amplitude. It also had the advantage that the kicks to the beam were localized at one point in the ring, and were of sufficient magnitude to dominate over other sources of dipole kicks, thus simplifying the analysis. Applying the above theory to this experiment, it was found that the predicted and measured emittance growth rates were in agreement.

Acknowledgements

S.R.M. thanks C. Ankenbrandt, D.A. Edwards and L. Michelotti. G.J. thanks Q. Kerns and J. McCarthy for helping with the experiment, and D. Finley and J. Gannon for supporting the flying wire development. This work was supported by the Universities Research Association Inc., under Contract DE-AC02-76CH03000 from the Department of Energy.

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