# MINI-β INSERTION AND LUMINOSITY FOR THE RHIC LATTICE*

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## Abstract

Requirement and implication of the mini-β insertion, where a pair of common quadrupoles are inserted between the interaction point and the frist beam crossing dipole, in RHIC lattice is studied. For the heavy ion operation, we find that $\beta^* = 1$m can be achieved with a ±5m free space for experimental setup and $\hat{\beta} = 500$m. The beam size limitation is located at the second beam crossing dipole BC2, where 8cm coil i.d. is considered. When the machine is operating at the proton-heavy ion collision mode, these common quadrupoles should be removed physically in order to let both beams crossing the center line at 3.4 mrad. These mini-β insertion can increase the luminosity by a factor of 2 ~ 3.

For the proton-proton collision mode, we find that $\beta^* = 0.5$m is also operable without the mini-β quadrupoles due to smaller emittance for the proton beam. The corresponding $\hat{\beta}$ is 2400m. When the mini-β quads are used, $\beta^* = 0.25$m can be achieved. The ultimate luminosity for the proton-proton collision is of the order of $10^{33}$/cm$^2$sec, which may be useful in the quest of the B physics studies.

## 1. Introduction

The luminosity of the many colliders are limited by the beam-beam tune shift. When the linear beam-beam tune shift is large, the nonlinear driving term in the beam-beam force determines the life time of the storage beam. It is important to note however that the beam-beam tune shift is independent of the $\beta^*$ value at the IP. Thus most of the colliders are designed to operate at the smallest $\beta^*$ value achievable. To obtain small $\beta^*$ while limiting the maximum $\hat{\beta}$ value at the high β quads region, a MINI-β insertion is considered.

The normal operation range for the RHIC collider design is constrained by the available dynamical aperture at Q2 and Q3 in the insertion, where the β-function is high. In the norminal insertion, there are 9 quadrupoles in each side of the interaction point (IP) excited anti-symmetrically with respective to IP. When the $\beta^*$, the β value at the IP, is 3m, the maximum $\hat{\beta}$ values at Q2 and Q3 are 400m, while $\beta^* = 2$m, $\hat{\beta}$ becomes 625m. Table 1 compares the beam size with the magnet coil i.d. at various operational conditions. The ratio of the beam size to the magnet coil i.d. is a measure of the effect of the nonlinear field on the particle motion, which determines the long term stability of the circulating particles. For an accelerator with superconducting magnets, the beam stability obtained from the tracking calculations for accelerators such as SSC, TEVATRON, and RHIC requires that the ratio should be less than 0.6 ~ 0.7.

Table 1. Beam size vs. Coil i.d.[+] in Au on Au operation

| | NORMAL INSERTION | | MINI-β |
|---|---|---|---|
| $\beta^*$(m) | 3 | 2 | 1 |
| $\hat{\beta}$(m) | 400 | 625 | 500 |
| $\varepsilon_N(\mu - \text{m})$ | 33 | 33 | 33 |
| 6σ/coil i.d. (at βρ = 840 Tm) | 0.42 | 0.52 | 0.48 |
| 6σ/coil i.d. (at βρ = 420 Tm) | 0.59 | (0.73) | 0.68 |

[+] Coil i.d. of Q2 and Q3 are both 131mm

Based on the the tracking calculations of Dell and Parzen,[1] we expect that $\beta^* = 2$m can be achieved at the the top energy. Since $\hat{\beta}$ is approximately inverse proportional to $\beta^*$, $\hat{\beta}$ becomes too large for heavy ion operation in the $\beta^* < 2$m due to large intrabeam scattering. The only way to avoid the dynamical aperture problem in Q2 and Q3 is to resort to a MINI-β insertion, where a pair of quadrupoles are inserted in each side of the IP. Note that the free space available in RHIC standard insertion is ±10m. The mini-β insertion would require the experimental set up to be limited to an even smaller space or the mini-β quadrupoles, shared by both counter circulating beams, should be incorporated in the detector design.

This study is intended to investigate the possible mini-β insertions. Our aim is to leave as much of space as possible for the experimental set-up while keeping the ratio of (6σ/coil i.d.) less than 0.6 ~ 0.7 in order to maintain useful dynamical aperture. This rule of long term stability is established by many tracking studies of Tevatron, SSC, and RHIC. In section 2, we shall discuss the MINI-β insertion layout and find the requirement of these quadrupoles. Section 3 discusses the operable condition for the proton collision. Section 4 discusses the implication of mini-β insertions and the chromatic corrections. Section 5 gives the conclusion.

## 2. The Layout of MINI-β Insertion

To maintain the antisymmetry of the RHIC insertion, two pairs common quadrupoles, QX1 and QX2, should be placed in the interaction region between two common beam crossing dipoles, BC1, on both sides of the IP. To maximize the available free space for experimental set-up, the distance between QX2 and BC1 is set to 0.5m, which is needed for coil end of magnets. The beam line in the interaction region is arranged as following:

BC1I DX2 QX2I DX1 QX1I DX0
IP DX0 QX1O DX1 QX2O BC1O

Table 2. Mini-β insertion contraint for RHIC

| | | |
|---|---|---|
| β (m) | 1.5 | 1.0 |
| DX0 (m) | 5.0 | 5.0 |
| LQX1 (m) | 1.5 | 1.5 |
| DX1 (m) | 1.5 | 1.5 |
| LQX2 (m) | 1.5 | 1.5 |
| βmax (m) | 400. | 500. |
| GF (1/m) | 0.099520 | 0.099520 |
| GD (1/m) | 0.099650 | 0.099650 |
| K1I (1/m) | 0.135168 | 0.150904 |
| K2I (1/m) | 0.135652 | 0.147122 |
| G1I (1/m) | 0.137238 | 0.137711 |
| G2I (1/m) | 0.157785 | 0.156988 |
| G3I (1/m) | 0.051932 | 0.050933 |
| G4I (1/m) | 0.085321 | 0.076361 |
| G5I (1/m) | 0.103415 | 0.093903 |
| G6I (1/m) | 0.125538 | 0.121913 |
| G7I (1/m) | 0.138627 | 0.140795 |
| G8I (1/m) | 0.111907 | 0.112740 |
| G9I (1/m) | 0.085220 | 0.086562 |

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Let us choose the available free space to be ±5m and the length of quadrupoles to be 1.5m each. The result of the beam matching is given in the column 2 of table 2 for $\beta^* = 1.5$m and $\hat{\beta} =$ 400m. Since the dynamical aperture in this case is not limited by the quadrupoles Q2 and Q3, one can decrease further $\beta^*$ to 1m, shown in the COLUMN 3 of Table 2. Note that the gradient requirement of QX1 becomes 84.5 T/m at $B\rho = 840$ Tm. Fig. 1 shows the betatron amplitude functions in the insertion region. At $\beta^* = 1$m, the β-value at BC2 is about 350m. Thus the ratio of beam size to the magnet coil i.d. would be about 61% at BC2, where 8 cm coil i.d. is assumed. We conclude that β=1m has almost reached the capability of the mini-β insertion.

## 3. Luminosity in the Proton-proton Collision Mode

For the proton operation at top energy with $B\rho = 840$ Tm, the emittance is considerably smaller than that of heavy ions. Thus $\beta^* =$ 0.5m may indeed be achievable without mini-β insertion. Table 3 shows the beam size in this operation mode. To obtain an even smaller betavalue at the crossing point, the mini-β insertion discussed in section 2 can be used to lower the $\beta^* = 0.25$m. Fig. 2 shows the betatron amplitude function for $\beta^* = 0.25$m. Thus the luminosity for the proton will be $10^{32}$/cm$^2$sec in the mini-β configuration without any intensity upgrade to the RHIC conceptual design. To reach an even higher luminosity, more number of bunches and/or more number of particles per bunch are needed. It is however worth pointing out that the number of interaction per bunch crossing has already reach 1 at the luminosity of $10^{32}$/cm$^2$sec with the proposed 57 bunches in each ring. The proper design of detectors may be an important issue for higher luminosity.

For the future upgrade, the proton can be increased to $2 \cdot 10^{11}$ particles per bunch with 114 bunches in each ring. The corresponding luminosity becomes $10^{33}$/cm$^2$sec. At such a luminosity, there are 4 interactions per crossing. Detector design is a challange for the luminosity. Similar luminosity can also be achieved for the polarized proton operation.

Table 3. Beam size vs. Coil i.d.[a] in proton operation

| | NORMAL INSERTIOn | | | | MINI-β |
|---|---|---|---|---|---|
| $\beta^*$ (m) | 3 | 2 | 1 | 0.5 | 0.25 |
| $\hat{\beta}$ (m) | 400 | 625 | 1290 | 2460 | 2409 |
| $\varepsilon_N$(μ-m) | 20 | 20 | 20 | 20 | 20 |
| 6σ/coil i.d. ($B\rho$ = 840 Tm) | 0.21 | 0.26 | 0.38 | 0.54 | 0.54 |
| 6σ/coil i.d. ($B\rho$ = 420 Tm) | 0.29 | 0.37 | 0.54 | ---- | ---- |
| Chromaticity[b] (per insertion) | −5.5 | −8.5 | −17. | −32. | −32. |
| Luminosity[c] ($10^{32}$/cm$^2$sec | .095 | .143 | .285 | .57 | 1.14 |

[a] Coil i.d. of Q2 and Q3 are both 131mm

[b] The total contribution of six arcs is −24 unit of chromaticity.

[c] Based on $10^{11}$ particles per bunch and 57 bunches per ring. When the intensity is increased by a factor of two, the luminosity is increased by a factor of 4. When 114 bunches are circulating in a ring the luminosity is increased by another factor of 2. In this case the luminosity can be as high as $10^{33}$/cm$^2$/sec.

## 4. Implication and Sexupole Requirement

The mini-β insertion in the present study requires two pairs of common quadrupoles in each of mini-β section. Because of the extra quadrupoles, the phase advance of the insertion is increased by about 0.06. To prevent the low order resonances in the collider, the tune of the machine is maintained by readjusting the rest of the accelerator. Thus it is necessary to setup the accelerator before the mini-β run. Because of the high gradient requirement for these common quadrupoles, smaller coil i.d. magnet is conceived. This limits the crossing angle configuration to be less than 2mrad at at 100 GeV/amu for heavy ion beams. For proton beam, the requirement is relaxed slightly because of smaller beam size.

Because of the large chromaticity of these low β insertions, careful sextupole correction scheme is needed. Let S, $S_F$ and $S_D$ be the integrated sextupole strengths for the systematic $b_2$ in the RHIC dipoles, the chromatic sextupoles located at the focusing and defocusing quadrupoles in the arc respectively. The chromaticity of the machine can be expressed as

$$C_x = C_x^o\ (\beta^*) + 751.22\ S + 432.07\ S_F + 42.280\ S_D$$

$$C_y = C_y^o\ (\beta^*) - 583.08\ S - 83.83\ S_F - 218.07\ S_D$$

where $C_x^o$ and $C_y^o$ are the natural chromaticity of the machine without any sextupoles (see table 3). By using $S_F$ and $S_D$, the chromaticites of the machine can be adjusted. The strengths of $S_F$ and $S_D$ depend on the systematic sextupoles S which varies with the beam energy and the dipole design.[2] Fine adjustment for minimizing the half integer stopband can be achieved by splitting the $S_F$ and $S_D$ into two families as $S_F \pm \Delta S_F$ and $S_D \pm \Delta S_D$. Fig. 3 shows the saturation sextupole, S, vs the Bρ value. The corresponding $S_F$ and $S_D$ needed for $C_x = C_y = 0$ are also shown for various natural chromaticities. The sextupole strength needed is well within the capability of the design specification.

The sextupole correction induces however the second order octupole effect of the tune vs amplitude dependence as

$$\nu_x = \nu_x^o + \alpha_{xx}\varepsilon_x + \alpha_{xy}\varepsilon_y$$

$$\nu_y = \nu_y^o + \alpha_{xy}\varepsilon_x + \alpha_{yy}\varepsilon_y$$

where $\alpha_{xx}$ ,$\alpha_{xy}$ and $\alpha_{yy}$ in $[\pi\text{mm-mrad}]^{-1}$ are given by

$$10^{-3}\alpha_{xx} = -190\ S^2 - 204SS_F - 32\ SS_D - 44\ S_F^2 - 19.6S_FS_D - 0.324S_D^2$$

$$10^{-3}\alpha_{xy} = 136S^2 - 119.4\ SS_F - 47.6\ SS_D - 16.2\ S_F^2 - 33.9S_FS_D - 2.38S_D^2$$

$$10^{-3}\alpha_{yy} = -138S^2 - 44.4SS_F - 97.7SS_D - 1.67S_F^2 - 19.7S_FS_D - 8.57S_D^2$$

Because the sextupoles are located in the arcs, the coefficients in the above equations does not depend on the $\beta^*$ values. Using a realistic estimate of the emittances obtained from the intrabeam scattering, we expect that the tune spread to be about 0.005. This is to be compared with the beam-beam tune shift of 0.005 per crossing. Thus the beam stability would be maintained in the mini-β insertions.

We have discussed only the basic mini-β lattice. Since the $\hat{\beta}$ remains small, we expect that the dynamical aperture is changed appreciably for the mini-β insertion. The half integer stop band can be minimized by the sextupole families.[3] Tracking calculation should be used to estabilish the available dynamical aperture. In order to optimize the configuration, these common quadrupoles should be incorporated into the detector design, where the high luminosity is required.

## 5. Conclusion

In conclusion, we have found some possible mini-β insertion to the present IR design. For 10 hours of heavy ion operation, it is possible to obtain $\beta^*$ value of 1m without much beam size constraint to the rest of the machine. Two pairs of common quadrupoles are needed. The available free space for the experimental instrument becomes ±5m. Because of the smaller aperture of these common quadrupoles, smaller crossing angle is required in the mini-β insertion.

For the proton operation, $\beta^* = 0.5$m is operable without mini-β insertion at the top energy. When mini-β insertion is used, the luminosity can increase by a factor of 2. The corresponding luminosity becomes $10^{32}$/cm$^2$sec where the number of interactions per crossing becomes 1. In the future upgrade, the number of protons per bunch will be $2 \times 10^{11}$ and number of bunches per ring will be 114. The corresponding luminosity can be as high as $10^{33}$/cm$^2$sec. In this high luminoisty operation mode, there are 4 collisions per crossing. Proper detector design may be an important issue.

At the unequal species operational mode, these common quadrupoles should be removed for beam clearence.

## Reference

[1] F.G. Dell and G. Parzen, private communications.

[2] P. Thompson, private communication

[3] S.Y. Lee, G.F. Dell, H. Hahn and G. Parzen, Proc. IEEE Part. Acc. Conf. p1328, 1987.

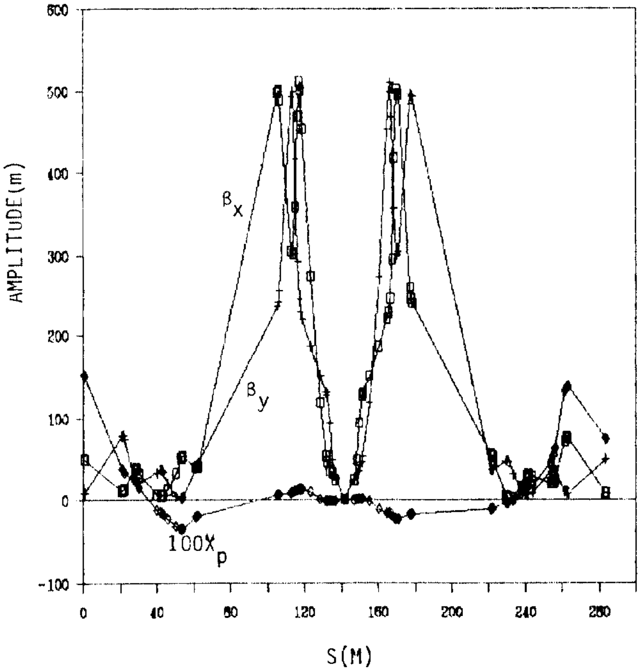

Fig 1 The betatron amplitude function for $\beta^* = 1$m is shown for the MINI-β insertion

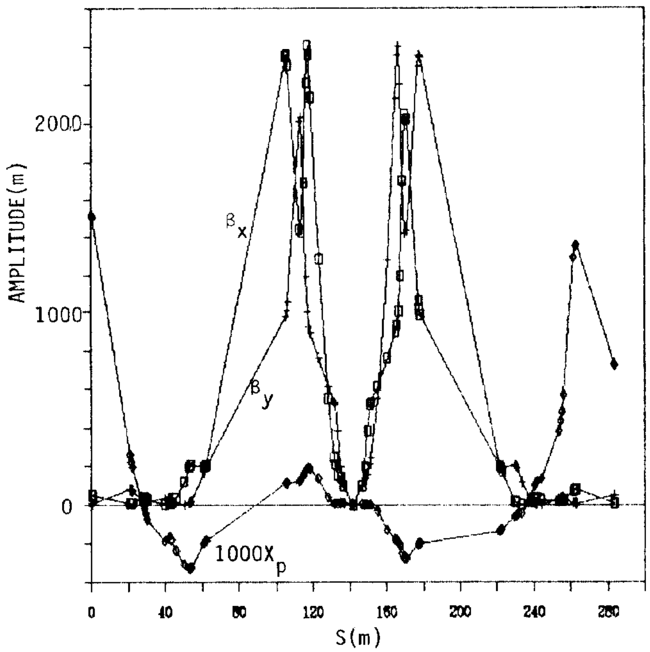

Fig. 2 Similar to that of Fig. 1 but for $\beta^* = 0.25$m

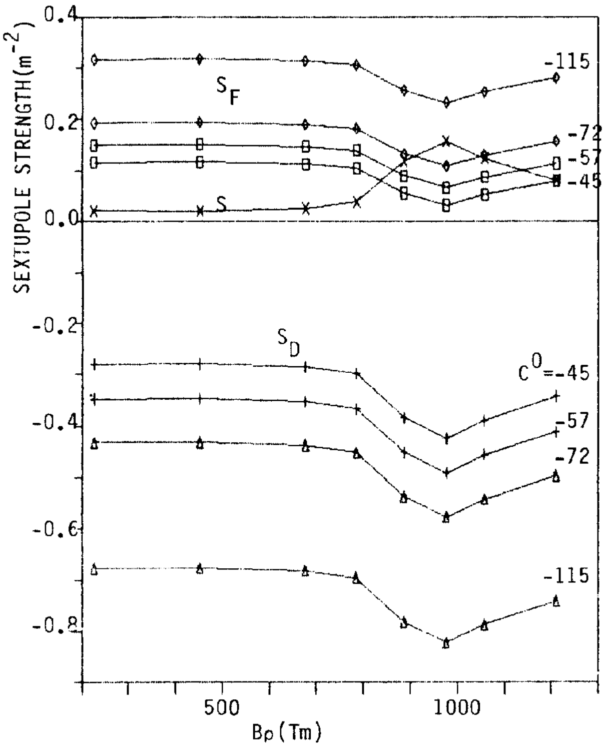

Fig. 3 The integrated Sextupole strengths needed for correcting the chromaticity to zero value are shown for various natural chromaticities as a function of the Bρ values.