Asymptotic Growth of Cumulative and Regenerative Beam Break-up Instabilities in Linear Accelerators

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Abstract

With the use of a simple model, it is shown that the asymptotic growth of the cumulative beam break-up instability is independent of the linear transverse focusing. The analysis is extended to include the transition from the cumulative to the regenerative type, both in the presence and absence of a focusing magnetic field.

Text

The beam break-up instability (BBU) continues to be a critical factor which places a limit on the current and on the pulse length in both rf and induction accelerators. [1-11] Depending on whether a wave with negative group velocity is present to provide feedback, BBU may either be regenerative or cumulative. [2,4,6]

Much theoretical effort on BBU in the past twenty years has been devoted to the cumulative type, [2,3,5,7-11] where the accelerating units are assumed to be decoupled from each other electromagnetically. Information is carried only by the beam. Under this assumption, Panofsky and Bander [2] found that the transverse displacement of the beam grows asymptotically like $\exp(at)^{1/3}$, at a given distance downstream, when the focusing magnetic field is absent. Somewhat later, Neil, Hall and Cooper [5] used an entirely different approach and found that the asymptotic growth of the cumulative BBU behaves instead like $\exp(bt)^{1/2}$ in the presence of a strong solenoidal magnetic field. Here, a, b are parameters proportional to the beam current. These peculiar time dependences, at first sight, are not expected from the usual experience of beam-circuit interaction.

In this paper, we use the continuum model and adopt a mode coupling analysis. The asymptotic growth is established analytically for both cumulative and regenerative BBU, in the presence of a general focusing magnetic field. This work was motivated by an attempt to understand the origin of the asymptotic dependences mentioned above, and by the need to assess the importance of the BBU in the two beam accelerator concept [12] currently explored at the Naval Research Laboratory. To isolate the growth mechanism, we may, for convenience, ignore the damping due to the finite quality factor Q of the accelerating units, even if Q has always played an extremely important role in the control of BBU growth. Bearing this in mind, the present analytic theory yields three specific results which hitherto were not given in the literature. First, the asymptotic growth $\exp(bt)^{1/2}$ exhibited in the cumulative BBU in the case of a strong solenoidal magnetic field [5] is a result of the coupling between the slow beam-cyclotron mode and the cavity mode. Second, this growth is reduced to $\exp(at)^{1/3}$ as $t \to \infty$. That is, the asymptotic growth of the cumulative BBU is independent of the focusing magnetic field---as if the focusing magnetic field were absent. Third, the treatment of the cumulative BBU is extended to the regenerative type with the inclusion of a negative group velocity v_g. The exponentiation factor is modified by a quantity which depends only on the group velocity, but is independent of the other properties of the structure. Here, we shall present the model and the results. The implications will be discussed and the details will be given elsewhere.

Consider a continuous beam with coasting velocity v, relativistic mass factor γ, and current I streaming in a focusing magnetic field of betatron frequency ω_c inside a series of identical accelerating units. Let $\xi(z,t)$ be the transverse displacement of the beam from the axis, $q(z,t)$ be a measure of the deflecting force produced by the non-axisymmetric mode (with $e^{i\omega_o t}$ dependence) in the individual accelerating units. In the continuum treatment of the cumulative BBU, the governing equations for ξ and q may be written as [2,3,13]

$$\left(\frac{\partial}{\partial t} + v\,\frac{\partial}{\partial z}\right)\left[\gamma\left(\frac{\partial \xi}{\partial t} + v\,\frac{\partial \xi}{\partial z}\right)\right] + \gamma\omega_c^{\,2}\xi = q(z,t), \quad (1)$$

$$\frac{\partial q(z,t)}{\partial t} - i\omega_o q(z,t) = -i\gamma\omega_o^{\,3}\varepsilon\xi(z,t). \quad (2)$$

where ε is the dimensionless coupling constant proportional to the beam current. [14]

Assuming a dependence $\exp(i\omega t - ikz)$ for the solutions, Eqs. (1) and (2) yield the dispersion relation

$$D(\omega,k) \equiv \left[\left(\omega - kv\right)^2 - \omega_c^{\,2}\right]\left(\omega - \omega_o\right) - \omega_o^{\,3}\varepsilon = 0, \quad (3)$$

which describes the coupling [15,16] between the cavity mode ($\omega = \omega_o$) and the fast and slow beam-cyclotron mode ($\omega - kv = \pm\omega_c$). The asymptotic growth of disturbances may be determined from the Green's function [15]

$$G(z,t) = \int_\Gamma d\omega \int_{-\infty}^{\infty} dk\, e^{i\omega t - ikz}/D(\omega,k) \sim \int_\Gamma d\omega\, e^{i\omega t - ik(\omega)z}, \quad (4)$$

where the Bromwitch contour Γ lies sufficiently far in the lower half complex ω plane, and $k(\omega)$ is the meaningful solution obtained from the dispersion relation $D(\omega,k) = 0$.

When the focusing magnetic field is absent, $\omega_c = 0$ and $k(\omega) = \omega/v \pm (\omega_o/v)\,[\omega_o\varepsilon/(\omega-\omega_o)]^{1/2}$. Substitution of this $k(\omega)$ in (4) yields the following asymptotic formula (from a saddle point calculation [2]):

$$\left|G_n(z,t)\right| \sim \exp\left(1.64W^{\,1/3}\right), \quad (5)$$

where

$$W = \varepsilon\left(\omega_o^{\,3}z^2/v^2\right)(t - z/v), \qquad (t > z/v > 0). \quad (6)$$

The asymptotic solution (5) was first obtained by Panofsky and Bander [cf. Eq. (33) of Ref. (2)]. In the other limit, where a strong focusing magnetic field is present, the dominant interaction is expected to be between the (positive energy) cavity mode ($\omega \simeq \omega_o$) and the negative energy beam-cyclotron mode, [15-17,9] for which $\omega - kv \simeq -\omega_c$. In this case, the dispersion relation (3) may be approximated by $(\omega - kv + \omega_c)(\omega - \omega_o) \simeq -\omega_o^{\,3}\varepsilon/2\omega_c$, yielding $k(\omega) \simeq (\omega + \omega_c)/v + \varepsilon\omega_o^{\,3}/(2v\omega_c(\omega - \omega_o))$. Substituting this $k(\omega)$ in Eq. (4) and performing a saddle point calculation similar to that given in Ref. (2), one obtains the asymptotic solution

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$$|G(z,t)| = |G_s(z,t)| \sim \exp\left[(2\rho W/z)^{1/2}\right], \quad (7)$$

where $\rho = v/\omega_c$ and W is given by Eq. (6). The asymptotic formula (7), exhibiting $\exp(bt)^{1/2}$ dependence, is easily shown to be identical to the growth factor given in Eq. (5.13) of Neil et al. [5]

For general values of ω_c, Eq. (3) gives $k(\omega) = \omega/v \pm [\omega_c^2 + \epsilon\omega_o^3/(\omega-\omega_o)]^{1/2}/v$ and the saddle point contribution may also be calculated analytically. The dominant contribution to (4) gives

$$|G(z,t)| \sim \exp\left\{\mathrm{Re}\left[i(z/\rho)\omega_s\tau\,(3 + 2\omega_s)\right]\right\}, \quad (8)$$

where the dimensionless time τ is

$$\tau = W(\rho/z)^3, \quad (9)$$

and ω_s is the root of the fourth degree polynomial:

$$\omega_s^{\,3}\,(1+\omega_s) = (1/2\tau)^2. \quad (10)$$

It is easy to show that there is one and only one root of ω_s in Eq. (10) with Im $\omega_s < 0$ for all values of $\tau \neq 0$, and we should use that root of ω_s in (8).

The solution (8) implies that, given a focusing magnetic field, the asymptotic growth is independent of the strength of the magnetic field. To see this, consider a time long enough so that $2\tau >> 1$. Then Eq. (10) gives $\omega_s \simeq (1/2\tau)^{2/3}\, e^{-i2\pi/3}$ and the solution (8) reduces to (5), the formula corresponding to zero focusing magnetic field. This is a rather surprising result, obtained directly from the model of Panofsky and Bander, [2] but is, at first sight, contradictory to the findings of Neil et al. [5]

The above paradox may be resolved by rewriting (8) as

$$|G(z,t)| \sim \exp\left[(2\rho W/z)^{1/2} p(\tau)\right] \quad (11)$$

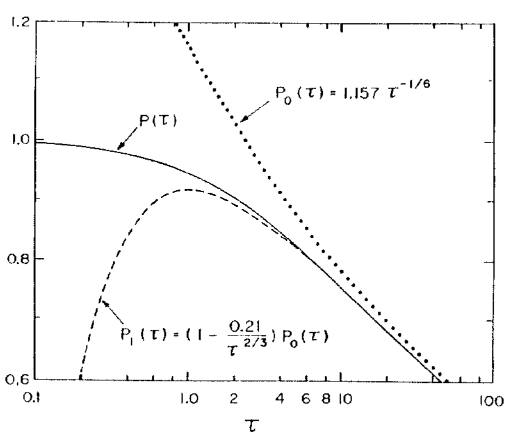

Fig. 1 Comparison of the general solution of $p(\tau)$ with the zero focusing solution $p_o(\tau)$, weak focusing solution $p_1(\tau)$, and the strong focusing solution $p(\tau) \to 1$.

where $p(\tau) = -(2\tau)^{-1/2}\,\mathrm{Im}[\omega_s\tau(3 + 2\omega_s)]$ is shown in Fig. 1. In this figure, the strong focusing solution (7) corresponds to $p(\tau) \to 1$ for $\tau << 1/2$. The zero focusing solution (5) corresponds to $P(\tau) \to p_o(\tau) = 1.1573/\tau^{1/6}$, and the weak focusing solution [Eq. (61) of Panofsky and Bander [2]] to $p(\tau) \to p_1(\tau) = (1 - 0.21/\tau^{2/3})p_o(\tau)$. The domains of validity of these asymptotic solutions and their transitions at $\tau \sim 1/2$ can readily be identified in Fig. 1.

To include the effects of a non-zero group velocity, we replace the factor $(\partial/\partial t - i\omega_o)q$ in Eq. (2) by $[(\partial/\partial t - i\omega_o) + v_g(\partial/\partial z + ik_o)]q$, where (ω_o, k_o) may now be taken as the point of intersection of the dispersion curves $\omega = \omega(k)$ in the (ω,k) plane between the "beam line" and that of the slow wave structure formed by the accelerating units, and $v_g = \partial\omega/\partial k$ is the group velocity of the structure mode in the absence of the beam. With this replacement, the Green's function (4) may again be re-evaluated. In the case of no focusing magnetic field, the method of steepest descent gives

$$|G_n(z,t)| \sim \exp\left\{1.64 W^{1/3}\ \left(\frac{1}{1-\beta_g}\right)\left(1-v_g t/z\right)^{2/3}\right\}, \quad (12)$$

whereas in the case of strong focusing, we have

$$|G_s(z,t)| \sim \exp\left\{(2\rho W/z)^{1/2}\ \left(\frac{1}{1-\beta_g}\right)\ \left(1-v_g t/z\right)^{1/2}\right\}, \quad (13)$$

where $\beta_g \equiv v_g/v$. It is obvious that (12) and (13) reduce to (5) and (7), respectively, as $v_g \to 0$. Since $\beta_g < 0$ for regenerative BBU, it is easily seen from Eqs. (12) and (13) that both G_n and G_s grow like a simple exponential function of time [i.e., exp(Ct)] as $t \to \infty$ when $v_g < 0$. This, of course, is consistent with what is expected from the outset when a backward wave interacts with an electron beam. [15,16] It also reaffirms the potential danger of the regenerative BBU, as exp(Ct) grows considerably faster than either $\exp(at)^{1/3}$ or $\exp(bt)^{1/2}$ for large t. Equations (12) and (13) are also valid when $\beta_g > 0$. In that case, the BBU becomes convective [15] and the Green's function shows growth only for $z/v < t < z/v_g$ at a given position z by causality. Equations (12) and (13), when compared with (5) and (7), suggest that the modification in the exponentiation due to a non-zero group velocity depends only on v_g and is otherwise independent of the accelerating structure. The value of Q required to render BBU harmless is determined [13] using these simple expressions.

Finally, we note the following. First, without the help from a finite Q, phase mixing, (due to a spread in the betatron frequency, for instance), by itself, cannot be expected to suppress BBU in a long pulse machine. The reason follows. Since the asymptotic growth of the cumulative BBU is shown here to be independent of the focusing field, a spread in the betatron frequency, as long as it is finite, cannot alter this long-time behavior. A direct calculation of the Green's function for a model which explicitly includes a betatron frequency spread has supported this intuitive argument. Second, large convective growth of BBU might occur when the beam velocity v happens to be synchronous with the group velocity v_g, as can be seen from Eqs. (12) and (13) in the limit $\beta_g \to 1$. This synchronous interaction is unimportant for electron linacs but is perhaps worthy of some attention in certain ion accelerators (in which case the theory needs to be appropriately modified.) Third, we have calculated analytically the asymptotic growth of the cumulative BBU for an accelerating beam, with constant acceleration, in a general focusing magnetic field. The results will be given elsewhere. Here, we only report that the asymptotic growth is also independent of the transverse focusing, in the sense described in this paper.

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13. Equation (2) is actually an approximation to the second order equation $(\partial^2/\partial t^2 + \omega_o^2)q = 2\gamma\omega_o^4\varepsilon\xi$, when the solution behaves like $\exp(i\omega_o t)$ times a slowly-varying function of t. Likewise, the effect of finite Q enters in Eq. (2) by adding the term $(\omega_o/2Q)q$ to the left-hand side, and the solutions (5), (7), (8), (11), (12), (13) are simply multiplied by the well-known decay factor $\exp(-\omega_o t/2Q)$.
14. The coupling constant ε depends on the accelerating structure and on the deflecting mode under consideration. The configuration treated in Ref. 3 consists of a circular waveguide loaded with identical apertured disks along the guide axis. In that case, $\varepsilon = 0.422(I/17kA)\beta/\gamma$ if the deflecting mode within the individual cavities is the TM_{110} mode. Here, $\beta = v/c$, c is the speed of light. In the more general configuration treated in Refs. (2) and (5), the accelerating units are separated by a distance L and the n-th unit is located at z = nL. In that case, $\varepsilon = (v^2\kappa/\omega_o^2 L)$ $I/(17\beta\gamma kA)$ where κ is proportional to the "transverse impedance" of the structure. [κ has a unit of inverse length; it is identical to the k defined in Eq. (3.11) of Ref. (5)]. Note that ε is independent of the focusing magnetic field but is inversely proportional to γ.
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