LONGITUDINAL EMITTANCE IN HIGH-CURRENT LINEAR ION ACCELERATORS*

T. P. Wangler, T. S. Bhatia, G. H. Neuschaefer, and M. Pabst†

ABSTRACT

The control of longitudinal emittance in an ion linear accelerator is important for minimizing both chromatic aberrations and beam halo. The root-mean-square (rms) longitudinal emittance grouth can result from either the nonlinear rf focusing fields or the nonlinear space-charge fields. We will present conclusions based on numerical beam-dynamics studies for both the radio-frequency quadrupole (RFQ), and the drift-tube linac (DTL). We will discuss the scaling of longitudinal emittance produced during the adiabatic bunching in an RFQ and will show the benefits of ramped DTL accelerating field designs to maintain high longitudinal focusing strength with increasing particle energy.

INTRODUCTION

The control of rms longitudinal emittance growth in linear ion accelerators[1] can be important for two reasons. First, longitudinal emittance growth produces a beam halo in longitudinal phase space that, because of coupling from longitudinal to transverse motion, also may result in a beam halo in transverse phase space. This can cause undesirable beam spill that may be a serious limitation for linacs with high average current. Second, for some applications, the output beam-optics requirements put limits on longitudinal emittance to minimize chromatic aberration effects. Designers of ion linacs need to know how to limit the growth of rms longitudinal emittance. However, few measurements of longitudinal emittance have appeared in the published literature, and in spite of progress in the understanding of emittance growth effects, there is insufficient guidance from analytic theory.

Nevertheless, we can use the numerical simulation codes PARMTEQ[2] and PARMILA[3] to study emittance growth phenomena that are induced by both external and space-charge fields, under a variety of conditions. These codes make use of a particle-in-cell (PIC) approach and have been subjected to experimental tests with a good record of consistency between theory and experiment. Therefore, we can use the codes as a tool to allow a study of different emittance growth mechanisms and to determine approximate scaling laws that will aid the linac designer.

A typical configuration of a high-current, high-energy proton linac is shown in Fig. 1. The dc injector supplies a (50- to 100-keV) beam to the radio-frequency quadrupole (RFQ)[4,5] linac. The RFQ adiabatically bunches the dc beam and accelerates it to an energy of about 2 MeV. In this energy range, the conventional drift-tube linac (DTL) can begin to provide strong transverse focusing and more efficient acceleration. Above about 100 MeV, a class of accelerating structures called coupled-cavity linacs (CCL) become more efficient than the DTL.

Fig. 1. Block diagram of an ion-linac configuration for high beam-current applications.

* Work supported by Los Alamos National Laboratory Institutional Supporting Research, under the auspices of the U. S. Department of Energy.

† On leave from KFA Jülich, West Germany.

Longitudinal emittance is often quoted in either energy·time or energy·degrees, or in length units for comparison with transverse normalized emittance. Useful conversions for a beam particle of mass m are ε_ℓ (MeV·s) = mc ε_ℓ (cm) = ε_ℓ(MeV·deg) λ/360c, where c is the speed of light and λ is the rf wavelength. In this paper, we report on longitudinal emittance growth mechanisms for two cases: (1) the bunching section of the RFQ and (2) the DTL accelerator.

Nonlinear external or space-charge fields can cause a growth in the rms emittance. Two common mechanisms of space-charge-induced emittance growth have been identified. First, a rapid redistribution of charge occurs as a result of internal oscillations of the beam density as the beam becomes internally matched to the accelerator.[6-8] Within a quarter plasma period, the beam emittance growth can occur as space-charge field energy is converted to thermal energy. Second, for beams with asymmetric properties in the three degrees of freedom, energy exchange can occur through the coupling terms of the space-charge force.[9,10] This is found to lead toward an approximate equipartitioning of the thermal energy components, which is more pronounced as the beam intensity increases.

LONGITUDINAL EMITTANCE GROWTH FROM BUNCHING IN THE RFQ

The RFQ forms the bunches from an input dc beam and determines the longitudinal emittance through the bunching process.[5] The bunching is done over many cells (adiabatically) as the accelerating field and synchronous phase are varied. Both nonlinear rf fields and nonlinear space-charge forces act on the beam.

We have studied the RFQ bunching using the PARMTEQ numerical simulation code with 2500 macroparticles per run in an initial 4-D waterbag distribution for the transverse coordinates and momenta (uniform filling within a 4-D hyperellipsoid). The initial transverse ellipse parameters were chosen to produce an rms match into the RFQ radial matching section, and the particles in longitudinal space were monoenergetic and spaced uniformly over 360° of phase. An earlier study[11] has shown that the beam characteristics after bunching in the RFQ are very insensitive to the initial beam distribution. In Figs. 2a through 2c, we show longitudinal phase-space plots at the end of the bunching section (0.244 MeV) for three different beam currents for a 200-MHz RFQ with an injection energy of 0.05 MeV and a current limit of 200 mA. These plots show a decreasing emittance as beam current is increased, in contrast to the space-charge-induced growth of emittance that is a characteristic result for transverse phase space in accelerators and beam transport systems. In Figs. 3 and 4, we show the rms longitudinal and transverse emittances versus cell number in the RFQ bunching section for the same three beam currents. The correlation between transverse emittance growth and longitudinal emittance decrease as the current increases is evident in the figures and corresponds to energy transfer from the longitudinal to the transverse planes. The ratio $r = \sigma_t\varepsilon_t/\sigma_\ell\varepsilon_\ell$, where ε_ℓ and ε_t are the rms normalized longitudinal and transverse (averaged over x and y) emittances and σ_ℓ and σ_t are the longitudinal and transverse phase advances per rf period (with space charge) of an equivalent uniform beam, should equal unity if the beam is equipartitioned. At the end of the bunching section, this ratio is r = 0.32, 1.1, and 2.1 for the I = 0, 50, and 100-mA cases.

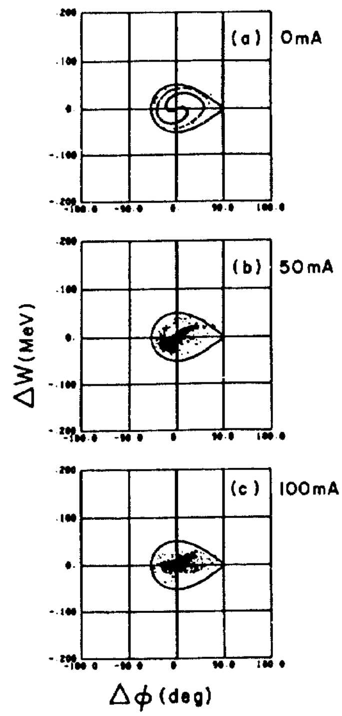

Fig. 2. Longitudinal phase-space plots at end of RFQ bunching section.

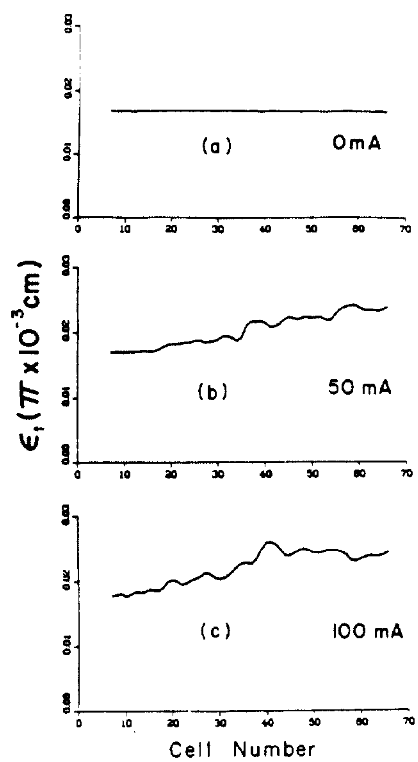

Fig. 4. The rms normalized transverse emittance vs RFQ cell number.

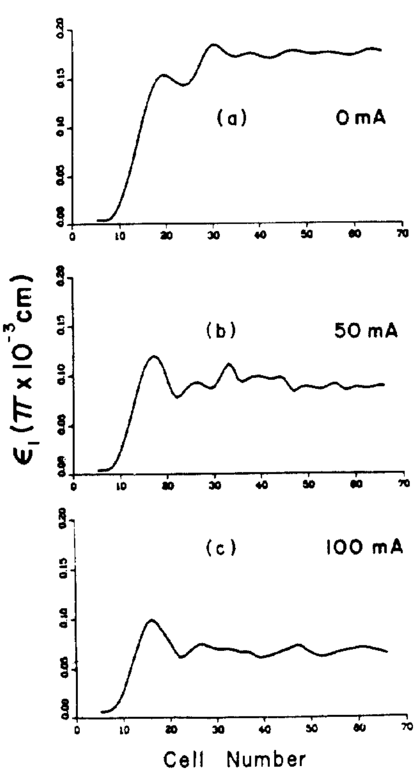

Fig. 3. The rms normalized longitudinal emittance vs RFQ cell number.

In an earlier work,[11] the scaling of transverse rms emittance growth was reported. In this paper we present the results of a systematic study of RFQ designs with different beam currents, current limits, injection energies, and rf frequencies to determine a scaling law for rms longitudinal emittance after bunching. We find that the rms longitudinal emittance ε_ℓ at the end of the bunching section can be approximately expressed as a product

$$\varepsilon_\ell = \varepsilon_s \eta \ . \qquad (1)$$

The quantity ε_s is equal to the product of the horizontal and vertical semiaxes of the zero-current separatrix area and is given in energy-time phase space by

$$\varepsilon_s = \delta W \delta t = \frac{3\lambda}{2\sqrt{2}\,\pi c}\sqrt{qAVW\phi^2(\phi\cos\phi - \sin\phi)} \qquad (2)$$

where q is the charge per particle, V is the intervane voltage, λ is the rf wavelength, c is the speed of light, and A, ϕ, and W are the acceleration efficiency, synchronous phase in radians, and energy at the end of the bunching section. The factor η depends on the ratio I/I_ℓ, where I is the captured beam current at the end of the bunching section, and I_ℓ is the longitudinal current limit, which is calculated from the uniform 3-D ellipsoid model.[12] In Fig. 5 we show $\eta = \varepsilon_\ell/\varepsilon_s$ as a function of I/I_ℓ for two different RFQ linacs. The decreasing value of η as I/I_ℓ increases is the result of increasing energy transfer to transverse motion. This curve may depend somewhat on the details of the design procedure.

LONGITUDINAL RMS EMITTANCE GROWTH IN THE DTL

We have generated a 200-MHz DTL for acceleration of a proton beam from 2 to 50 MeV to study longitudinal rms emittance growth for low-emittance bunched beams. Numerical simulation studies have been made with PARMILA with 2000 macroparticles per run. Two different initial, random particle distributions have been generated, both of which were rms-matched to the DTL in all three

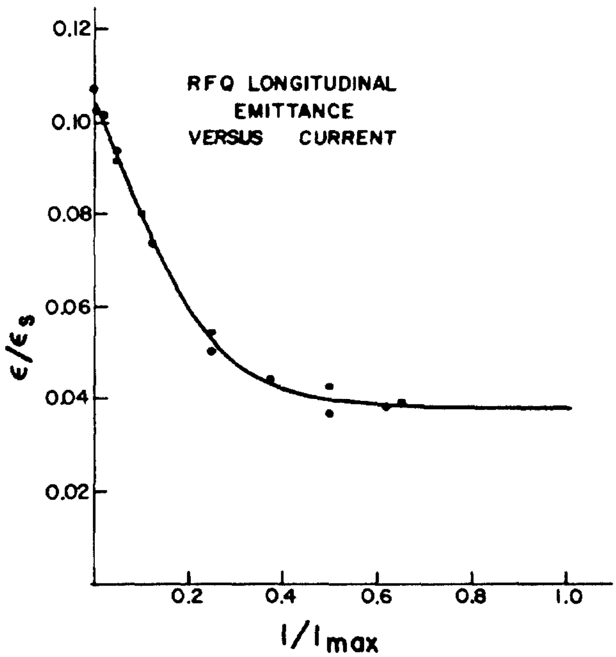

Fig. 5. RFQ longitudinal emittance ratio vs normalized current as described in text.

planes using the program TRACE.[13] The first distribution was Gaussian in 6-D space and truncated at four standard deviations. The second distribution was uniform within a 3-D ellipsoid in real space; it was slightly modified from a 3-D uniform ellipsoid in momentum space to produce elliptical boundaries in the 2-D phase-space projections. Both the axial accelerating field and the synchronous phase were chosen to be constant along the linac at $E_0T = 1.6$ MV/m and $\phi = -40°$. The zero-current phase advances per $2\beta\lambda$ period in the transverse planes were held constant throughout the linac at $\sigma_{t0} = 60°$ and for the longitudinal motion $\sigma_{\ell 0} = 46°$ at 2 MeV, decreasing to 19° at 50 MeV. The phase advances were significantly depressed by space charge. The initial σ/σ_0 ratios were 0.19 and 0.17 for the transverse and longitudinal planes, respectively, and the beams were initially equipartitioned so that the ratio $r = 1$.

Figure 6a shows the rms longitudinal emittance versus cell number for the initial Gaussian beam at $I = 0$ and 100 mA. For $I = 0$, no longitudinal rms emittance growth is observed, which implies that the nonlinear external field has a negligible effect on the emittance. At 100 mA, a very rapid emittance growth of about a factor of 2 is observed in the first cell, which is followed by beam oscillations and a slow growth throughout the remainder of the DTL. The corresponding transverse emittance plots (Fig. 6b) show the presence of the rapid initial growth, but not the slow growth component. The beam becomes nonequipartitioned; at 50 MeV the ratio $r = 3.3$. We attribute the rapid growth to charge redistribution of the beam as the beam matches itself internally to the accelerator, and field energy is transformed to particle kinetic energy. This conclusion is supported by the following facts: (1) the observed growth rate occurs within a beam plasma period, (2) the magnitude of the observed growth in all three planes is nearly equal to that expected for a Gaussian spherical bunch,[14] and (3) the rapid growth is not observed for the initial uniform distribution (see Fig. 7a), which has no available field energy for the rapid emittance growth. Figure 7a shows, however, that the slow growth is still observed for an initial uniform beam.

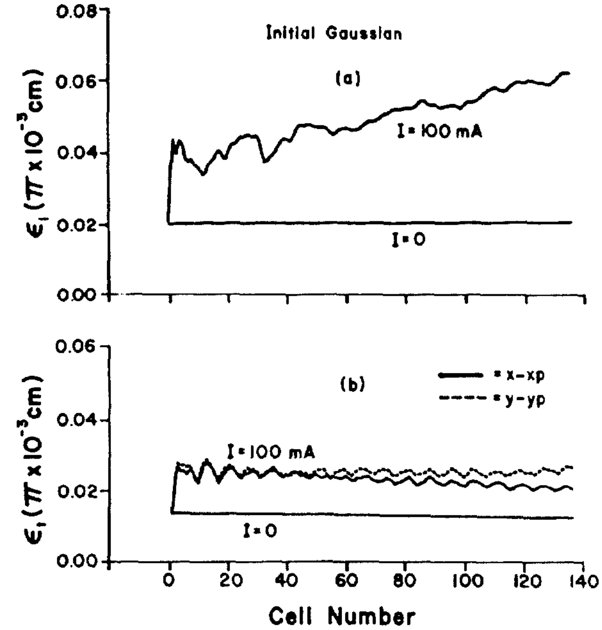

Fig. 6. DTL rms normalized emittance vs cell number for initial Gaussian beams: (a) longitudinal and (b) transverse.

Figure 7b again shows that the slow growth is suppressed for the uniform beam in transverse phase space.

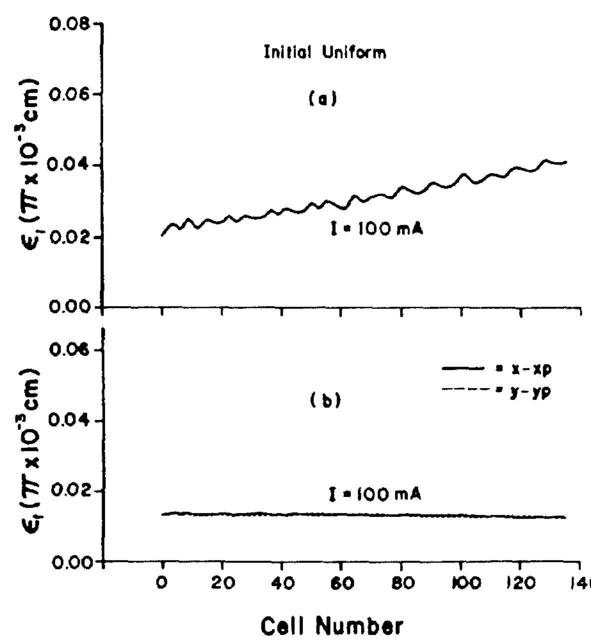

Fig. 7. DTL rms normalized emittances vs cell number for an initial uniform beam: (a) longitudinal and (b) transverse.

We interpret the slow growth as caused by a slow charge redistribution as the bunch length expands during acceleration. Also, we cannot exclude a contribution resulting from energy transfer from the transverse motion as the beam becomes more nonequipartitioned during the expansion. The expansion occurs because of a decrease in the longitudinal focusing force as the beam velocity β increases. This effect can be seen in the longitudinal oscillation frequency $\omega_{\ell 0}^2 = 2\pi q\, E_0T \sin(-\phi)/\gamma^3 m\beta\lambda$. When E_0T and ϕ are constant, $\omega_{\ell 0}$ and the external focusing force decreases with increasing β. We find that the slow longitudinal emittance growth rate can be significantly reduced by ramping the accelerating field as β increases, to keep $\omega_{\ell 0}$ constant. This improvement can be seen in Fig. 8, which shows the longitudinal emittance versus cell number for a uniform beam, when $\omega_{\ell 0}$ is kept constant. The result for constant E_0T is also shown for comparison. The practicality of ramping E_0T during acceleration is related to issues of rf electric breakdown and structure cooling. The transverse emittance for constant $\omega_{\ell 0}$ (not shown) shows no significant growth and the resulting beam remains equipartitioned.

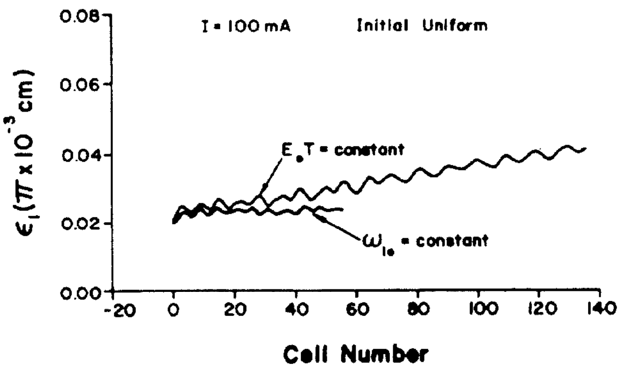

Fig. 8. DTL rms normalized longitudinal emittance vs cell number for an initial uniform beam.

Although an initial uniform beam provides protection from the rapid growth, it does not prevent the slow growth. We believe the explanation for the slow growth is that the initial uniform beam does not generally remain uniform for a finite emittance beam. An initial rapid charge redistribution occurs, which produces no initial emittance growth but does produce Debye-length tails and creates available field energy for subsequent emittance growth.

The conventional linac design generally keeps E_0T constant throughout most of the linac. An approximate empirical formula for the slow emittance growth rate in the constant E_0T linacs of this PARMILA study is,

$$\frac{\varepsilon_\ell}{\varepsilon_{\ell i}} = \sqrt{1 + c_\ell \lambda E_0 T N_c \upsilon_i} \tag{3}$$

where $c_\ell = 0.00027\ \mathrm{MV}^{-1}$, N_c is the cell number, and v_i is the initial value of a space-charge parameter, given in terms of the initial rms radius a, bunch length b, and rms longitudinal emittance $\varepsilon_{\ell i}$, and $Z_0 = 376.73\ \Omega$ as

$$\upsilon_i = \frac{1}{20\sqrt{5}\,\pi} \frac{eIZ_0}{mc^2} \frac{\lambda b^2}{a\varepsilon_{\ell i}^2} . \tag{4}$$

We have checked Equation 3 against our numerical simulation results at constant $\phi = -40°$ and for various values of λ, E_0T, N_c, and υ_i for prolate bunches. We have not yet determined the scaling of emittance growth with respect to ϕ. Equation 3 is consistent with the earlier observation by Pabst and Bongardt.[15]

SUMMARY

We have studied longitudinal emittance growth in RFQ and DTL linacs. For the RFQ bunching, we find that (1) nonlinear rf and space-charge fields are both important, (2) ε_ℓ scales as the product of zero-current separatrix area times a current-dependent factor that decreases with increasing current, and (3) the decrease of ε_ℓ with current is correlated with transverse emittance growth. For the DTL, both rapid and slow charge-redistribution emittance growth mechanisms are observed. The rapid growth is consistent with the charge redistribution mechanism studied previously. The slow growth is caused by a gradual weakening of the longitudinal focusing force with increasing beam velocity and can be controlled if the accelerating field can be ramped to compensate.

REFERENCES

1. T. P. Wangler, "Physics Design of Linear Accelerators For Intense Ion Beams," submitted to 1988 Linear Accelerator Conference, Williamsburg, VA, October 3-7, 1988.
2. K. R. Crandall and T. P. Wangler, "PARMTEQ—A Beam Dynamics Code for the RFQ Linear Accelerator," Proc. Workshop on Linear Accelerator and Beam Optics Codes, San Diego, CA, January 19-21, 1988, AIP Conf. Proc. No. 177, 22.
3. G. P. Boicourt, "PARMILA—An Overview," Proc. Workshop on Linear Accelerator and Beam Optics Codes, San Diego, CA, January 19-21, 1988, AIP Conf. Proc. No. 177, 1.
4. I. M. Kapchinskij and V. A. Teplyakov, "Linear Ion Accelerator With Spatially Homogeneous Strong Focusing," Prib. Tekh. Eksp. 2 (1970), 19.
5. Richard H. Stokes and Thomas P. Wangler, "Radio-Frequency Quadrupole Accelerators and Their Applications," in *Annual Review of Nuclear and Particle Science*, Vol. 38, 1988.
6. J. Struckmeier, J. Klabunde, and M. Reiser, "On the Stability and Emittance Growth of Different Particle Phase-Space Distributions in a Long Magnetic Quadrupole Channel," Particle Accelerators 15, 47 (1984).
7. T. P. Wangler, K. R. Crandall, R. S. Mills, and M. Reiser, "Relationship Between Field Energy and RMS Emittance in Intense Particle Beams," IEEE Trans. Nucl. Sci. 32, 2196 (1985).
8. T. P. Wangler, K. R. Crandall, R. S. Mills, and M. Reiser, "Field Energy and RMS Emittance in Intense Particle Beams," Proc. of Workshop on High Current, High Brightness, High Duty Factor Ion Injectors, San Diego, CA, May 21-23, 1985, AIP Conf. Proc. No. 139, 133 (1986).
9. R. A. Jameson, "Equipartitioning in Linear Accelerators," Proc. 1981 Linear Accelerator Conf., Los Alamos National Laboratory report LA-9234-C (1982), 125.
10. I. Hofmann and I. Bozsik, "Computer Simulation of Longitudinal-Transverse Space-Charge Effects in Bunched Beams," Proc. 1981 Linear Accelerator Conf., Los Alamos National Laboratory report LA-9234-C (1982), 116.
11. T. P. Wangler, R. S. Mills, and K. R. Crandall, "Emittance Growth in Intense Beams," Proc. 1987 Particle Accelerator Conf., IEEE Catalog No. 87CH2387-9 (1987), 1006.
12. T. P. Wangler, "Space-Charge Limits in Linear Accelerators," Los Alamos National Laboratory report LA-8388, July 1980.
13. K. R. Crandall, "TRACE, An Interactive Beam Dynamics Program," Proc. Workshop on Linear Accelerators and Beam Optics Codes, San Diego, CA, January 19-21, 1988, AIP Conf. Proc. No. 177, 29.
14. T. P. Wangler, K. R. Crandall, and R. S. Mills, "Emittance Growth From Charge Density Changes in High-Current Beams," Proc. International Symposium on Heavy Ion Fusion, Washington, D. C., AIP Conf. Proc. 152 (1986), 166.
15. M. Pabst and K. Bongardt, "Longitudinal Trapping and Particle Loss of high Intensity Bunched Beams," Proc. 1986 Linear Accelerator Conf., Stanford Linear Accelerator Center report SLAC-303 (1986), 50.