COMPARISON OF GABOR LENS, GAS FOCUSING, AND ELECTROSTATIC QUADRUPOLE FOCUSING FOR LOW-ENERGY ION BEAMS*

M. Reiser
Laboratory for Plasma Research
University of Maryland, College Park, MD 20742

Abstract

When a particle beam propagates through a background gas, a plasma is formed by collisional ionization resulting in (partial) neutralization of the beam's space charge and decrease of the beam radius. This "gas focusing" effect occurs naturally and is often utilized to improve high-current beam transport. Gabor, in 1947, proposed a nonneutral electron plasma confined in a magnetron-type trap as an effective "space-charge lens" for positive ion beams. This "Gabor lens", which offers better control and focusing strength than both gas focusing and applied fields, has been investigated by several research groups since its invention. So far, however, the experimental results have been inconclusive. In this paper, we will present a theoretical reevaluation of the Gabor lens and a comparison with an electrostatic quadrupole (ESQ) doublet. It will be shown that the focusing strength of the Gabor lens depends on the electron trapping efficiency and is significantly higher than that of the ESQ doublet if the trapping efficiency is close to the theoretical Brillouin limit. On the other hand, an ESQ doublet with equivalent geometry and voltage parameters performs better than the Gabor lens if the trapping efficiency is below a certain threshold, as appears to be the case in experiments so far.

Introduction

Focusing of high-brightness low-energy ion beams, such as H^+ and H^- in the range of 10 kV to 2 MV, 10 mA to 200 mA, is a problem that has not yet found a satisfactory solution. The approach adopted in most laboratories is to use charge neutralization in the background gas in combination with magnetic lenses (solenoids or quadrupoles). The degree of charge neutralization is measured by the factor $f_e = |\rho_e/\rho_b|$ where ρ_b and ρ_e are the space-charge densities of the ion beam and the neutralizing particle distribution (with opposite charge polarity), respectively. For positive-ion beams, e.g. H^+, the neutralization is incomplete, i.e., $f_e < 1$, which is why the additional focusing by magnetic lenses is needed. Ionizing collisions between the beam ions and the atoms or molecules of the background gas produce electron-ion pairs. Due to the positive space charge of the beam in this case, the secondary positive ions are expelled from the beam and the electrons are trapped. However, the electrons are born in the collisions with kinetic energy and, therefore, can escape from the potential wall of the beam before full neutralization is achieved. By contrast, in the case of a negative ion beam, e.g. H^-, overneutralization, i.e., $f_e > 1$, can be obtained.[1] However, here too the net focusing may be inadequate, or one may in fact stay away from the $f_e > 1$ state to avoid electron stripping, so that additional magnetic focusing is required.

"Gas focusing," as explained above, has been known and utilized from the early days of electron optics.[2] Its inadequacy and the well-known weakness of conventional lenses apparently motivated D. Gabor to propose in 1947 a "space-charge lens"[3] consisting of an electron distribution, or nonneutral plasma, whose density can be controlled externally such that a desired value of $f_e > 1$ can be achieved. The electrons are trapped in a solenoidal magnetic field that confines them radially and with an electrostatic potential well that prevents them from escaping axially. Such a "Gabor lens" with electrons can be used to focus a positive ion beam. (In principle, one could also apply it to negative ion beams by using a positron plasma in place of the electrons.) Although theoretically, the focusing capabilities of a Gabor lens look very promising, relatively little experimental research has been performed to test and develop this device for practical use. In 1966-1969 some work – both experimental and theoretical – was performed by Morozov, Lebedev, et al. in the Soviet Union.[4–6] Following this work, the two authors published several related theoretical papers in 1974-1976.[7–9] During the late seventies, two experimental groups conducted research on the Gabor-lens concept in the United States: Booth and Lefevre at Livermore[10,11] and Mobley, Gammel and Maschke at Brookhaven.[12]

All of this past work in the Soviet Union and the United States can be characterized as exploratory. While focusing was observed in these experiments, it is fair to say that the results were inconclusive.

More recently new interest in the Gabor lens developed at Fermilab in connection with the upgrade of the linear accelerator.[13,14] In first experiments by Palkovic, et al., with a 30 kV H^+ beam from a duoplasmatron ion source an emittance growth by a factor of 3 to 4 was observed.[13] Like in the previous experiments in the USSR and USA the Gabor lens at the Fermilab is operated with a gas discharge to obtain the desired nonneutral electron distribution — in contrast to Gabor's original proposal of a pure magnetron-type electron beam.

The purpose of this paper is to present a brief theoretical analysis of gas focusing and of the Gabor lens and to compare the latter with an electrostatic quadrupole (ESQ) doublet. In the low-energy ion regime of interest here the ESQ lens provides the strongest focusing of any conventional lens that employs applied electric or magnetic fields. For our analysis in the following sections we will assume an H^+ beam and electrons for charge neutralization. Application to H^- beams will be briefly discussed at the end.

Gas Focusing

Consider an H^+ beam in a drift tube filled with a background gas at low pressure. The beam particles ionize the gas, i.e., they create electron-ion pairs. The positive ions from the collisions are driven to the wall and the electrons remain trapped in the beam's positive space charge well. Within a time of typically a few tens of microseconds (depending on the gas pressure) after the beam front has entered the drift tube, a quasi-steady state of maximum charge neutralization is reached where $f_e = |\rho_e/\rho_b| < 1$ remains more or less constant.

*Research supported in part by ONR and Los Alamos National Laboratory. Institutional Supporting Research under DOE.

Assuming an axisymmetric, uniform-density beam and electron plasma in a coaxial cylindrical drift tube, one can calculate the net radial electric field at radius r in the beam region as

$$E_r = \frac{\rho_b + \rho_e}{2\epsilon_0} r = \frac{\rho_b(1-f_e)}{2\epsilon_0} r, \tag{1}$$

where ρ_b and ρ_e can be expressed in terms of the beam and electron densities as $\rho_b = en_b$ and $\rho_e = -en_e = -f_e n_b$. The beam space charge density, ρ_b can be related to the beam current, I, radius R, and particle velocity v, by

$$\rho_b = \frac{I}{R^2 \pi v}. \tag{2}$$

As an example, the space charge density for a 100 mA, 30 kV ($v = 8 \times 10^{-3}$ c) H^+ beam of radius $R = 3$ mm is $\rho_b = 1.474 \times 10^{-3}$ C/m^3. The corresponding particle density in this beam is then $n_b = \rho_b/e = 9.2 \times 10^{15}$ m^{-3}, or $n_b = 9.2 \times 10^9$ cm^3.

Since $f_e < 1$ for the H^+ beam, the radial force is defocusing and the resulting equation for the particle trajectories in the "gas focusing" region of the drift tube is

$$r'' - k^2 r = 0. \tag{3}$$

The constant k^2 can be expressed in terms of the beam current I as

$$k^2 = \frac{eI(1-f_e)}{2\pi\epsilon_0 m_i R^2 v^3}, \tag{4}$$

where m_i is the ion mass.

The partial charge neutralization, as defined by the factor 1 - f_e, reduces the radial expansion of the beam radius due to the space-charge repulsion. "Gas focusing" is thus a somewhat misleading description of this effect since no net focusing occurs — only a reduction of the divergence of the beam. Still, the effect is very pronounced and may result in a substantial increase of beam current that can be transported through a drift tube of a given length: without the partial charge neutralization the beam would simply blow up and most of the current would be lost to the wall near the entrance of the drift tube.

The great advantage of gas focusing is that one can transport the beam by adding solenoids or magnetic quadupole lenses which, if used alone, would not provide sufficient focusing.

The Gabor Lens

The design concept of a Gabor lens for a positive ion beam is illustrated somewhat simplistically in Fig. 1. A solenoid with axial field strength B provides radial confinement of the electron cloud. An electrode configuration like the one shown in the figure (or a variation thereof) with a positive voltage of V_0 on the center electrode provides the axial confinement for the electrons. As shown in Fig. 1, a second shorter solenoid located on one side of the main solenoid producing a magnetic field in the opposite direction. Gabor placed a ring-shaped thermionic cathode at the midplane ($B = 0$) of the cusp field that is created by this coil arrangement. The advantage of this configuration is that the electrons emitted by the cathode are born in a region with $B = 0$, i.e., their canonical angular momentum, p_θ, is zero, or

$$p_\theta = mr^2 v_\theta + \frac{1}{2} eBr^2 = 0, \tag{5}$$

assuming that $v_\theta = 0$ at the cathode. With such an electron beam, launched from a cathode with $p_\theta = 0$, one can achieve, according to the theory a Brillouin-flow (or "rigid-rotor") equilibrium. This equilibrium state has the property that for a given voltage, V_0, the current, I, and hence the electron density, n_e, is a maximum. The Brillouin-flow condition is usually expressed in terms of the electron plasma frequency, ω_p, and the electron cyclotron frequency, ω_c, as

$$2\omega_p^2 = \omega_c^2, \tag{6}$$

or, in view of $\omega_p^2 = e^2 n_e/\epsilon_0 m_e$, where m_e is the electron mass,

$$n_e = \frac{\epsilon_0}{2m_e} B^2 = 4.86 \times 10^{18} B^2 \tag{7}$$

in MKS units. Thus to achieve an electron density of $n_e = 9.5 \times 10^{15}$ m^{-3} one needs a magnetic field of $B = 4.42 \times 10^{-2} T$, or 442 Gauss. Given the electron density n_e, one readily obtains the radial electric field which, by analogy to Eq. (1), is

$$E_r = \frac{\rho_e}{2\epsilon_0} r = -\frac{en_e}{2\epsilon_0} r. \tag{8}$$

Let us now assume that the electron plasma uniformly fills a cylindrical region of length ℓ and radius a of the Gabor lens, as shown in Fig. 2 (top). A positive ion (H^+) of mass m_i passing through this column will experience a linear focusing force $F_r = -eE_r$ and its motion will be described by the nonrelativistic trajectory equation

$$r'' + k_G^2 r = 0. \tag{9}$$

The Gabor-lens focusing constant in this equation is defined as

$$k_G^2 = \frac{e^2 B^2}{4 m_e m_i v^2} = \frac{B^2 c^2}{8(m_e c^2/e) V_b}, \tag{10}$$

where $V_b = m_i v^2/2e$ is the beam voltage.

An ion entering the Gabor lens at radius r_0 with slope $r_0' = 0$ at $z = 0$ will, according to (9), emerge at $z = \ell$ with a slope

$$r_e' = r_0 k_G \sin k_G \ell. \tag{11}$$

If $k_G \ell \ll \pi/2$ (thin-lens approximation) one can define the focal length of the Gabor lens, f_G, as

$$\frac{1}{f_G} = \frac{r_e'}{r_0} = k_G^2 \ell = 2.2 \times 10^{10} \frac{B^2 \ell}{V_b}. \tag{12}$$

As an example, for $B = 1.0 \times 10^{-2} T$, $\ell = 0.2$ m, and V_b = 100 kV, one finds $f_G = 0.227$ m, which shows the strong focusing capability of the Gabor lens.

In the following we will expand on Gabor's theory to obtain a relation for k_G and f_G that contains the electrode voltage V_0. To achieve this goal let us integrate E_r to obtain the potential difference across the electron column:

$$\Delta V = V_a = -\int_0^a E_r d_r = \frac{en_e a^2}{4\epsilon_0} \tag{13}$$

For the ideal Brillouin flow envisioned by Gabor and discussed above, one finds that there is a maximum current where

$$\Delta V = V_a = \frac{2}{3} V_0. \tag{14}$$

Substituting this result into Eq. (13) and solving for the electron charge density yields

$$\rho_{e,max} = en_{e,max} = \frac{8}{3} \frac{\epsilon_0 V_0}{a^2}. \tag{15}$$

Using this relation between electron density and electrode voltage in the ideal Gabor lens we obtain the following simple expressions for k_G^2 and $1/f_G$:

$$k_G^2 = \frac{2}{3}\frac{V_0}{V_b a^2}, \tag{16}$$

$$\frac{1}{f_G} = \frac{2}{3}\frac{V_0}{V_b}\frac{\ell}{a^2}. \tag{17}$$

For the above example (100 kV H^+ ions) one finds $V_0/a^2 = 3.3 \times 10^6$ V/m^2 to obtain the focusing strength of $f_G = 0.227$ m. Thus, if one chooses $a = 1$ cm, the voltage would be $V_0 = 330$ V.

In practice it is very difficult to achieve the ideal Brillouin flow envisioned by Gabor. Non-zero magnetic field at the cathode, finite electron temperature ($kT_e > 0$), instabilities and other effects may significantly reduce the electron density and hence the focusing strength of the lens. We will therefore introduce a parameter α that measures the (trapping) efficiency with respect to the ideal Brillouin case and rewrite Eqs. (21) and (22) as follows:

$$k_G^2 = \frac{2}{3}\alpha\frac{V_0}{V_b a^2}, \tag{18}$$

$$\frac{1}{f_G} = \frac{2}{3}\alpha\frac{V_0}{V_b}\frac{\ell}{a^2}, \tag{19}$$

where $0 < \alpha < 1$.

The efficiency factor α can also be expressed as

$$\alpha = \frac{n_e}{n_{e,max}}, \tag{20}$$

so that $\alpha = 1$ represents the ideal Brillouin flow of Eq. (15), where the electron density reaches the maximum value, $n_{e,max}$. For the Fermilab experiment, Palkovic estimated[15] that $\alpha = 7 \times 10^{-2}$.

Comparison of Gabor Lens and Electrostatic Quadrupole Doublet

To our knowledge, no systematic comparison has yet been made between the Gabor lens and conventional lenses using applied electromagnetic fields. Since the strongest focusing for ions at low energies is provided by electrostatic quadrupoles (ESQ), we will compare the ESQ lens with the Gabor lens. The focusing strength of an ESQ of length ℓ_Q, aperture radius a_q, and electrode voltage V_q for an ion with voltage V_b is given by

$$k_q^2 = \pm\frac{V_q}{V_b}\frac{1}{a_q^2}, \tag{21}$$

and

$$\frac{1}{f_q} = \pm\frac{V_q}{V_b}\frac{\ell}{a_q^2}. \tag{22}$$

The signs indicate the fact that quadrupole fields are focusing in one plane and defocusing in the other. To obtain a net focusing effect in both directions one needs at least a doublet configuration consisting of a focusing and defocusing lens separated by a distance d. The total net focusing strength of such a doublet is given by

$$\frac{1}{F_q} = \frac{d}{f_q^2} = \left(\frac{V_q}{V_b}\right)^2\frac{d\ell_q^2}{a_q^4}. \tag{23}$$

Let us now consider an ESQ doublet that is connected to the same power supply and occupies the same space as the Gabor lens, as shown in Fig. 2 (bottom). For the doublet the two output terminals of the power supply must be at a polarity of $+V_0/2$ and $-V_0/2$ providing a total voltage of $\Delta V = V_0$ as with the Gabor lens. (For the Gabor-lens operation, the negative terminal is at ground potential.) Thus, we have $V_q = V_0/2$. Furthermore, we will assume that $a_q = a$ and $d = \ell_q = \ell/2$. The focusing strength of the ESQ doublet is then

$$\frac{1}{F_q} = \frac{1}{32}\left(\frac{V_0}{V_b}\right)^2\frac{\ell^3}{a^4}. \tag{24}$$

Comparing (24) with relation (19) for the Gabor lens, we obtain

$$\frac{(1/F_q)}{(1/f_G)} = \frac{3}{64\alpha}\frac{V_0}{V_b}\frac{\ell^2}{a^2} = \frac{9}{128\alpha^2}\frac{\ell}{f_G}. \tag{25}$$

The focusing strength ratio is thus seen to depend inversely on the efficiency as α^{-2} and linearly on the length ℓ divided by the focal length f_G of the Gabor lens. As an example, for $\ell/f_G = 1$ and ideal Brillouin flow, i.e., $\alpha = 1$, we see that the Gabor lens exceeds the focusing strength of an equivalent ESQ doublet by a factor of $128/9 = 14.2$. On the other hand, one finds that the doublet is stronger than the Gabor lens when the efficiency α is less than

$$\alpha = \sqrt{\frac{9}{128}\frac{\ell}{f_G}}, \tag{26}$$

i.e., $\alpha \leq 0.265$ for $\ell = f_G$.

Conclusion

The above theoretical re-examination of Gabor's space-charge lens shows that this lens is capable of providing much stronger focusing than an equivalent electrostatic quadrupole doublet provided that a relatively "cold" electron beam is used which operates close to the ideal Brillouin-flow limit. If the electron densities are significantly below the ideal Brillouin limit, as appears to be the case in the experiments performed so far, the ESQ doublet would be a better choice.

Focusing strength alone is not the only consideration in this comparison. The Gabor lens is only attractive if a uniform electron density and hence force linearity ($E_r \sim n_e r$), can be achieved. The relatively large emittance growth observed in the Fermilab experiment[13] are an indication that strong nonlinear forces are acting on the H^+ beam in that system. Such nonlinear forces could be due to nonuniformity of the electron density to start with. But it could also develop due to the interaction between the space charge of the H^+ beam and the electron plasma. This interaction merits theoretical study if the Gabor lens is to be developed into a practical device. Equally important is the experimental realization of an ideal Brillouin electron beam. With the electrical discharge approach used in experiments so far it appears to be difficult to obtain the desired electron distribution envisioned by Gabor.

Lastly, we note that, in principle, a Gabor lens could also be used for negative-ion beams, e.g., H^-, provided that the electrons are replaced by positrons. Although some work on positron traps is in progress[16], it appears that positron densities achieved so far are many orders of magnitude below the desired levels required for effective beam focusing.

References

1. M. Reiser, C.R. Chang, D. Chernin, and E. Horowitz, SPIE Vol. 873, Microwave and Particle Beam Sources and Propagation (1988), p. 172 ff.

2. B. von Borries and E. Ruska, Z. Physik 76, 649 (1932).

3. D. Gabor, Nature 160, 89 (1947).

4. A.I. Morozov, Dokl. Akad. Nauk. SSR 163, 1363 (1965); Sov Phys. Doklady 10, 775 (1966).

5. S.V. Lebedev and A.I. Morozov, Zh. Tekh. Fiz. 36, 960 (1966); Sov. Phys. Tech. Phys. 11, 707 (1966).

6. V.V. Zhukov, A.I. Morozov and G. Ya. Shchepkin, ZhETF Pis. Red. 9, 24 (1969); JETP Lett. 9, 14 (1969).

7. S.V. Lebedev and A.I. Morozov, Zh. Tekh. Fiz. 44, 1547 (1974); Sov. Phys. Tech. Phys. 19, 966 (1975).

8. A.I. Morozov and S.V. Lebedev, Zh. Tekh. Fiz. 44, 1548 (1974); Sov. Phys. Tech. Phys. 19, 967 (1974).

9. S.V. Lebedev and A.I. Morozov, Zh. Tekh. Fiz. 46, 1571 (1976); Sov. Phys. Tech. Phys. 21, 903 (1976).

10. R. Booth and G.W. Lefevre, Nucl. Instr. & Meth. 151, 143 (1978).

11. H.W. Lefevre and Rex Booth, IEEE Trans. Nucl. Sci. NS-26, 3115 (1979).

12. R.M. Mobley, G. Gammel, and A.W. Maschke, IEEE Trans. Nucl. Sci. NS-26, 3112 (1979).

13. J.A. Palkovic, R. Hren, G. Lee, F.E. Mills, C.W. Schmidt, J. Wendt, and D.E. Young, "Measurements on a Gabor Lens for Neutralizing and Focusing a 30 kV Proton Beam," 1988 Linac Conference , Williamsburg, VA, October 3-7, 1988; to be published in conference proceedings.

14. J.A. Palkovic, F.E. Mills, C. Schmidt, D.E. Young, "Gabor Lens Focusing of a Negative Ion Beam," paper in these proceedings.

15. J.A. Palkovic, private communication.

16. C.M. Surko, M. Leventhal, A. Passner, and F.J. Wysocki, in "Non-Neutral Plasma Physics," AIP Conference Proceedings 175, 75 (1988); editors C.W. Roberson and C.F. Driscoll.

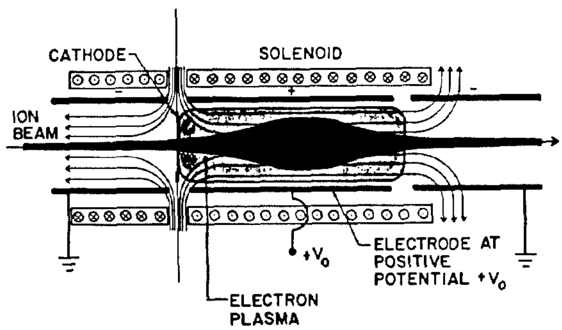

Fig. 1. Schematic of Gabor's space-charge lens showing electron plasma, solenoids with magnetic field lines for radial confinements, and electrode configuration for axial confinement of electrons.

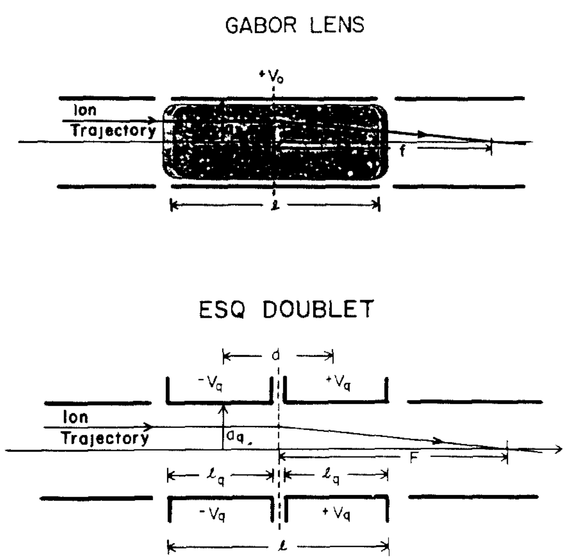

Fig. 2. Gabor lens with electron plasma (top) is replaced by electrostatic quadrupole doublet of same length, aperture, and voltage (bottom) to compare focusing strength for ion beam.