Applications of Modern Filtering to Accelerators

Shou-Yuan Zhang

AGS Department
Brookhaven National Laboratory
Upton, NY 11973

Abstract

In this paper we present the modern filtering formulation, the system modeling, and the applications of the modern filtering to the Accelerator technologies.

I. Introduction

The conventional filtering approaches (including the digital filter) postulate that the useful signals lie in a frequency band, while the noises lie in another. The modern filtering approaches however identify the signals from their statistical properties, and can be applied in the following categories: real time filtering, for control; smoothing, for analysis; and prediction, for control with delay factors. The data acquisition, computer calculation, system modeling, and noise statistics identification are required in the implementation of a modern filtering. For the detailed treatment of the modern filtering and its properties, readers are referred to [1]. For the various applications, readers are referred to [2].

In this paper, we shall present the modern filtering applications in the Accelerator technologies. To make the paper self-contained we shall go through every step for the design of a modern filtering from the filtering formulation, system modeling, to the application. In Section II, we present the modern filtering formulation for the simplest case of the single-variable filtering with a first order system. In Section III, we show the system modeling for the AGS Slow Extracted Beam (SEB) system. In Section IV, we show the SEB filtering. The last section is devoted for the discussion of other applications.

II. Modern Filtering

1. System Model

Consider a dynamic system represented in a state space form,

$$\frac{dx}{dt} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

where u is the system input, y is the output, and x is the state. We consider in this paper only the single-variable system, i.e. u and y are scalars. The dimension of the state depends however on the system order and therefore x is in general a vector. Thus, A, B and C are the matrices with admissible dimensions.

Let the system be sampled uniformly at the sampling time τ. It is known that without the effect of an input the state at the step $k+1$ can be determined from the state at k,

$$x_{k+1} = \phi x_k \tag{2}$$

where k is an integer, ϕ is called the transition matrix that can be calculated from the system state matrix A in (1),

$$\phi = e^{A\tau} \tag{3}$$

* Work performed under the auspices of the U.S. Department of Energy

2. Signal Model

With the transition matrix, the signal affected by both the system input noise w_k and the measurement noise υ_k can be described by

$$x_{k+1} = \phi x_k + Q w_k \tag{4a}$$

$$y_k = C x_k + \upsilon_k \tag{4b}$$

where Q is the coupling matrix from the noise w_k to the state. The statistics of the two noise models is described as follows. Both models are assumed to be the Gaussian white random precesses with zero means, i.e.

$$E\{w_k\} = E\{\upsilon_k\} = 0 \tag{5}$$

Here for simplicity we also assume that w_k is scalar. The two precesses are independent with each other. Their covariances, i.e. the quadratic mean deviations are written as

$$E\{w_k^2\} = \sigma_w^2 \tag{6}$$

$$E\{\upsilon_k^2\} = \sigma_\upsilon^2 \tag{7}$$

They are not necessarily the same.

3. Single-Variable Modern Filtering

Let $\hat{x}_k$ be the expected state at the step k. Then the key function of the modern filtering can be shown as

$$\hat{x}_{k+1} = \phi\hat{x}_k + K_{k+1}(y_{k+1} - C\phi\hat{x}_k) \tag{8}$$

where K_{k+1} is the weighting function. The filtering process is drawn in Fig.1. The system model (1), the signal model (4), and the statistics of the processes $\{w_k\}$ and $\{\upsilon_k\}$ are assumed to be known. Then, the filtering algorithm (8) gives the expectation for the state at the step $k+1$ according to the following informations:

i) One step transitioned state from the last expectation. This is $\phi\hat{x}_k$.

ii) Weighted difference of the measured $k+1$ step system output data y_{k+1} and the expected $k+1$ step output data $C\phi\hat{x}_k$.

The weighting function K_{k+1} plays an important role in the filtering, it is determined by the following equation.

$$K_{k+1} = m_{k+1}C/(C^2 m_{k+1} + \sigma_\upsilon^2) \tag{9}$$

where to provide convenience in writting we have further assumed that the system (1) is a first order system and therefore the state is also a scalar. In (9), we may find that if the measurement noise is high, i.e. the covariance σ_υ^2 is large, then the weighting function is reduced. Thus, the difference term in (8) is less appreciated. Also, the weighting function is affected by the $k+1$ step error covariance m_{k+1} that is described as the follows.

$$m_{k+1} = \phi^2 z_k + Q^2\sigma_w^2 \tag{10}$$

We may see that the error covariance m_{k+1} is in turn determined by

i) The input noise. If the input noise covariance σ_w^2 is large, the error covariance m_{k+1} is large.

ii) The quadratic mean error of the expectation at the step k, z_k, as

CH2669-0/89/0000-1626$01.00©1989 IEEE

$$z_k = (1 - K_k C) m_k \tag{11}$$

that can be calculated from the last step weighting function K_k and the last error covariance m_k.

The equations (8)-(11) consist of the single-variable modern filtering with a first order dynamic system under no effect of input.

4. Justification of Parameters

Using the system model (1) and the signal model (4), the parameters ϕ and C can be determined. Therefore, we only need to justify Q, σ_w^2, and σ_v^2. Let the process $\{x_k\}$ be stationary, and let the processes $\{x_k\}$ and $\{w_k\}$ be independent. Then we have

$$E\{x_k^2\} = E\{x_{k+1}^2\} = E\{w_k^2\} = \sigma_w^2 \tag{12}$$

Taking the covariance from (4a), it is easy to have

$$\phi^2 + Q^2 = 1 \tag{13}$$

Thus, Q can be determined. The input noise covariance σ_w^2 and the measurement noise covariance σ_v^2 should be determined from the test, that will be shown later.

III. AGS Slow Extracted Beam Model

In this section, we discuss the AGS Slow Extracted Beam model that is used for the modern filtering application. The study is aimed at the beam behavior under the spill servo. Therefore, other factors may be overlooked and the filtering is assumed to be useful as the beam extraction observation and regulation.

The slow spill servo system at the AGS is designed for the purpose of maintaining a constant beam extraction during the spill period. The feedback signal is taken from the extracted beam. It is then compared with the desired reference signal. The difference is to drive the main magnet power supply to achieve an expected magnet current slope, that moves the circulating beam radially outward for the extraction. The model is plotted in Fig.2. In the following, we discuss each element in the system. The results presented are from the measurement test [3]. The transfer function T_1 represents the main magnet power supply voltage regulation loop. Under the operation condition, the gain of the multiphase rectifier remains constant, therefore the loop gain remains constant. Since the loop corner frequency ranges from 100 Hz to 200 Hz, that is higher than the frequency range of the possible regulation signal, we denote T_1 as a constant gain element whose gain is about 76, at the test. T_2 is the main magnet power supply filter with the corner frequency 300 Hz, it is therefore represented by a unity gain. The main magnet transfer function can be written as

$$T_3 = \frac{1.54}{s + 0.4} \tag{14}$$

where the input of T_3 is the power supply voltage and the output is the magnet current. T_4 is the spill model that includes a beam detector. The model contains the resonant extraction gain 0.00015, and a 1 ms delay factor. Since the spill is controlled by the main magnet current slope, it also contains an integration factor. Thus, we have

$$T_4 = 0.00015 s e^{-0.001 s} \tag{15}$$

T_5 is also a low pass filter and it is denoted as 100. The spill servo regulator is

$$T_6 = \frac{0.032s + 9.68}{s + 9.68} \tag{16}$$

T_7 is the loop gain control unit that has a gain 2.2 at the time of data acquisition. T_8 is again a filter that is denoted as 0.68.

The system suffers disturbances from the multiphase rectifier, the power line, the spill process, etc. To apply the filtering, we set a simulated system input noise in the front of the main power supply filter, outside the magnet voltage regulation loop T_1. The measurement noise is shown at the output of the spill model. Finally, we note that the spill reference is simply a constant slope, that can be disregarded in this test.

As far as the beam extraction process is concerned, the interested frequency band ranges from 0.1 Hz to 30 Hz. Therefore the delay factor of 1 ms may be disregarded in the filtering, and the magnet denominator s+0.4 and the spill integration factor s may be canceled with each other. After some manipulations, the system transfer function turns out to be

$$T = \frac{0.0231s + 0.2236}{1.084s + 35.09} \tag{17}$$

IV. Modern Filtering Application

To apply the modern filtering to the AGS SEB we first sampled the spill signal (after a low pass filter with the corner frequency at 5 KHz, denoted as T_5). The sampling time τ is 1 ms, and the resolution is 16 bit. Therefore, a data file that contains 1000 data is a 1 second scan on the beam spill.

The next step is to find the system model (1), the signal model (4), and to estimate the noises covariances. A state space form realization of T in (17) can be found as follows.

$$A = -32.37, \quad B = 1, \quad C = -0.48, \quad D = 0.021 \tag{18}$$

Since in this example the direct transmission part of the transfer function, D, is not important, it can be disregarded. The transition function is calculated as $\phi = e^{-32.37 \times 0.001} = 0.968$. From (13), we get Q=0.25. From the data observation, we find that the mean of the total scan is 0.0022, that is very close to zero. The covariance is found to be 0.1034. Note that we are interested in the beam spill, therefore we let the measurement noise covariance σ_v^2 be 0.1 and the input noise covariance σ_w^2 be 0.01. Thus, all parameters required in the equations (8)-(11) have been specified and the filtering algorithm is ready to apply.

The results are shown in Figs.3-5 for three runs. The beam signals are shown on the top. Below, the results of the application of the modern filtering (8)-(11) to the data. On the bottom, the results of the conventional filtering with the corner frequency 4 Hz. Note that for the purpose of low pass, the two filtering approaches result in a similar extent. The accompanied distortion and the delay of the filtered signals however are different. The modern filtering algorithm gives rise better results. Due to the fast convergence of the modern filtering, it can be noticed that the waveforms resulted by using the modern filtering in Figs.3-5 overcome the intrinsic delay of the conventional filtering. In Fig.3, for instance, between the zero time to 200 ms, the two filtered waveforms reached the bottoms that differ more than 50 ms on the time scale, see the blown up picture in Fig.6. The importance of this lies in that not only it provides better observation but it also provides opportunity to a better spill servo control.

It is pointed out in [2] that the major problems in the application of the modern filtering is the system modeling and the computation. In the presented example, we have shown that the model reduction of the sophisticated spill servo system, that is in fact a high order system with various nonlinear, nonuniformly sampled discrete, and time delay (dead time) elements, to a first order linear system satisfies our purpose. For this particular application, since for each sampling there is 1 ms for the signal process and the algorithm is simple, the computation should be implementable for real time calculation, and therefore it can be used for the real time control.

V. Other Applications

In this section, we briefly discuss other applications for the modern filtering.

i) Control of the system with time delay. It is known that, for instance, the AGS spill process has time delays from 1 ms to 3 ms. This single factor introduces additional 36 to 108 degrees phase shift for a 100Hz signal in the correction. By modifying the algorithm (8)-(11), we may use one to three step forward prediction instead of the present time estimation. This can be easily done by, for example for 1 ms delay, using

$$\overline{x}_{k+1} = \phi^2 \hat{x}_k \tag{19}$$

as the $k+1$ step expectation, and using $\{\overline{x}_k\}$ for the control.

ii) Signal Smoothing. This is for a close look and a detailed analysis at a sampled waveform. There are some slight modifications on the presented algorithm that estimates the system state by using both the signal history and future information.

iii) One advantage of the modern filtering is that for a very limited number of information data, it can figure out probably the best estimation value by a simple algorithm. This is particularly useful in the real time control by sampled data. We show an example. In the proposed multiphase rectifier subharmonic ripple reduction scheme [4] the subharmonic ripple information is attained by FFT. These information is used to decide the amplitude and the phase for each component of the correction signal. The data are corrupted by noise, and there is not much time that we can sample many times for the averaging to filter out the noise. A simple algorithm formed by (8)-(11) can then be applied. The result will be reported in [4].

Acknowledgment

The author would like to thank J. Geller for data acquisition and transformation.

References:

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering* , Englewood Cliffs, New Jersey: Prentice-Hall, 1979.

[2] *Special Issue on Applications of Kalman Filtering* , IEEE Transactions on Automatic Control, Vol. AC-28, No.3, 1983.

[3] S. Y. Zhang, A Soukas, A Feltman, and J. Gabusi, *SEB Slow Spill Servo System Measurement* , AGS Studies Report, No.243, BNL, Sep. 1988.

[4] S. Y. Zhang and A. Soukas, *Multiphase Rectifier Subharmonic Ripple Reduction* , in preparation.

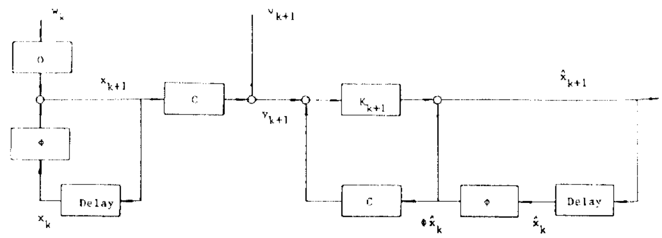

Fig.1

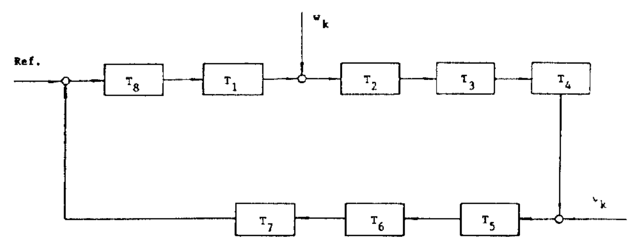

Fig.2

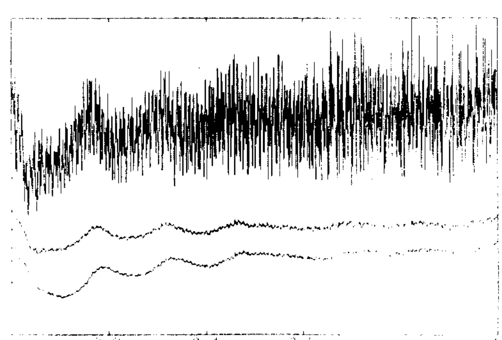

Fig.3

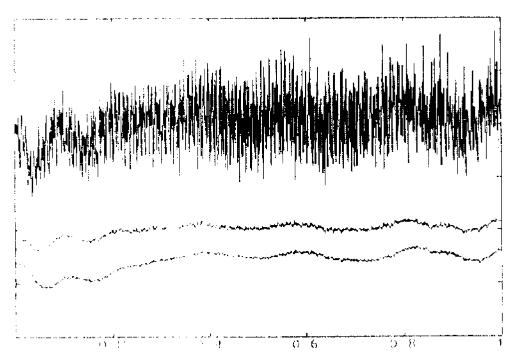

Fig.4

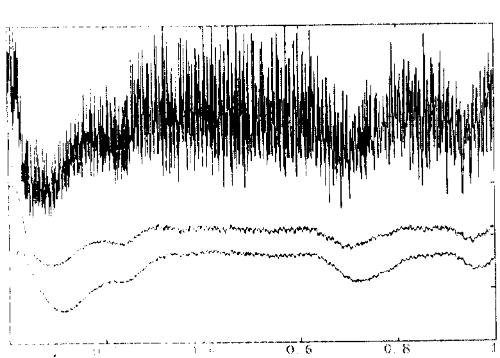

Fig.5

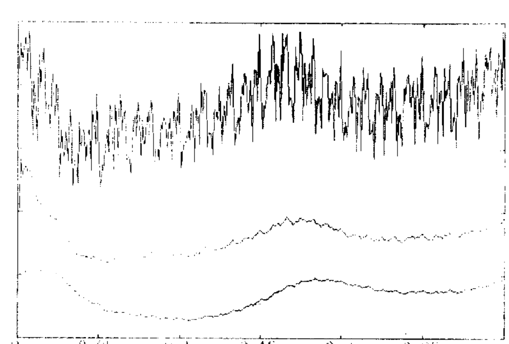

Fig.6