

## PHASE SPACE PROFILE MEASUREMENT USING AN X-RAY PINHOLE CAMERA

A. OGATA, T. MITSUHASHI, T. KATSURA,  
N. YAMAMOTO AND T. KAWAMOTO

*National Laboratory for High Energy Physics  
Oho, Tsukuba-shi, Ibaraki-ken, 305, Japan*

### ABSTRACT

Transverse beam profiles in the real  $(x,y)$  and phase  $(y,y')$  spaces are obtained by an X-ray pinhole camera sensitive to synchrotron radiation in the TRISTAN Accumulation Ring (AR). Vertical Twiss parameters, and the vertical and horizontal emittances are derived from the profiles. A pinhole cannot vertically see the whole beam, because of the small cone angle of the radiation. The following two measurements were tried to solve this problem. The first measurement was made by moving the pinhole vertically. The second was made by using multiple pinhole plate. An image on a fluorescent screen is observed by a CCD camera, digitized and stored. The phase space and the real space profiles are then reconstructed. A non-linear least square program fits the resultant profiles to a two dimensional Gaussian distributions to derive the Twiss parameters and emittances.

### 1. Introduction

We can observe transverse beam profile in a storage ring focusing the synchrotron radiation. Use of the visible radiation is straightforward, but a quantitative analysis is not easy because of the diffraction. Such diffraction is negligibly small in the X-ray region. We have a good old technique to image the X rays; that is a pinhole camera.

There is, however, another problem instead. The opening angle of the synchrotron radiation in the X-ray region is so small that a pinhole cannot vertically accept the radiation from all particles vertically distributed. Two methods are tried to solve this problem in the TRISTAN AR. One is to move the pinhole vertically during a course of measurement.<sup>[1]</sup> The other is to make images simultaneously by several pinholes which are vertically scattered. Both methods enable not only the profile reconstruction but also the calculations of the vertical Twiss parameters and beam emittance.

Section 2 describes the principle. The experimental setups are described in section 3. Results of the measurement are given in section 4.

### 2. Principle

Let us denote the distance between the radiation source and the pinhole  $s$ , the distance between the pinhole and the fluorescent screen  $t$ , and the height of the pinhole  $h$ , as shown in Fig. 1. Let the radiation intensity distribution in the phase space at the source be  $I(y, y')$ , and that at the screen be  $I_1(y, y'; h)$ .

A point  $(y_p, y'_p)$  at the pinhole is transformed to the screen by the relation:

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_p \\ y'_p \end{pmatrix}.$$

Putting  $y_p = h$ , we have

$$y_1 - t y'_1 - h = 0.$$

The relation between  $I$  and  $I_1$  then becomes

$$I_1(y_1, y'_1; h) = \delta(y_1 - t y'_1 - h) I(y, y').$$

Use of the  $\delta$ -function means that we neglect the size of the pinhole.

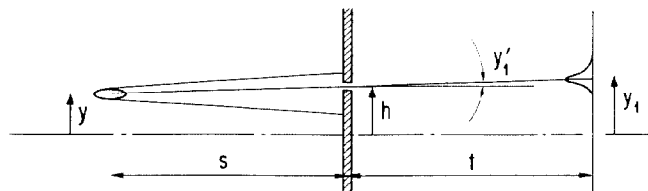


Fig. 1. Geometrical relation among the radiation source, the pinhole and the screen.

A point on the source is transformed to the screen by the relation

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & s+t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}.$$

The image on the screen  $I_1(y_1; h)$  is the integration of  $I_1(y_1, y'_1; h)$  over  $y_1$ ; i.e.,

$$\begin{aligned} I_1(y_1; h) &= \int_{-\infty}^{+\infty} dy'_1 I_1(y_1, y'_1; h) \\ &= \int_{-\infty}^{+\infty} \delta(y_1 - t y'_1 - h) I(y_1 - (s+t)y'_1, y'_1) dy'_1 \\ &= \frac{1}{t} I\left(-\frac{s}{t} y_1 + \frac{s+t}{t} h, \frac{y_1 - h}{t}\right). \end{aligned} \quad (2.1)$$

The lefthandside of the equation is the original datum, which we can regard as the datum on the phase space at a point given in the righthandside.

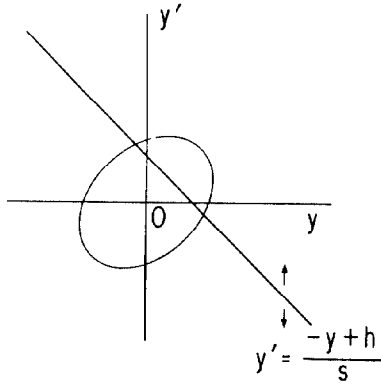


Fig. 2. Synchrotron radiation and a pinhole in the phase space.

The points on the source corresponding to the pinhole are expressed by a line  $y' = (-y + h)/s$  in the phase space. As given in Fig. 2, a pinhole image is a cross section of the synchrotron radiation ellipse along this line. To cover the whole area of the ellipse, we have to move the pinhole.

### 3. Experimental Setups

#### 3.1. SINGLE PINHOLE METHOD

A pinhole was placed at a half way to the imaging fluorescent screen from the source point. A TV camera was used to observe the image. The pinhole is made on a  $2mm$  thick lead plate. It is cone-shaped with minimum diameter of approximately  $10\mu m$ . The lead plate is sandwiched by two water-cooled  $2mm$  thick copper plates. Copper plate absorbers were placed in front of the pinhole to avoid the saturation of the observation system, the number of which is changed according to the beam energy. The pinhole is mounted on a remotely-controllable x-z table.

The fluorescent screen is made from rare earth materials, product of Kasei Optonics K.K.. It is most sensitive to the radiation energy of  $120keV$ . The screen image is observed by a CCD camera, WV-CD60 of Matsushita Tsusin Kogyo K.K.. The AGC function is disconnected to ensure the linearity of the camera. The difference of sensitivity of each pixel was ignored. The image is recorded by the use of a video tape recorder.

The magnification of the imaging system was measured by using a modified trigonometric survey. The distance from the pinhole to the source point was obtained as a crossing point between the two passages, each of which corresponds to a different pinhole position. In the present case, we found that the distance between the light source and the pinhole is  $5.25m$ . The distance between the pinhole and the screen is measured to be  $6.3m$ . The magnification rate is 1.20.

The image data are digitized into  $256ch \times 256ch \times 8bits$ . The intrinsic resolution is  $74\mu m/ch$ . The measurement in the absence of the beam gives the background. A series of data is obtained changing the pinhole height  $h$  with  $50\mu m$  steps. The vertical phase space profile is then calculated according to eq. (2.1).

To obtain the Twiss parameters and emittance, we fit the trajectory of the points deviated by one standard deviation from the peak of the two dimensional distribution to the ellipse:

$$\frac{1}{1 - P^2} \left( \frac{y^2}{\Sigma_y^2} - \frac{2Pyy'}{\Sigma_y \Sigma_{y'}} + \frac{y'^2}{\Sigma_{y'}^2} \right) = 1. \quad (3.1)$$

The Marquardt algorithm is used to make the nonlinear least square fitting. The  $\Sigma_y, \Sigma_{y'}, P$  thus obtained are expressed by the following relations,

$$\begin{aligned} \Sigma_y^2 &= \sigma_y^2, \\ \Sigma_{y'}^2 &= \sigma_{y'}^2 + \sigma_{syn}^2, \\ P^2 &= \frac{\rho^2 \sigma_{y'}^2}{\sigma_{y'}^2 + \sigma_{syn}^2}, \end{aligned} \quad (3.2)$$

assuming that the distribution of the synchrotron radiation is approximately Gaussian with the standard deviation  $\sigma_{syn}$ . Following Green,<sup>[2]</sup> we approximate

$$\gamma \sigma_{syn} = 0.565(\lambda/\lambda_c)^{0.425},$$

where  $\gamma$  is the beam energy in the rest mass unit and  $\lambda_c$  is the wavelength of the critical radiation. We suppose that the X rays have an energy equal to the critical using this equation.

The Twiss parameters and the emittance can be obtained by using relations:

$$\begin{aligned} \sigma_y^2 &= \beta_y \epsilon_y, \\ \sigma_{y'}^2 &= \gamma_y \epsilon_y, \\ \rho^2 &= \alpha_y / (\beta_y \gamma_y). \end{aligned} \quad (3.3)$$

#### 3.2. MULTI-PINHOLE METHOD

The method to move a single pinhole is inadequate in the cases where the beam changes during a measurement due to instabilities, very short lifetime, etc.. One method to solve this problem is to make a number of pinholes.

We have tried it using six pinholes, each of which is similar to the one used at the single pinhole measurement. We found that it is difficult to make pinholes with identical diameter at specified positions. A computer code taking account of the nec-

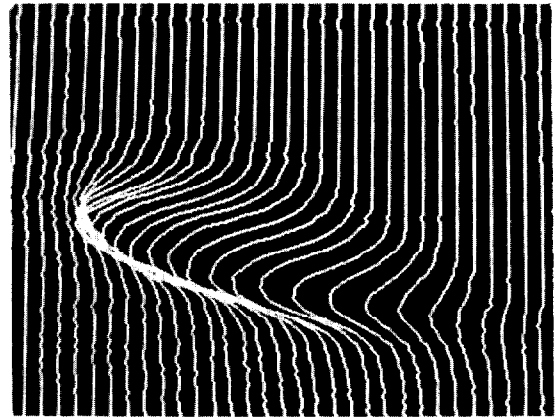


Fig. 3. Vertical histograms taken at a scan of pinhole height.

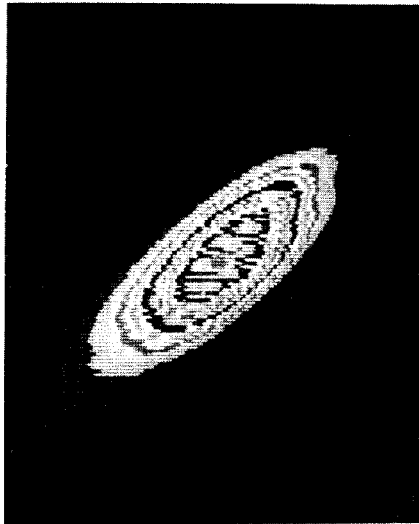


Fig. 4. Phase space profile in  $(y, y')$  plane obtained during  $6.5\text{GeV}$  operation of the TRISTAN AR. This is a photograph of a color-coded CRT display.

essary calibration is under development to analyze fluorescent screen images, which are digitized with better resolution, into  $512\text{ch} \times 512\text{ch}$ .

#### 4. Results and Discussion

An example of a series of data is given in Fig. 3, each line of which is a histogram of radiation intensity in the vertical direction at a certain pinhole height. They are taken during  $6.5\text{GeV}$  operation for synchrotron radiation users. Fig. 4 gives the vertical phase space profile reconstructed from Fig. 3. The  $\alpha_y$  and  $\beta_y$  values given by the profile are  $1.819$  and  $5.73\text{m}$ , respectively. The corresponding design values are  $1.492$  and  $6.914\text{m}$ , respectively.

The vertical emittance derived from the profile is  $0.38 \times 10^{-7}\text{mrad}$ . The horizontal emittance, given as the difference between the designed natural emittance value and the vertical obtained above, becomes  $2.51 \times 10^{-7}\text{mrad}$ . Experimentally obtained horizontal beam size also gives the horizontal emittance, if the beam width due to the dispersion is correctly subtracted. The emittance thus obtained becomes  $1.73 \times 10^{-7}\text{mrad}$  using the designed dispersion value, which is much smaller than given by the previous method.

We have assumed that the opening angle of the synchrotron radiation equals to that with the critical energy,  $11.8\text{keV}$  at  $5\text{GeV}$  of beam energy. The radiation accepted by the screen however has a wide spectrum. In addition, the screen is most sensitive to the energy much higher than the critical. It is necessary to monochromatize the X ray to define the opening angle correctly.

The work using the multi-pinholes has just begun. Fig. 5 shows some images taken for various optics of the AR. The designed pinhole arrangement is also given as Fig. 5(a). Fig. 5(b) gives an image where the x-y coupling is fairly large. We have

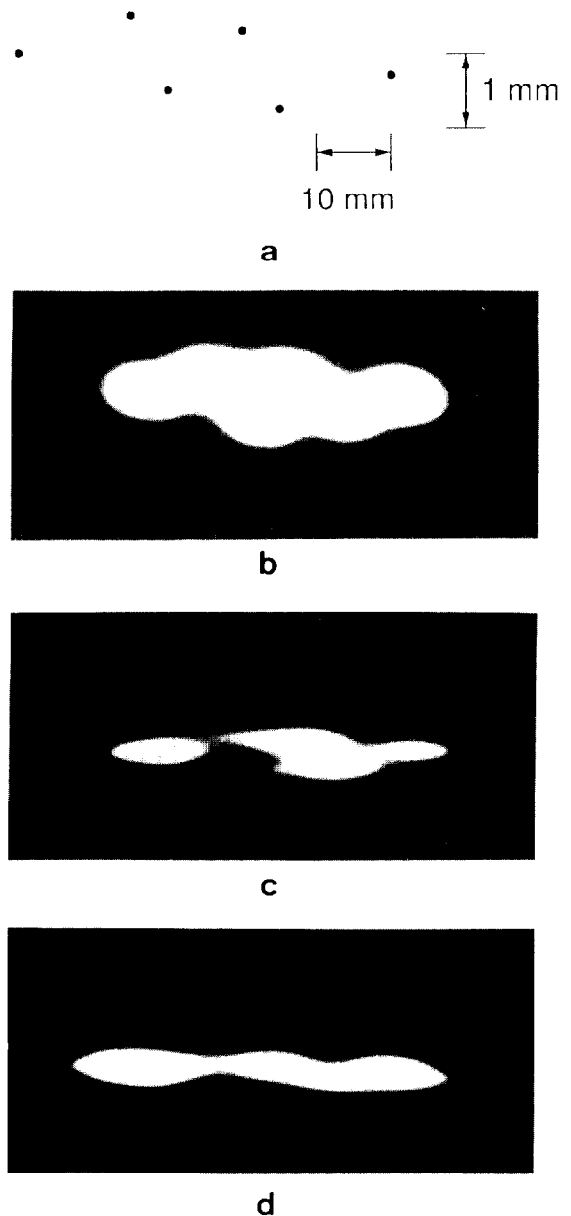


Fig. 5. (a) Designed arrangement of pinholes. (b)-(d) Examples of images made by multi-pinholes. Beam energies are  $5.8\text{GeV}$  in (b),  $5\text{GeV}$  in (c) and  $6.5\text{GeV}$  in (d). Copper filters,  $1.6\text{mm}$  thick in total, are inserted in front of the pinhole at (d).

found that the number of the pinholes is too small for this optics. To the contrary, only four or five pinholes can make images in Fig. 5(c) and (d), where the coupling is much smaller.

#### REFERENCES

1. G. P. Jackson, R. H. Siemann and D. M. Mills, "Vertical Emittance Measurement of the CESR Electron Beam Using Synchrotron Radiation", Proc. 12th. Conf. High-Energy Accelerators, Fermilab, 1983, p217.
2. G. K. Green, "Spectra and Optics of Synchrotron Radiation", BNL50522 and TID4500 (1976).