Multipole Correction in Synchrotrons*

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Abstract

New methods of correcting dynamic nonlinearities resulting from the multipole content of a synchrotron are presented and discussed. In a simplest form, correction elements are placed at the center (C) of the accelerator half-cells as well as near the focusing (F) and defocusing (D) quadrupoles. In a first approximation, the corrector strengths follow Simpson's Rule. The F, C, and D correctors also permit direct control of horizontal, coupled, and vertical motion. For example, second-order sextupole nonlinearities can be corrected with F, C, and D octupoles. Generalizations and variations of the methods are described and applications to the SSC, LHC, AHF, and HERA projects are discussed. Correction by three or more orders of magnitude can be obtained, and simple solutions to a fundamental problem in synchrotrons are demonstrated.

Introduction

Future synchrotrons will use high-field conductor-dominated superconducting dipoles and these magnets have relatively large nonlinear (multipole) fields from persistent-current, conductor-placement, and saturation effects. The greatly increased circumferences of the highest energy machines magnify the nonlinear effects, while forcing the designs toward smaller aperture, more nonlinear magnets. Beam stability demands highly linear motion and therefore linear fields. In the SSC, linear motion tolerances for multipole content are at the 10^{-6} cm^{-n} level while expected strengths of the lower multipoles are near the 10^{-4} level. Correction of b_2, b_3, b_4, and possibly b_6 (sextupole, octupole, 10-pole, and 14-pole) is necessary.[1]

Previously, synchrotron dynamics has been dominated by dipole, quadrupole, and first-order sextupole effects, and corrector elements near focusing (F) and defocusing (D) quadrupoles are adequate. However, correctors near the quadrupoles are completely ineffective for higher orders. Prior to the discoveries described in this paper, it was believed necessary to include internal b_2, b_3, and b_4 trim coils along the length of every dipole for local cancellation of nonlinearities.[2] However, such internal coils greatly complicate the dipoles and may be impractical, particularly in a large, small-aperture synchrotron such as the SSC.

In May 1987, the author considered the possibility of including correctors in the center (C) of accelerator half-cells (see Fig. 1a) and immediately discovered enormous improvements, including the elimination of any need for internal trim coils.[3] In further elaborations, the author and his collaborators have found that the methods are much more general and powerful than the initial evaluations and have firmly connected them with basic physical principles.[4-8] The methods, generalizations and variations, physics bases and applications are described below.

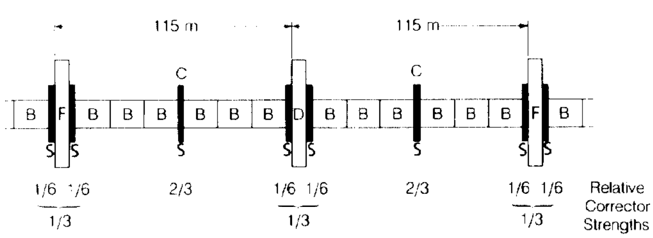

Fig. 1a. A symmetrical cell. The element labels are: B – dipoles, F and D – quadrupoles, S – slots for correctors, C – center slot. Correctors on opposite sides of the quads may be combined in units on either side, and exact symmetry is not necessary.

The Correction Methods

The nonlinear fields in the dipoles may be represented by

$$B_y + iB_x = B_0\left[1 + \sum (b_n + ia_n)(x+iy)^n\right] \quad . \qquad (1)$$

where B_0 is the bending field, and $b_n(s)$ and $a_n(s)$ are the normal and skew multipole components. The amplitude- and momentum-dependent tune shifts $\Delta\nu_x$ and $\Delta\nu_y$ are particularly sensitive measures of nonlinearity, because the phase shifts add coherently around the large synchrotron circumferences. In first order in the terms $b_n(s)$ and $a_n(s)$, only systematic normal multipoles ($\bar{b}_n$) contribute. The first-order tune shifts that are due to b_2, b_3 and b_4 are

$$\begin{aligned}\Delta\nu_x = \langle b_2\beta_x\eta\delta\rangle + \langle\frac{3}{4}b_3\beta_x^2 I_x - \frac{3}{2}b_3\beta_x\beta_y I_y + \frac{3}{2}b_3\beta_x\eta^2\delta^2\rangle \\ + \langle 3b_4\beta_x^2\eta I_x\delta - 6b_4\beta_x\beta_y\eta I_y\delta + 2b_4\beta_x\eta^3\delta^3\rangle \\ \Delta\nu_y = -\langle b_2\beta_y\eta\delta\rangle + \langle\frac{3}{4}b_3\beta_y^2 I_y - \frac{3}{2}b_3\beta_x\beta_y I_x - \frac{3}{2}b_3\beta_y\eta^2\delta^2\rangle \quad (2) \\ + \langle 3b_4\beta_y^2\eta I_y\delta - 6b_4\beta_x\beta_y\eta I_x\delta - 2b_4\beta_y\eta^3\delta^3\rangle \quad .\end{aligned}$$

The $I_i = A_i^2/2\beta_i$ are the actions, the A_i are the amplitudes, and $\delta = \Delta p/p$.

Requiring total tune spread $\Delta\nu \lesssim 0.01$ over the SSC design aperture of $A_x, A_y \lesssim 0.5$ cm and $\delta \lesssim \pm 0.001$ sets tolerances of $\bar{b}_n \lesssim 10^{-6}\text{cm}^{-n}$, an impractical limit demanding active correction (see Table I).

Linearity limits on the allowable "smear" ($\Delta A/A$) of particle orbits set less severe constraints on multipole content; however, some compensation of the random multipoles $b_{2,\text{rms}}$ and possibly $a_{2,\text{rms}}$ may be desirable for the SSC.

Previously, correctors have been placed near the F and D quadrupoles, and such correctors are sufficient for b_0, b_1, and first-order b_2 correction but are ineffective for higher multipoles. Consequently, trim coils within every dipole were believed necessary.

However, a great improvement is obtained by adding correctors to the centers of the half-cells (Fig. 1b), and that improvement is based on fundamental physics. For example, a b_3 tune-shift term from Eq. 2 may be written as

$$\Delta\nu_x = \frac{3I_x}{4L}\left\{\int_0^L b_3(s)ds + \left[\frac{S_{3,F}}{B_0}\beta_x^2(0) + \frac{S_{3,C}}{B_0}\beta_x^2(L/2) + \frac{S_{3,D}}{B_0}\beta_x^2(L)\right]\right\} \quad . \qquad (3)$$

All first-order terms are of similar form. The corrector strengths $S_{n,i}$ are defined by $S_{n,i} \equiv B_{n,i}l_i = -f_{n,i}B_0\bar{b}_n L$, where $B_{n,i}$ and l_i are the corrector lengths and strengths, and L is the half-cell length. The correction is equivalent to approximating integrals of powers of betatron functions by a sum over discrete points. The Simpson's Rule[9] integration solution is $f_\text{F} = f_\text{D} = 1/6$ and $f_\text{C} = 4/6$ per half-cell; it corrects nonlinearities by two orders of magnitude. Optimization about

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TABLE I. First-order correction of b_3, b_4, and second-order b_2 correction. The correction factor is the ratio of uncorrected to corrected $\Delta\nu_{\max}$. The tolerance is the maximum corrected b_n permitted under the criteria that $|\Delta\nu| \leq 0.005$ for all trajectories with $A_x < 0.5$ cm, $A_y < 0.5$ cm, and $|\Delta p/p| \leq 0.001$. The criteria are somewhat stricter than those currently used for the SSC.

Correction Condition	Correction Factor	Tolerance (10^{-4}cm^{-n})
b_3 (Octupole) Correction		
No correction	1.0	0.018
F, D chromatic correction only ($f_F = 0.28$, $f_D = 0.70$)	1.9	0.033
C corrector only ($f_C = 1$)	3.0	0.054
F, C, D Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	93	1.6
F, C, D correction (0.165, 0.66, 0.165)	370	6.7
b_4 (Decupole) Correction		
No correction	1.0	0.029
F, D chromatic correction only (f_F, = 0.24, $f_D = 0.93$)	1.4	0.04
C corrector only ($f_C = 1$)	2.2	0.064
F, C, D Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	31	0.9
F, C, D correction (0.158, 0.663, 0.168)	800	24
Second-order b_2 (Sextupole) Correction		
No correction	1.0	1.2
F, D chromatic b_2 correction only	5.1	2.7
F, C, D chromatic b_2 correction, equal weights ($f_C = 0.5$)	24	5.9
F, C, D chromatic correction, Simpson's Rule ($f_C = 0.667$)	23	5.7
F, D first-order b_2 correction ($f_{C,2} = 0$), and F, C, D octupoles	120	13
F, C, D first-order b_2 correction ($f_{C,2} = 0.5$ to 0.67), and F, C, D octupoles	700	32

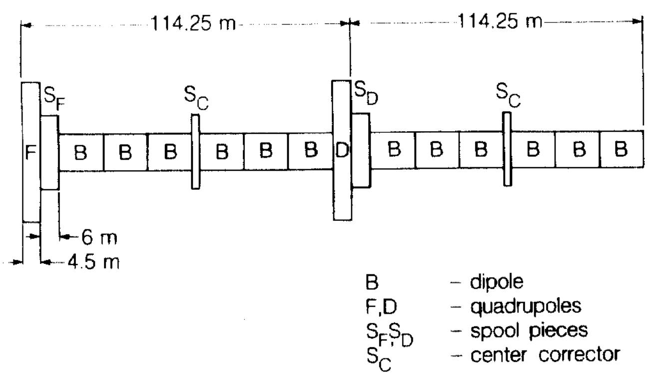

Fig. 1b. An SSC cell with the added center corrector.

that solution permits another order of magnitude reduction (see Table I). Second-order tune shifts and orbit distortions (such as smear) are also reduced by large factors. For instance, the Simpson's Rule correction of b_2 reduces the first-order Collins distortion functions[10] completely to zero at the half-cell level because they can all be represented as integrals of third-order polynomials. The correction is also insensitive to lattice errors.

Forest and Peterson[5] have extended the method to correct random multipole content. The F, C, and D corrector strengths are set by requiring that the lowest-order moments of the dipole multipole content plus the correctors be zero (as in the $b_2 = \bar{b}_2$ Simpson's Rule) on the half-cell level. After consideration of measurement and corrector "binning" errors, the method was found to be as effective and more efficient than placing trim-coil correctors within every dipole.

For both systematic and random correctors, corrections on opposite sides of the thin quadrupoles may be combined in units on either side; there are only two correctors per half-cell.

Superconducting magnets may have a very large b_2 content, and second-order terms may be nonnegligible, even after (F, C, D) b_2 correction. From perturbation theory, second-order sextupole terms have the same form as first-order octupole terms:

$$\Delta\nu_x = aI_x + bI_y + c\delta^2 \quad \text{and} \quad \Delta\nu_y + dI_y + bI_x + c\delta^2 \quad . \quad (4)$$

Second-order b_2 correction can be greatly improved by using the (F, C, D) octupoles.[4] The correction strategy is to use the C octupoles to correct the coupled-motion b and ϵ terms of Eq. (4) and to use the F and D octupoles to correct the horizontal- (a and c) and vertical-motion (d) terms (Fig. 2). In the SSC, choosing F, C, and D strengths in the ratio (1, −2.7, 1) reduces second-order $\Delta\nu$ by $\sim$30×, increasing b_2 tolerances to $\gtrsim 30 \times 10^{-4}$cm^{-2} (see Table I). Fine-tuning of lattice parameters can provide complete second-order cancellation. The use of F, C, and D octupoles to control second-order sextupole nonlinearities is similar to the use of F and D sextupoles to correct quadrupole chromaticity. Figure 2 displays graphically why the (F, C, D) correctors are naturally well matched to the correction task.

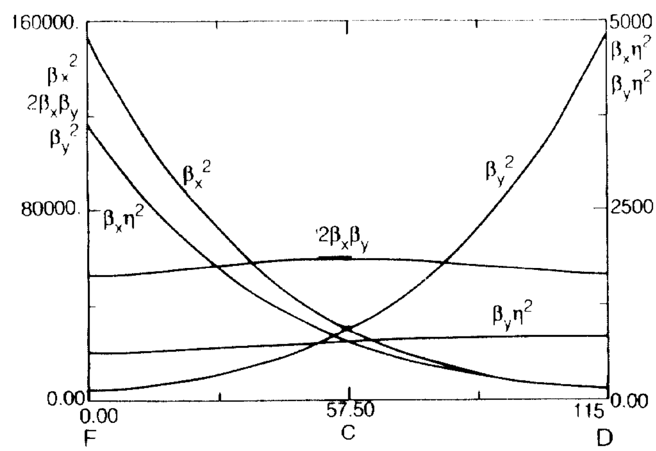

Fig. 2. Octupole $\Delta\nu$ functions on a half-cell. The β_x^2 and $\beta_x\eta^2$ terms are derived from x^4 in the Hamiltonian and are controlled by the F element; the 2 $\beta_x\beta_y$ and $\beta_y\eta^2$ are derived from $(-x^2y^2)$ and are controlled by the C corrector, and $\beta_y{}^2$, from y^4, is controlled by the D element.

Table I summarizes some results on $\Delta\nu$ correction of $\bar{b}_2$, $\bar{b}_3$, and $\bar{b}_4$ in the SSC. Correction by about three orders of magnitude is readily obtained. Field tolerances can be increased from the impractical 10^{-6} level to $\gtrsim 10^{-3}$ cm^{-n}. Similar corrections can be obtained for all nonlinearities in all machines.

Physical Bases of the Correction

The three-point correction (F, C, D) is universally effective because it incorporates basic physical principles; variations that use the same physics also will be effective.

Choice of Simpson's Rule strengths for the multipole corrections means that the correctors provide a general cancellation of all the effects of a multipole content, because the correctors then form an optimal three-point cancelling approximation to the continuous multipole content. Application of this principle on a cell-by-cell basis provides complete quasi-local correction,[5,6] and this principle has been extended to correct in other geometries and to correct varying multipole content. Adequate cancellation does require including correctors in the interior of the half-cells; correctors only near quadrupoles are inadequate.

The F, C, and D correctors are optimal locations for separate control of horizontal-, coupled-, and vertical-motion parameters, and these are precisely the operational observables. The separate tunability has been used above in the correction of b_2^2 effects by F, C, and D octupoles and also can be used in improving correction from initial integration-rule approximations. Previously, F and D correctors provided only the control of horizontal and vertical uncoupled motion; that is inadequate beyond first-order sextupole tune shifts.

The combination of accurate nonlinearity cancellation with the ability to control the observables of motion makes an F, C, and D geometry particularly effective.

Extensions and Elaborations

At the SSC Central Design Group (CDG), variations on corrector configurations are being studied. Variations with more correctors can be more accurate, but not much more for SSC parameters.[8] One interesting variation places two correctors in each half-cell at the locations for Gaussian Quadrature.[6] It shows similar $\Delta\nu$ correction, similar "smear" reduction but inferior tunability to the (F, C, D) correction.

If only first-order effects are important, placing correctors every N^{th} cell at N times greater strengths is equally effective. At SSC parameters, b_3, b_4, and b_6 need only first-order correction; correctors every N = 5 or 10 cells may be adequate.[9] However, closed-orbit sensitivity and higher order effects are then enlarged, particularly near resonances, and must be carefully evaluated.

The strong focusing and large betatron functions in interaction-region quadrupoles magnify their nonlinearities. The present methods can be adapted to provide quasi-local correction of quadrupole fields by orders of magnitude.

Applications to Other Accelerators

While the initial evaluations are of the SSC, the same methods can and should be applied to any synchrotron or transport, with similar improvements. A key ingredient in the present methods, which should be applied generally, is the inclusion of correctors at the half-cell centers (where $\beta_x \cong \beta_y$) for the coupled motion.

An initial evaluation for the LHC has been obtained; the correction is adequate enough to permit weaker focusing and therefore more dipole length, even after allotting space for correctors.[7]

The HERA design was largely completed before the present solutions were known; it includes sextupole-quadrupole trim coils within the dipoles. Recent developments indicate that it is also necessary to correct b_4; an initially proposed solution follows the present methods by placing b_4 coils at the centers of the half-cells.[11] However, placing correctors only near the center provides inadequate $\Delta\nu$ correction and should be supplemented with some correctors near the quadrupoles (Table I).

The RHIC design unfortunately contains correctors only near the quadrupoles, and, hence, those correctors will be ineffective.[12] The design should be modified to include some corrector slots near the half-cell centers.

Butler has applied the present methods to correct large b_2 in an advanced hadron facility; the nonlinearities are effectively removed.[13]

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