NONLINEAR DYNAMICS EXPERIMENT IN THE TEVATRON

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Abstract

Results of the continuing analysis of the nonlinear dynamics experiment E778 are presented. Sixteen special sextupoles introduced nonlinearities in the Tevatron. 'Smear,' which is one of the parameters used to quantify the degree of nonlinearity, was extracted from the data and compared with calculation. Injection efficiency in the presence of nonlinearities was studied. Measurements of the dynamic aperture were performed. The final results in one degree of freedom of the smear, the injection efficiency and the dynamic aperture are presented. Particles captured on nonlinear resonance islands were directly observed and measurements were performed. The capture efficiency was extracted from the data and compared with prediction. The influence of tune modulation on the stability of these islands was investigated. Plans for future measurements are discussed.

Introduction

Experiment E778, performed in the Fermilab Tevatron, studies the nonlinear dynamics of transverse particle oscillations. Though mainly motivated by planning for the Superconducting Super Collider (SSC), E778 is an experimental investigation of the phase-space of nonlinear oscillations in general [1]. In order to simulate the nonlinear features anticipated in the SSC, sixteen special sextupoles were introduced in the Tevatron. The expected phase-space structure is illustrated in Fig. 1, where particles of various amplitudes have been tracked through an accurate representation of the Tevatron.

At sufficiently small amplitudes the motion is still linear to a good approximation and the one-degree-of-freedom trajectories are circles. At larger amplitudes deviation from circularity due to nonlinearities becomes apparent. 'Smear' quantifies the magnitude of the distortion. It is defined as the fractional rms deviation from a circle. At intermediate amplitudes one finds the 5-th order resonance island chain of Fig. 1. Finally, at large amplitudes the regularity is lost and the motion becomes chaotic. The largest regular contour defines the dynamic aperture of the accelerator.

Most of the features of Fig. 1 have been demonstrated and measured. Up to date the studies have been restricted to one degree of freedom. Existing plans for continuation of E778 include studies in both transverse planes.

Smear/Injection Efficiency/Dynamic Aperture

The first purpose of E778 was to determine if smear is predictable from tracking calculations. The second purpose was to find the correlation between smear and accelerator performance measures such as injection efficiency and particle lifetime. Measurements of the dynamic aperture were performed and comparisons with tracking calculations were made.

The final results in one degree of freedom of the smear, the variation of tune with amplitude, and the dynamic aperture are summarized in

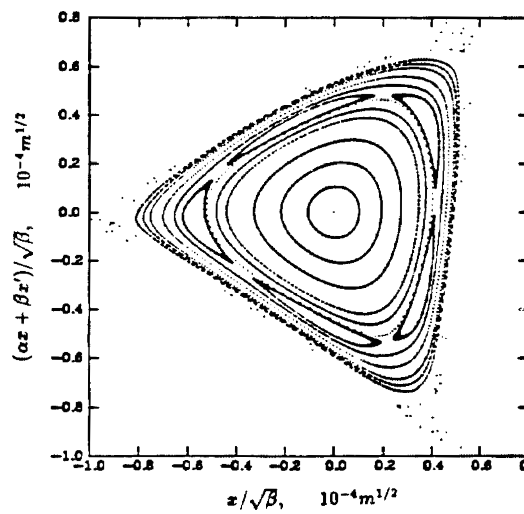

Figure 1: Normalized phase space plot obtained by numerical tracking of particles through an accurate representation of the Tevatron (β = 100 meters).

Fig. 2. The horizontal axis is the product of the sextupole strength, the particle's oscillation amplitude and the beta function at the observation point. Smear and tuneshift as predicted from lowest order perturbation considerations are plotted with solid and dashed lines respectively. The data points are measured values for one of many sets of experimental conditions at a horizontal tune of 19.38. The comparison between calculated and observed smear and tuneshift shows excellent agreement.

The vertical lines correspond to the measured (left) and predicted (right) dynamic aperture. The thickness of the vertical line quantifies the degree of accuracy of the measurements. The tracking calculations were limited to a few hundred turns whereas the data points were extracted after millions of turns. This is considered the major reason for the disagreement between measured and calculated aperture. It should be noted that the dynamic aperture measurements and calculations were performed at a base tune of 19.37.

Finally, the injection experiment suggests that injection diagnosis and correction can function satisfactorily up to the largest sextupole settings for a typical injection error of 1.5 mm. Slow beam loss related to the sextupole excitation was observed right after injection into the Tevatron. However, turning the RF off and cooling the cavities improved the situation dramatically. The underlying mechanism causing these phenomena is yet to be understood.

Resonance Islands

The existence of stable nonlinear resonance islands was demonstrated experimentally by directly observing particles captured into them. Although particle trapping was observed on other resonance is-

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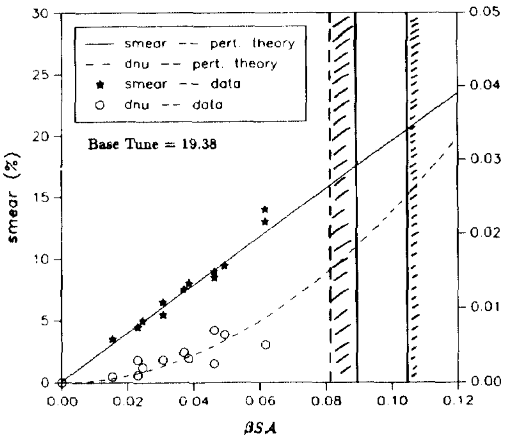

Figure 2: Smear, tuneshift, measured and calculated dynamic aperture plotted as functions of the product of the sextupole strength, the particle's amplitude and the beta function at the observation point. Data points are measured values; curves are predictions.

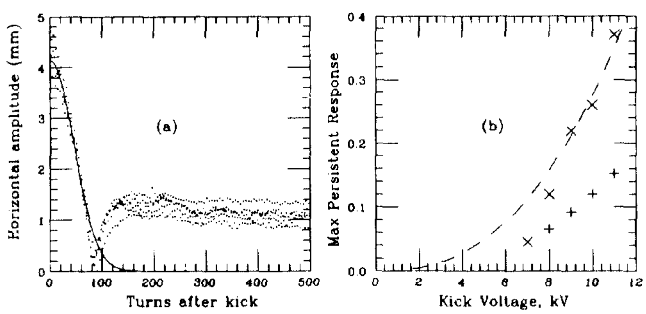

Figure 3: (a) Amplitude of the betatron oscillation vs turn number for an initial kick of 4 mm with sextupoles excited to 25 amperes. The solid line is a Gaussian fit to the data. (b) Maximum persistent response vs kick amplitudes, for experimental data (diagonal crosses) and simulated data (vertical crosses).

lands such as 3/7, 3/8 and 5/13, systematic data taking was restricted to the 2/5 resonance. After the initial Gaussian decoherence, due to the tune spread across the beam, a persistent signal remains, illustrated in Fig. 3(a). The ratio of the persistent signal to the kick amplitude approximately equals the fraction of the beam trapped in the island. The maximum persistent response for different kick amplitudes has been calculated for simulated data and compared with the experimental observations. The result is shown in Fig. 3(b). The dashed line is a fit to the data from a simple theoretical model. The two disagree by roughly a factor of 2. The disagreement is attributed to the sensitivity of these measurements to lattice function errors. It has been demonstrated using tracking calculations (though not observed experimentally) that if one of the sextupoles is turned off then the size of the islands, and hence the captured fraction, increases dramatically.

Successive canonical transformations led to the derivation of a Hamiltonian which describes a system under the action of the isolated resonance 2/5 generated by the sextupole term x^3 [2]:

$$H_5 = \delta I + \frac{1}{2}cI^2 + \epsilon_0 I^{5/2}\cos 5\psi \qquad (1)$$

where δ is the tune distance from the resonance,

$$\delta = Q - \frac{97}{5}. \qquad (2)$$

An expression for the island tune, Q_I, defined as the average number

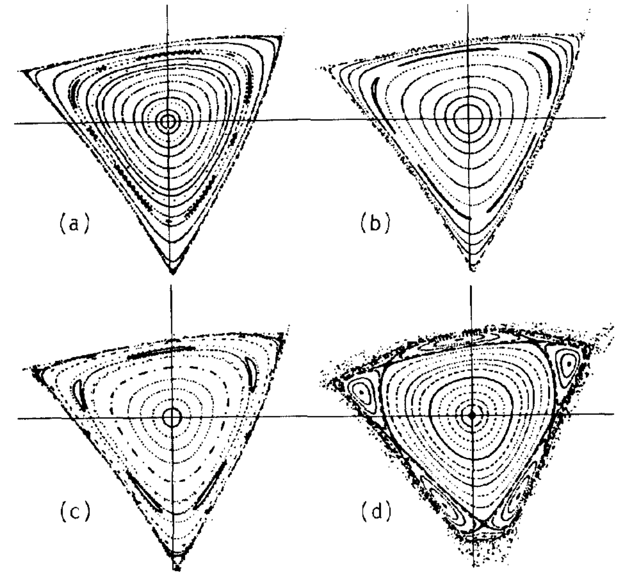

Figure 4: (a) Normalized phase space plot obtained by tracking through a twofold symmetric Tevatron, illustrating the 10-island chain. In the other 3 plots the symmetry has been broken by adding (b) $.001\times 2\pi$, (c) $.01\times 2\pi$, and (d) $.1\times 2\pi$ to the phase of one of the sextupoles.

of revolutions around the island per turn around the accelerator, was derived in terms of the sextupole strength and configuration. For a sextupole excitation of 25 amperes and initial tune of 19.415, the island tune is calculated to be

$$Q_I = 4.9 \times 10^{-5} \mathcal{A}_r^{5/2}, \qquad (3)$$

where $\mathcal{A}_r$ is the resonance amplitude. The coefficient 4.9×10^{-5} is expressed in $mm^{-5/2}$. Single particle tracking calculations were performed to extract Q_I. The result is

$$Q_I = 3.8 \times 10^{-5} \mathcal{A}_r^{5/2}, \qquad (4)$$

in very good agreement. A report on the experimental determination of Q_I can be found elsewhere [3].

If the Tevatron exhibited a twofold symmetry, with the present sextupole configuration ($+ - + - \ldots$), then the 5 island chain would be prohibited. However the 4/10 resonance island chain would be possible. Fig. 4(a) illustrates the phase space as calculated numerically for the E778 sextupole configuration in an ideal ring. The ten islands belong to the same chain. However if the twofold symmetry is broken, for instance by adding $.001 \times 2\pi$ to the phase of one of the sextupoles, then Fig. 4(b) emerges with the five thin islands clearly shown. As we break the symmetry by larger amounts, the size of the islands increases. Hence in Figs 4(c) and 4(d), the phase of the same sextupole has been increased by $.01 \times 2\pi$ and $.1 \times 2\pi$ respectively. Notice that there is a smooth transition from smaller to larger islands.

In any realistic situation the symmetry is broken, often by design, and hence 5-th order resonance is the dominant feature, which is what we observed.

Tune Modulation

The decay mechanism for loss of particles out of the stable islands was investigated as part of E778. This is the subject of the tune modulation experiment [3]. A set of quadrupoles was perturbed by a small sinusoidal current, causing the base tune to modulate according to

$$Q = Q_0 + q\sin(2\pi Q_M t) \qquad (5)$$

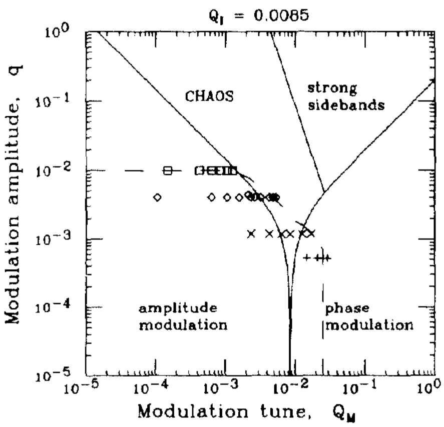

Figure 5: Four dynamical phases in the q, Q_M parameter space, for a typical island tune $Q_I = 0.0085$. The dashed line represents the region accessible to the experiment.

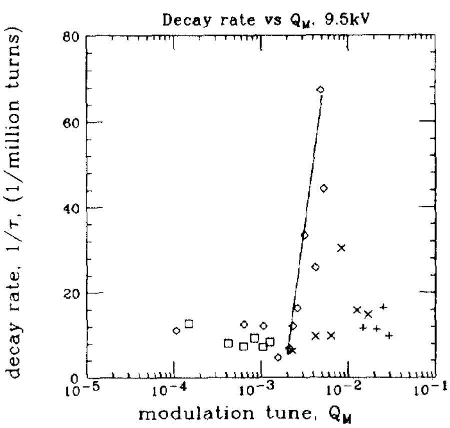

Figure 6: Effect of tune modulation on the persistent signal for the data plotted in the previous plot. The decay rate is significantly larger in the chaotic region.

where q and Q_M are the tune modulation amplitude and tune. In the presence of tune modulation, the isolated resonance Hamiltonian for the 5-th order resonance becomes [4]

$$H_5 = 2\pi q \sin(2\pi Q_M t) I + \frac{1}{2} c I^2 + \epsilon_0 I^{5/2} \cos 5\psi. \qquad (6)$$

It can be shown [3] that at constant q, the action I approaches $(2\pi q)/c$ for small Q_M and zero for large Q_M. The phase ψ on the other hand, goes to zero and q/Q_M for slow and fast modulation respectively. These two circumstances are referred to as the 'amplitude modulation' and 'phase modulation', in Fig. 5, which shows the four dynamical phases in the q, Q_M parameter space.

The other two phases of the q, Q_M 'phase diagram' arise as follows. In a tune modulation system the solutions correspond to a family of resonance sidebands. The sidebands are isolated from each other if their separation in action is larger than the sideband width. Chaos appears if the sidebands overlap, spanning the action range of sidebands of significant size. Sideband overlap is expected if

$$Q_M^{3/4}(5q)^{1/4} < \frac{4}{\pi^{1/4}} Q_I. \qquad (7)$$

This boundary is the nearly vertical solid line in Fig. 5.

Finally Fig. 6 shows the effect of tune modulation on the decay rate of a persistent signal. Data taken at four values of q reaches from the amplitude modulation region just into the phase modulation region, and into the chaos region. The decay rate of the persistent signal increases significantly as the boundary between amplitude modulation and chaos is crossed.

Future Plans

Future plans on the continuation of E778 include the study of the following projects. In one degree of freedom, a further study of the resonance islands will be attempted, both statically — observation of particle trapping and measurements of island width and island tune — and dynamically — exploration of the stability of the islands under tune modulation.

Furthermore the smear measurements will be repeated with different sextupole configurations and hence different values of the smear and the tuneshift. Until now we have worked with only one sextupole configuration and hence one relation between the smear and the amplitude-dependent tuneshift. It is desirable to explore different regions in the smear versus tuneshift space.

With the one-degree-of-freedom study more or less completed, the next step is to proceed to the more realistic, and hence more relevant question of nonlinear behavior in two degrees of freedom. An attempt will be made to repeat the smear and the injection experiment. There are no difficulties expected in performing and analyzing the results of the injection experiment.

The smear experiment in two degrees of freedom will require installation of the vertical kicker and commissioning of the skew sextupoles which are already installed in the Tevatron. The first aim of the smear experiment will be the understanding of the more complicated phase space near a third order resonance such as $\nu_x + 2\nu_y$. A judicious choice of variables will be needed, such that the two degree of freedom phase space can be projected onto two or three dimensions in such a way that all the important information is revealed.

Extraction of the smear and tuneshift parameters from the beam position monitor data will also present some challenge. Coupling between the two planes complicate the situation resulting in a non Gaussian decoherence. The assumption of a Gaussian decoherence was a critical element for the extraction of smear and tuneshift from the one degree of freedom data. In two degrees of freedom, extraction of the same information will require a completely different approach: working in the frequency domain with the Fourier spectra of the signals. A substantial offline effort will have to be put into this subject, before any further conclusions can be drawn.

References

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