

# Measurements of $\beta$ in the TEVATRON and Comparisons with Calculations

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## Abstract

A computer model of the Tevatron based on the tracking code TEVLAT has been developed. The model incorporates the basic lattice structure of the Tevatron, the correction elements and the high order multipoles of the Tevatron dipoles and quadrupoles, as measured at cryogenic temperatures at the Fermilab Magnet Test Facility. The model also includes the nominal closed orbit distortions. Measurements of the lattice amplitude function  $\beta$  will be compared with the values calculated by the model.

## INTRODUCTION

The knowledge of the betatron amplitude function  $\beta$  at specific points in the Tevatron lattice is essential if we are to calculate the luminosity of the collider. It is not practical to measure  $\beta$  at these points so that we need to rely on lattice calculations for our knowledge of  $\beta$ . In order to check the model of the Tevatron used in the calculations we measured the horizontal and vertical  $\beta$  at four locations in the Tevatron lattice. These locations have relative phase advances so that simple  $\beta$  beats could be discovered. The value of  $\beta$  has been measured for 4 different lattices:

1. The fixed target injection lattice.
2. The low  $\beta$  lattice with  $\beta^* \approx 1.0\text{m}$
3. The low  $\beta$  lattice with  $\beta^* \approx 0.5\text{m}$ .
4. The low  $\beta$  lattice with  $\beta^* \approx 0.5\text{m}$  with four of the correction quadrupoles (at A44, A45, B15 and B16) adjusted to match the  $\beta$  functions of the insertion lattice. (The modified  $0.5\text{m } \beta^*$  lattice.)

## EXPERIMENTAL METHOD OF DETERMINING $\beta$

Protons bunches were stored in the Tevatron. The tunes were measured by analyzing the output of the transverse Schottky plates in a spectrum analyzer. Prior to measuring  $\beta$  the horizontal and vertical tunes were separated to avoid complications due to coupling. To determine  $\beta$  the tune of the Tevatron beam was measured as the currents in selected quadrupoles (located in the second correction package spool pieces at F27, F28, F33, F34) were varied. The change in tune  $\Delta\nu$  is given by

$$\Delta\nu = \frac{\beta}{4\pi} \times \int \frac{B'(l)}{[B\rho]} dl$$

where  $B'(l)$  is the gradient in the quadrupole. The values of the  $\beta$  function at the locations of the quadrupoles were determined from the value of the slope, at zero current, of the measured values of the tune vs. the current in the spool pieces.

Measurements were made for the fixed target injection lattice, the  $1\text{m } \beta^*$  lattice and the modified  $0.5\text{m } \beta^*$  lattice in July, 1988. The measurements for the  $0.5\text{m } \beta^*$  lattice were made in March 1989.

## THE MODEL OF THE TEVATRON

The tracking code TEVLAT was used to model the Tevatron. The model incorporates the Tevatron dipoles, quadrupoles and correction quadrupoles. TEVLAT also can include in its calculations:

1. A closed orbit which does not pass through the centers of the magnetic elements.
2. The actual measured (at the Magnet Test Facility (MTF) at Fermilab) high order multipoles for the actual dipoles and quadrupoles in the lattice at the time the measurements of  $\beta$  were made.

3. The measured strengths (again at MTF) of the lattice quadrupole magnets.

It is very important in doing the calculations that we use a description of the lattice which is as accurate as possible. In the case of these measurements the tuning quads at A44, A45, B15 and B16 were on different circuits from the other tuning quads and the tuning quads at E26 and E28 were not working. The lattice description reflected these conditions.

## CALCULATIONS

TEVLAT can calculate the values of the amplitude function  $\beta$  from our knowledge of the elements of the Tevatron lattice in two different ways:

1. TEVLAT is used to calculate the transfer matrix of the lattice and extract from the matrix the values of  $\beta$  at the desired locations.
2. TEVLAT is used to duplicate the experimental procedure. The kick from the quadrupoles in the second correction package spool piece is varied and the change in tune calculated. From the change in tune and the known size of the kick the value of  $\beta$  is calculated.

For the  $1\text{m } \beta^*$  lattice a comparison was made of the two techniques. Both methods gave the same results for  $\beta$ . The method using the transfer matrix was used to calculate the values of  $\beta$  presented here.

## RESULTS and DISCUSSION

The calculations were done for the four different modes of the lattice using the tunes at which the measurements were done. The comparison of the calculated values of  $\beta$  with the measured values can be found in figures 1-4.

Tables I-IV give the results of the calculations<sup>1</sup> along with the measured values. We estimate that the error on the experimental determination of  $\beta$  is  $< 2\text{m}$  independent of  $\beta$ . It must be remarked that there are no parameters to adjust in the calculations. We also note the inclusion in the calculation of the measured strengths of the lattice quads has a totally negligible effect on the results for the fixed target lattice but is significant for the  $1\text{m } \beta^*$  and  $0.5\text{m } \beta^*$  lattices.

It is obvious that the disagreement between the calculated  $\beta$  and the measured value is, in most cases, much greater than the measurement error of  $\approx 2\text{m}$ . This discrepancy is presumably due to our incomplete knowledge of the strengths of the quadrupoles in the lattice, either the lattice quadrupoles or the correction quadrupoles. We can attempt to quantify the discrepancy with a statistical analysis.

We can compute the uncertainty  $\sigma$  in the value of  $\beta$  that would give a statistical agreement between the measured and calculated values. The analysis yields the values for  $\sigma$  of  $5.0\text{m}, 7.7\text{m}, 5.5\text{m}$  and  $8.6\text{m}$  for the four lattices (fixed target injection, the  $1\text{m } \beta^*$ ,  $0.5\text{m } \beta^*$  and the modified  $0.5\text{m } \beta^*$  lattices) which is of course greater than  $2\text{m}$ .

In general this is not too bad. For the fixed target injection lattice and the  $0.5\text{m } \beta^*$  lattice, where the nominal value of  $\beta$  is  $\sim 100\text{m}$  this discrepancy is  $\approx 5\%$ . On the other hand there are known problems in reconciling the measurements of the longitudinal emittance with the flying wire system (which relies on our knowledge of  $\beta$  and the dispersion at the wire positions) with the direct measurements of bunch lengths. This may be due to our inability to calculate with sufficient accuracy  $\beta$  in at all points in the lattice.

<sup>1</sup>The calculated values represent the TEVLAT results using the nominal closed orbit, the measured high order multipoles of the dipoles and quadrupoles and the measured strengths for the lattice quads.

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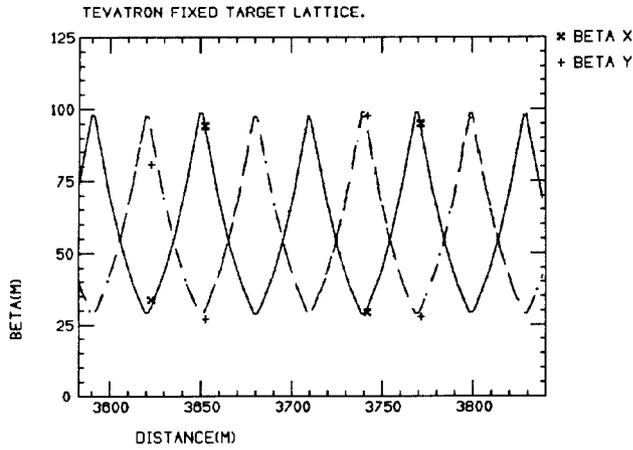


Figure 1: Calculated and measured values of  $\beta$  for the Fixed Target Lattice. The solid curve and the dashed curve are for a linear lattice. The curves with the points include the measured quadrupole strengths and high order multipoles.

Table I.  
**FIXED TARGET LATTICE**

Location	$\beta_x$		$\beta_y$	
	Measured	Calculated	Measured	Calculated
F27	33.7	31.1	80.7	92.0
F28	94.3	93.0	27.1	30.8
F33	29.4	30.9	97.7	91.9
F34	95.0	92.8	27.8	30.9

$$\sum (\beta_{meas} - \beta_{calculated})^2 = 200.2$$

$$\sigma = \sqrt{200.2/8} = 5.0$$

Table II.  
**1m LOW  $\beta$  LATTICE**  
Nominal Low  $\beta$  Quadrupole Strengths

Location	$\beta_x$		$\beta_y$	
	Measured	Calculated	Measured	Calculated
F27	37.0	30.4	91.9	100.3
F28	83.1	79.2	33.7	31.3
F33	39.4	38.1	83.1	98.9
F34	104.1	96.3	41.0	36.2

$$\sum (\beta_{meas} - \beta_{calculated})^2 = 470.3$$

$$\sigma = \sqrt{470.3/8} = 7.7$$

TABLE III.  
**THE 0.5m LOW  $\beta$  LATTICE**  
Nominal Low  $\beta$  Quadrupole Strengths

Location	$\beta_x$		$\beta_y$	
	Measured	Calculated	Measured	Calculated
F27	8.34	9.72	132.0	129.0
F28	74.7	88.2	77.2	75.6
F33	74.4	76.3	35.9	40.4
F34	77.6	72.8	33.0	31.1

$$\sum (\beta_{meas} - \beta_{calculated})^2 = 246.2$$

$$\sigma = \sqrt{246.2/8} = 5.5$$

TABLE IV.  
**THE MODIFIED 0.5m  $\beta$  LATTICE**  
Nominal Low  $\beta$  Quadrupole Strengths

Location	$\beta_x$		$\beta_y$	
	Measured	Calculated	Measured	Calculated
F27	29.3	28.8	68.7	89.6
F28	85.3	85.2	26.1	31.7
F33	28.4	34.8	91.3	94.5
F34	92.0	86.5	25.2	31.9

$$\sum (\beta_{meas} - \beta_{calculated})^2 = 594.8$$

$$\sigma = \sqrt{594.8/8} = 8.6$$

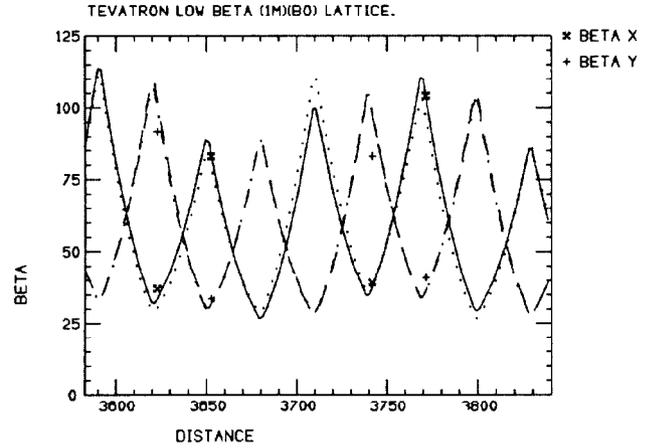


Figure 2: Calculated and measured values of  $\beta$  for the 1m  $\beta^*$  Lattice. The solid curve and the dashed curve are for a linear lattice. The curves with the points include the measured quadrupole strengths and high order multipoles.

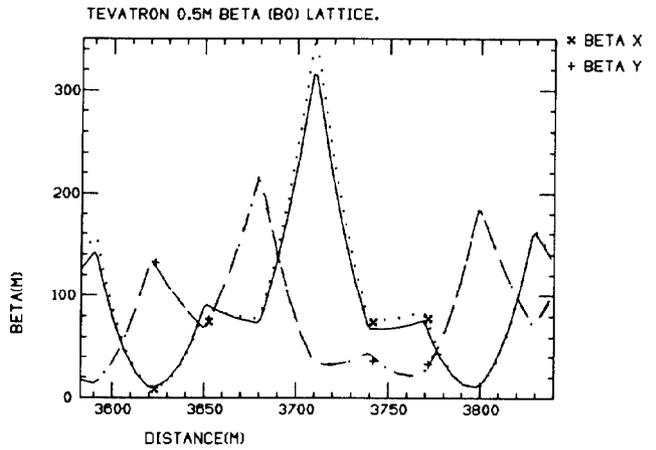


Figure 3: Calculated and measured values of  $\beta$  for the 0.5m  $\beta^*$  Lattice. The solid curve and the dashed curve are for a linear lattice. The curves with the points include the measured quadrupole strengths and high order multipoles.

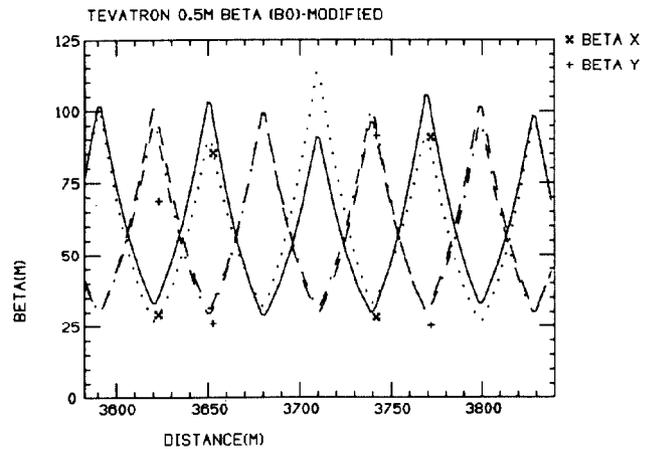


Figure 4: Calculated and measured values of  $\beta$  for the modified 0.5m  $\beta^*$  Lattice. The solid curve and the dashed curve are for a linear lattice. The curves with the points include the measured quadrupole strengths and high order multipoles.