MULTIPLE EQUILIBRIUM STATES OF ELECTRON FLOW IN A MAGNETRON GUN

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Abstract

The equilibrium states of a rotating electron flow inside a cylindrical coaxial diode imbeded in a longitudinal external magnetic field, on condition that magnetic flux is conserved are investigated. It is shown that in the case of a given magnetic field a multiplicity of equilibria can be realized. These equilibria differ from each other by the number of revolutions of electrons around the cathode and the emission current. In the case of a large number of revolutions kinetic electron flow passes over to Brillouin parapotential flow.

Introduction

The initial phase of magnetron operation consists of a magnetically insulated rotating electron flow. Since in this phase the electron flow essentially does not interact with HF waves, we consider a smooth-bore magnetron as a form of coaxial diode imbeded in an external magnetic field.

Two classes of solutions that have received the most attention are Brillouin or parapotential flow [1 - 3], in which electrons move with drift velocities $v_\vartheta \sim E_r/B_z$ parallel to the electrode surface, and the double-stream model [4, 5], in which the electrons move in cycloidal orbits begining and ending on the cathode surface. In the latter case, it is assumed that all electrons perform a single turn along such a trajectory, starting and finishing on the surface of the cathode independent of diode geometry.

It is easily verified that in the plane diode the net force acting on the electrons at the turning point, where transverse mechanical pulse $p_r = 0$, also equals zero. Under such conditions, it is possible to continue from the top of the trajectory symmetrically downward (and this is usually done) or upward to the next turning point etc., i.e. the top of the trajectory is a branching point.

On the other hand, we shall show that in a coaxial diode these degenerate trajectories are absent due to cylindrical geometry. At the points where the net force equals zero electrons have a nonzero transverse momentum, and at the turning point, where $p_r = 0$, a return force acts on electrons towards the cathode, i.e. the trajectory will be of a definite form.

Theoretical model of rotating electron flow

Electron flow fills up the region from $r = R_c$ to $r = r_e \le R_a$, where R_c, r_e, R_a are the radii of cathode, electron flow and anode, respectively. A voltage V_o and homogeneous external longitudinal magnetic field B_{zo} are impressed on the diode.

It is assumed that electron flow consists of electrons rotating and moving in the r-direction. For this case, the distribution function can be written as

$$f = F\cdot\delta(H - H_o)\cdot\delta(P_\vartheta - M)\cdot\delta(P_z - P_o), \qquad (1)$$

where $H = (m_e^2c^4 + c^2p^2)^{1/2} + e\varphi$ – hamiltonian, $P_\vartheta = r(p_\vartheta + eA_\vartheta/c)$ and $P_z = p_z + eA_z/c$ are the full momenta of an electron, F, H_o, P_o, M are constants.

Integrating (1) to calculate charge and current density of electrons, we obtain

$$j_\vartheta = \partial\Psi/\partial(eA_\vartheta/c),\ j_r = \pm eF/r,\ j_z = \partial\Psi/\partial(eA_z/c),$$

$$\varrho = -\partial\Psi/\partial(e\varphi),$$

where

$$\Psi = \frac{eF}{r}\cdot\left[\left(\frac{H - e\varphi}{c}\right)^2 - m_e c^2 - \left(\frac{M}{r} - \frac{e}{c}A_\vartheta\right)^2 - \left(P_o - \frac{e}{c}A_z\right)^2\right]^{1/2}.$$

Choosing the constants according to $H_o = P_o = M = 0$, we can express the energy and mechanical momenta as functions of electrostatic φ and vector A_ϑ potentials as follows

$$m_e c^2\gamma = -e\varphi,\ p_\vartheta = -eA_\vartheta/c,\ p_z = -eA_z/c = 0.$$

It is assumed that on the cathode surface

$$-e\varphi(R_c) = m_e^2 c\ ,\ A_\vartheta(R_c) = 0,$$

and on the anode surface

$$-e\varphi(R_a) = m_e c^2 + eV_o.$$

In this case, the equations which describe the steady state solutions are

$$\begin{cases} x_c \le x \le x_e \\ \dfrac{1}{x}\dfrac{d}{dx}x\dfrac{d\gamma}{dx} = \dfrac{F}{x}\dfrac{\gamma}{\sqrt{\gamma^2 - A^2 - 1}}, \\ \dfrac{d}{dx}\dfrac{1}{x}\dfrac{d}{dx}xA = \dfrac{F}{x}\dfrac{A}{\sqrt{\gamma^2 - A^2 - 1}}, \end{cases} \quad \begin{cases} x_e \le x \le 1 \\ \dfrac{1}{x}\dfrac{d}{dx}x\dfrac{d\gamma}{dx} = 0, \\ \dfrac{d}{dx}\dfrac{1}{x}\dfrac{d}{dx}xA = 0, \end{cases} \qquad (2)$$

where $x = r/R_a$, $A = eA_\vartheta/m_e c^2$, and the appropriate boundary conditions are

$$\gamma(x_c) = 1,\ \gamma(1) = \gamma_o = 1 + eV_o/m_e c^2,\ d\gamma/dx(x_c) = 0, \qquad (3)$$
$$A(x_c) = 0,\ A(1) = A_o.$$

If the duration of the electron pulse is much less than the diffusion time of magnetic field through the electrodes, then image currents are generated in the anode and the flux of magnetic field Φ inside the anode-cathode gap is conserved. Since $B_z = (1/r)d(rA_\vartheta)/dr$,

$$\Phi = 2\cdot\pi\cdot\int_{R_c}^{R_a} B_z r dr = \pi B_{zo}(R_a^2 - R_c^2) = const = 2\pi R_a A_o.$$

This equation gives the boundary condition for A_o

$$A_o = (eB_{zo}R_a/2m_e c^2)\cdot(1 - R_c^2/R_a^2). \qquad (4)$$

Main results

Analysis of the solution of eq. (2) under the conditions (3, 4) shows that for fixed values of B_{zo} and V_o there exists a multiplicity of equilibrium states, differing as to structure of electron trajectories and emission current density.

In Fig. 1,a two dependences of radial momentum of electrons on radius are presented. The first (1) is for the case of $B_{zo} = B_c$ (B_c is the critical field of magnetic insulation). The second (2) is for the case of $B_{zo} = B_1 > B_c$. Provided $B_{zo} = B_1$, the net force and the value of p_r are exactly equal to zero at the turning point. If $B_{zo} > B_1$ solutions with one maximum of p_r (k=1) do not exist and only solutions with several maxima can exist. In Fig. 1,b a more complicated trajectory (k = 8) is shown at the same magnetic field $B_{zo} > B_1$.

For the case k = 1, the displacement of electrons around cathode is less than 2π and for the case k = 8 the electrons perform more than 4 revolutions (Fig. 1,c,d), i.e., electrons stay inside the anode-cathode gap for a longer time-interval in the last case. Therefore the emission current has to be less to satisfy the condition of space-charge flow on the cathode surface.

It is clear that for the case k >> 1 the structure of electron flow approaches that of Brillouin flow. This is attested by the decrease of p_r with increasing k (note the scales in Fig. 1,a,b). That is, the full energy of electrons is distributed between kinetic energy of rotation and potential energy while the energy of radial motion decreases.

The variations of the radius of electron flow and nondimensional current density on the cathode with external magnetic field are illustrated in Fig.2 for different values of k and $\gamma_o = 3$. A multiplicity of equilibria is possible at a given magnetic field, differing as to values of k and emission current. It is instructive to compare them with that of the Brillouin equilibrium [6]. The latter is shown by the heavy line. All the kinetic solutions lay above and tend to the parapotential solution as k increases. Here, kinetic flow passes over to Brillouin flow.

It follows from these results that to choose the equilibrium it is necessary to single out the value of emission current, which depends on the history of a transition process, real emission capability etc.

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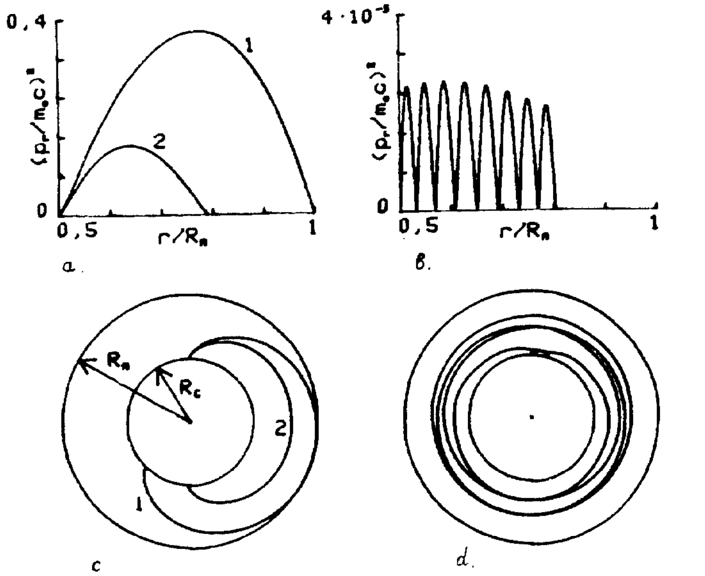

Fig. 1. The dependences of radial momentum of electrons (a) for the case $B_{zo} = B_c$ (1) and $B_{zo} > B_c > B_1$ (2) and the corresponding trajectories in r-ϑ plane (c). More complicated dependences (b, d) are for the case $B_{zo} > B_1$.

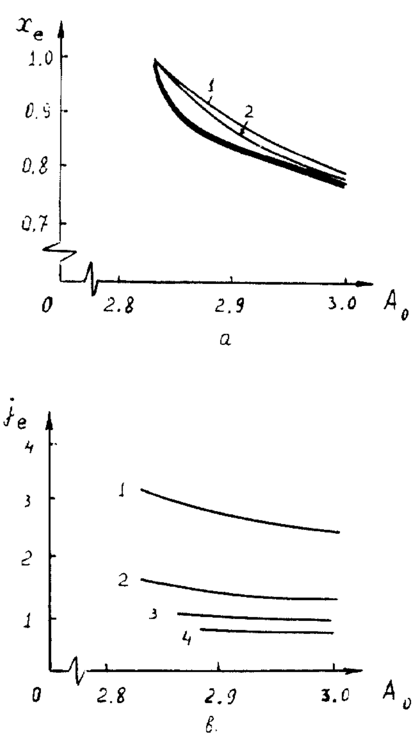

Fig. 2. The dependences of radii of electron flows on the external magnetic field for different values of k (a) and the corresponding dependences of emission current (in arbitrary units) (b).