# STUDIES ON MAGNET SHUFFLING FOR RHIC*

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## Abstract

Shuffling of arc dipoles is performed on realistic RHIC lattices; $\Delta\beta/\beta$, $\Delta X_p/\sqrt{\beta_x}$, and $Y_p/\sqrt{\beta_y}$ from random $a_1$ and $b_1$ multipoles are reduced to acceptable levels. Even before shuffling, the contribution from the arc dipoles is less than the contribution from the insertion magnets. In the insertions, the contribution from the random $b_1$'s is removed by the independent powering of all insertion quadrupoles; the contribution of random $a_1$'s to coupling will be corrected with skew quadrupole correction elements.

## Introduction

RHIC is a separated function, heavy ion storage accelerator that utilizes superconducting dipoles and quadrupoles. It consists of two separate rings that intersect at six equally spaced locations. Each ring has three identical superperiods consisting of an inner arc with 12 cells, an IN-to-OUT insertion, an outer arc of 12 cells, and an OUT-to-IN insertion. Each cell has a phase advance of nearly 88.0°, and each insertion has a phase advance of 667°. The nominal tune is $\nu_x$=28.825 and $\nu_y$=28.823; provision is made to change the tune by ±0.5 units by changing both the phase advance per cell and the phase advance across the insertions.

The scheme for correcting chromaticity consists of six families of sextupoles -- two focusing families and one defocusing family in the inner arcs and one focusing family and two defocusing families in the outer arcs. The strengths of the two focusing families in the inner arcs and two defocusing families in the outer arcs have been selected to minimize the dependence of tune on momentum. Even without the random $a_1$ and $b_1$ errors, a pronounced $\Delta\beta/\beta$ remains when $\Delta P/P\neq 0$; the shuffling studies have been made at $\Delta P/P=0$ where the $\Delta\beta/\beta$ is small.

The multipoles used are random components due to rms construction tolerances of 0.002 inch in angular orientation, subtended angle, inner radius, and radial thickness of the current blocks. Each of these tolerances is independent of the others, so the random multipoles are assumed to be uncorrelated. The multipoles are listed in Table 1.

| Order | Dipoles[1] | | Quadrupoles[2] |
|---|---|---|---|
| n | $\sigma b_n'$ | $\sigma a_n'$ | $\sigma b_n'=\sigma a_n'$ |
| 1 | 2.1 | 4.3 | 4.0 |
| 2 | 4.6 | 1.3 | 3.7 |
| 3 | 1.3 | 2.2 | 2.3 |
| 4 | 2.2 | 0.57 | 2.2 |
| 5 | 0.53 | 0.91 | 1.2 |
| 6 | 0.83 | 0.23 | 0.85 |
| 7 | 0.18 | 0.34 | 0.60 |
| 8 | 0.28 | 0.084 | 0.41 |
| 9 | 0.061 | 0.12 | 0.27 |
| 10 | 0.093 | 0.029 | 0.18 |
| 11 | 0.020 | 0.039 | |

Table 1 Random multipoles in primed units for arc dipoles and quadrupoles -- the multipoles are expressed in terms of $B_o$ for both the dipoles and the quadrupoles. Units are $10^{-4}$ at 25mm reference radius.

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## Magnet Shuffling

Studies by S. Ohnuma[3-5] on magnet shuffling for RHIC have been reported previously. Shuffling of arc dipoles on random $a_1$ and $b_1$ multipoles resulted in reductions of $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, $\Delta X_p/\sqrt{\beta_x}$, and $Y_p/\sqrt{\beta_y}$ by factors of 4 to 5 compared with values expected for a random distribution of these multipoles. Dipoles were shuffled on a "local" basis of four or six cells; the equations for variations produced from a localized region containing M dipoles are:

$$(\Delta\beta/\beta)-i(\Delta\alpha-\alpha\,\Delta\beta/\beta) = \frac{-e^{i2\pi\nu}}{2\sin(2\pi\nu)}\sum_{k=1}^{M}\left(\theta\,\beta\,b_1\right)_k e^{i2\psi_k} \qquad (1)$$

$$\Delta X_p/\sqrt{\beta_x} + i\,\sqrt{\beta_x}\,(\Delta X_p' + \alpha_x\,\Delta X_p/\beta_x) = \frac{-e^{i\pi\nu(x)}}{2\sin(\pi\nu(x))}\sum_{k=1}^{M}\left(\theta\,\sqrt{\beta_x}\,X_p\,b_1\right)_k e^{i\psi_k(x)} \qquad (2)$$

$$\Delta Y_p/\sqrt{\beta_y} + i\,\sqrt{\beta_y}\,(\Delta Y_p' + \alpha_y\,\Delta Y_p/\beta_y) = \frac{-e^{i\pi\nu(y)}}{2\sin(\pi\nu(y))}\sum_{k=1}^{M}\left(\theta\,\sqrt{\beta_y}\,X_p\,a_1\right)_k e^{i\psi_k(y)} \qquad (3)$$

Equation (1) applies to the x and y planes when the appropriate $\alpha$, $\beta$, $\nu$, and $\psi$ are used; $\theta = \ell/\rho$ is the bending angle of each dipole. Since $\theta$ and $\beta$ are the same for all dipoles, the contribution from the M dipoles is zero outside the region when the following conditions are satisfied:

$$\sum_{k=1}^{M} b_1\,e^{i2\psi(x)} = 0\,, \qquad \sum_{k=1}^{M} b_1\,e^{i2\psi(y)} = 0\,,$$
$$\sum_{k=1}^{M} b_1\,e^{i\psi(x)} = 0\,, \quad\text{and}\quad \sum_{k=1}^{M} a_1\,e^{i\psi(y)} = 0\,. \qquad (4)$$

The present study has been made on realistic RHIC lattices with the aid of a tracking program. The kicks from multipoles are evaluated at the center of quadrupoles and at the center and both ends of dipoles. The same multipoles are used for all kicks in a dipole; the contribution to the total kick is assigned with a weight of 2/3 at the center and 1/6 at each end. Although several recipes for placing dipoles have been used with moderate success, the use of a recipe has been replaced with an auxilliary computer program that monitors the vectors of Eqn 4. Magnets selection is made so the four conditions are satisfied approximately throughout all six arcs of RHIC.

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PATRICIA[8] uses a library of multipoles for different types of magnets to generate multipole coefficients for each magnetic element. Random multipoles of order $1 \le n \le 11$ are generated according to a Gaussian distribution that is truncated at $\pm 3\sigma$. These multipoles are stored internally and are used throughout the tracking run. Magnet shuffling has been implemented by adding a provision to write the contents of the multipole array to an external file, to process it with a separate shuffling program, and to read and use the new file.

The multipole file contains multipole information through order m for each magnetic element. Each line includes the name of the element, its location in the element library, its location in the array of multipole elements, the order n of the multipole, and the value of $b_n$ and $a_n$. The identity of a magnet is maintained throughout the shuffling process; all multipoles of a magnet are moved when it is assigned to a new location.

The shuffling program processes the input multipole file according to a directions specified by the user. The file is scanned to locate all lines satisfying the input specifications; the information from each of these lines is stored for further processing. Also specified are the number of magnets required (ISHUF) plus the number of spare magnets (ISPARE) that are available. In the present study, ISHUF is 24 (one arc of RHIC), and ISPARE is also 24 -- the spares consist of the next ISPARE arc dipoles in the multipole file; the shuffling program accepts ISPARE in the range $0 \le$ ISPARE $\le 24$. Groups of ISHUF+ISPARE magnets are first ordered according to the descending value of the selected multipole, and the location of each dipole in this list is used as an intermediate identifier. Final assignment of magnet location can be made with a recipe, however an alternate method of assignment is based on the vectors of Eqn 4 and is discussed below.

Selection of locations for particular magnets is complicated by the requirement of reducing the effects of both the $a_1$'s and $b_1$'s at the same time. After the shuffling program has arranged the ISHUF+ISPARE magnets in terms of the decreasing value of the $b_1$ multipole, the $a_1, b_1$ multipoles are written to a short file that is easy to manipulate. A third program is used to aid in the selecting magnet placement. The phase advance $\Delta\psi_x$ and $\Delta\psi_y$ between elements of the cells and across the insertions is used to monitor the propagation of the following vectors:

$$\sum b_1\, e^{i\psi_x},\quad \sum b_1\, e^{i2\psi_x},\quad \sum b_1\, e^{i2\psi_y},\ \text{and}\quad \sum a_1\, e^{i\psi_y}.$$

Once a dipole is selected for the first magnet slot, the program is run to determine the moduli as well as the real component of the four vectors at the next magnet slot. The next dipole is selected to reduce the real component of as many as possible of the vectors; priority is given to vectors whose moduli are large. Selection is made to keep the moduli of all vectors less than $1.5\sigma$. In general, the moduli of the vectors become zero at least once in an arc, but usually this does not happen simultaneously. However, an effort is made to make the moduli of all four vectors as small as possible at the end of each arc.

When an acceptable assignment is obtained for all dipoles of an arc, it is used in the shuffling program, and the selected dipoles are moved to their new locations. The dipoles not used plus the next 24 arc dipoles of the multipole file form the inventory for the next arc. The process of running the shuffling program to; create a new magnet inventory, arrange the dipoles in descending order of the specified multipole, assign dipoles to particular positions, and remove the assigned dipoles from the inventory is repeated until dipoles have been selected for all six arcs. The shuffling program is then run to generate a final multipole file that can be used by PATRICIA; the results discussed in the rest of the paper have been obtained with such a file.

## Results

Evaluation of $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, and $\Delta X_p/\sqrt{\beta_x}$ is performed by PATRICIA in three steps: 1). the linear lattice is established, 2). the linear part of the multipoles is included, and the average values of $\beta_x$, $\beta_y$, and $X_p$ are determined separately at the focusing and defocusing quadrupoles of the inner and of the outer arcs, and 3). these averages are used to evaluate the rms values of $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, and $\Delta X_p/\sqrt{\beta_x}$ over all of the arc quadrupoles -- $Y_p/\sqrt{\beta_y}$ is not available from the lattice functions.

Although the shuffled multipole file contains multipoles of orders $1 \le n \le 11$ for all elements, scaling variables in the input permit setting multipoles to zero in selected elements without changing the order in other elements. Shuffling has been done when only the random $a_1, b_1$ multipoles in the arc dipoles are nonzero; there are no multipoles in other elements. Results of this study appear in Table 2.

| | $\Delta\beta/\beta)_x$ | $\Delta\beta/\beta)_y$ | $\Delta X_p/\sqrt{\beta_x}$ |
|---|---|---|---|
| $a_1=b_1=0$ | 1.70E-4 | 2.37E-4 | 3.63E-3 |
| $a_1, b_1\ne 0$ (Random) | 3.22E-2 | 3.22E-2 | 1.09E-2 |
| $a_1, b_1\ne 0$ (Unshuffled) | 3.03E-2 | 3.20E-2 | 6.9 E-3 |
| $a_1, b_1\ne 0$ (Shuffled*) | 4.96E-3 | 5.04E-3 | 3.96E-3 |
| $a_1, b_1\ne 0$ (Ohnuma+) | 7.4 E-3 | 6.9 E-3 | 2.1 E-3 |

Table 2 rms values of $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, and $\Delta X_p/\sqrt{\beta_x}$ at the arc quadrupoles. (*) indicates the present study; (+)indicates Ohnuma's "Shuffled" results.

The values of $\Delta\beta/\beta)_x$ and $\Delta\beta/\beta)_y$ for the unshuffled case of the present study are in good agreement with the expected values from a completely random distribution, but $\Delta X_p/\sqrt{\beta_x}$ is smaller than expected. Of interest is Ohnuma's best value of $\Delta X_p/\sqrt{\beta_x}$ = 2.1E-03; this is smaller than the value obtained in the present study when $a_1=b_1=0$. The printout of the RHIC lattice functions shows a wave in the $X_p$ function in the arcs that is responsible for the large residual $\Delta X_p/\sqrt{\beta_x}$; this large value masks the reduction of $\Delta X_p/\sqrt{\beta_x}$ from magnet shuffling.

In addition to determinations made for the arcs, the rms values of $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, $\Delta X_p/\sqrt{\beta_x}$, and $Y_p/\sqrt{\beta_y}$ have been determined from lattice functions and small amplitude tracking at all six crossing points. The displacements of the phase plots for tracking at $\Delta P/P$ = 0 and 0.05% are used to determine $X_p$ and $Y_p$. Results are tabulated in Table 3.

| | $\Delta\beta/\beta)_x$ | $\Delta\beta/\beta)_y$ | $\Delta X_p/\sqrt{\beta_x}$ | $Y_p/\sqrt{\beta_y}$ |
|---|---|---|---|---|
| $a_1=b_1=0$ | 3.5E-5 | 0.0 | 3.4E-4 | |
| $a_1, b_1\ne 0$ (F) | 4.6E-3 | 3.4E-3 | 7.3E-4 | |
| $a_1, b_1\ne 0$ (T) | | | 8.6E-4 | 2.25E-3 |

Table 3 rms values of $\Delta\beta/\beta$, $\Delta X_p/\sqrt{\beta_x}$, and $Y_p/\sqrt{\beta_y}$ at the crossing points for the $\beta^*$ = 3m lattice. (F) denotes lattice functions; (T) denotes tracking.

| Elem | $\beta^*$(m) | $\Delta\beta/\beta)_x$ | $\Delta\beta/\beta)_y$ | $\Delta X_p/\sqrt{\beta_x}$ |
|---|---|---|---|---|
| B | 2 | 4.98E-3 | 5.22E-3 | 3.76E-3 |
| B | 3 | 4.96E-3 | 5.04E-3 | 3.96E-3 |
| B | 6 | 9.39E-3 | 5.09E-3 | 4.04E-3 |
| All | 2 | 1.02E-1 | 8.93E-2 | 8.13E-3 |
| All | 3 | 8.42E-2 | 8.49E-2 | 4.77E-3 |
| All | 6 | 8.03E-2 | 3.74E-2 | 4.95E-3 |

Table 4 rms values for random $a_1$ and $b_1$ multipoles. "B" denotes arc dipoles; "All" denotes all elements.

## Arc Quadrupoles and Insertion Magnets

Inclusion of random $a_1$ and $b_1$ in all magnetic elements greatly increases $\Delta\beta/\beta)_x$ and $\Delta\beta/\beta)_y$; the $a_1$ and $b_1$ multipoles for all remaining elements were "turned on" with the scaling variables mentioned previously -- the assignment of multipoles to the arc dipoles were unchanged. Results for lattices with $\beta^*$ = 2m, 3m, and 6m are tabulated in Table 4.

The quadrupole triplet, Q1-Q3, makes the largest contribution to $\Delta\beta/\beta$, but dipoles BS1 and BS2 for dispersion suppression and BC1 and BC2 for steering are also important. Contributions to $\Delta\beta/\beta$ and $X_p/\sqrt{\beta_x}$ are listed in Table 5 when random $a_1$ and $b_1$ multipoles are nonzero in selected elements. Comparison of Item 1 and Item 2 indicates the contribution from the arc quadrupoles. Additional random errors arise from installation tolerances. Quadrupole correctors are present in the arcs. The $b_1$ correctors will be used for crossing the transition energy and may not be available for correcting random $b_1$'s. The contribution to $\Delta\beta/\beta$ from the uncorrected random $b_1$ is expected to degrade the values of Table 3 by a factor of two*. The random $a_1$'s will contribute to coupling; four families of skew correctors are included for its correction. All insertion quadrupoles can be adjusted independently; thus Item 8 rather than Item 7 indicates the expected rms values of $\Delta\beta/\beta$ and $\Delta X_p/\sqrt{\beta_x}$. The difference between Item 8 and Item 9 indicates the contribution from dipoles in the insertions.

## Placement of Q1-Q3

Item 5 of Table 5 indicates a strong contribution to $\Delta\beta/\beta$ from the Q1-Q3 quadrupole triplet. An additional contribution to the random $a_1,b_1$ of these quadrupoles comes from a rotational error during installation. Since the strength of these quadrupoles can be adjusted independently, the $b_1$ component of this error can be removed. However, the rotational error introduces a random $a_1$ that is not compensated. Rotation of a normal quadrupole by 45° converts it to a skew quadrupole. Using $k_o$ to denote the strength of the quadrupole, the strength of the skew component is:

$$\Delta k_{skew} = k_o \sin 2\theta.$$

The equivalent $a_1$ multipole coefficient is obtained from $\Delta k_{skew} X = B_o a_1 X/(B_o \rho)$ or:

$$a_1 = k_o \rho \sin 2\theta.$$

With $\theta$ = 1 mradian, $\rho$ = 243.2 m, and $k_o$=0.064 $m^{-1}$, $a_1$ = 0.032 $m^{-1}$; this is 2.3 times the usual rms $a_1$. The impact of this random $a_1$ has been determined by tracking when the $a_1$ multipoles in quadrupoles Q1 to Q3 are scaled by a factor of 2.3; results are listed in Table 6. The contribution to $Y_p/\sqrt{\beta_y}$ from rotational errors is small. The contribution to coupling is important; skew quadrupole correctors in the insertions will be used to correct coupling from this source.

* Estimates made at BNL by G. Parzen and A.G. Ruggiero.

| # | Elements | $\Delta\beta/\beta)_x$ | $\Delta\beta/\beta)_y$ | $\Delta X_p/\sqrt{\beta_x}$ |
|---|---|---|---|---|
| 1 | B | 4.96E-3 | 5.04E-3 | 3.96E-3 |
| 2 | B,QF,QD | 9.08E-3 | 2.36E-2 | 4.55E-3 |
| 3 | B,BS1,BS2 | 1.22E-2 | 1.44E-2 | 4.35E-3 |
| 4 | B,BS1,BS2,BC1,BC2 | 3.09E-2 | 3.18E-2 | 4.21E-3 |
| 5 | B,Q1-Q3 | 7.48E-2 | 5.59E-2 | 3.99E-3 |
| 6 | B, All Quads | 8.14E-2 | 7.35E-2 | 4.32E-3 |
| 7 | All Elements | 8.42E-2 | 8.49E-2 | 4.77E-3 |
| 8 | ($b_1$=0,$a_1$≠0) IQ ($a_1$,$b_1$)≠0 QF,QD & all dipoles | 3.08E-2 | 4.77E-2 | 5.00E-3 |
| 9 | ($b_1$=0,$a_1$≠0) IQ, ($a_1$,$b_1$)≠0 B,QF,QD. | 9.08E-3 | 2.36E-2 | 4.55E-3 |

Table 5 rms values of $\Delta\beta/\beta$ and $\Delta X_p/\sqrt{\beta_x}$ when randon ($a_1$,$b_1$) multipoles are present in various lattice elements. IQ denotes all of the insertion quadrupoles; other names have been defined previously.

| $\beta^*$ | $Y_p/\sqrt{\beta_y}$ a1 x1.0 | $Y_p/\sqrt{\beta_y}$ a1 x2.3 |
|---|---|---|
| 2 | 1.17E-3 | 3.04E-3 |
| 3 | 1.24E-4 | 2.59E-4 |
| 6 | 8.00E-4 | 1.64E-3 |

Table 6 rms values of $\Delta Y_p/\sqrt{\beta_y}$ at the crossing points for nominal random $a_1$ multipoles (Scale = 1.0) and for random $a_1$ multipoles arising from a 1 mradian rms roll of the Q1 to Q3 quadrupoles (Scale = 2.3).

## Summary

Shuffling is successful in reducing the $\Delta\beta/\beta)_x$, $\Delta\beta/\beta)_y$, $\Delta X_p/\sqrt{\beta_x}$, and $Y_p/\sqrt{\beta_y}$ from random $a_1$ and $b_1$ multipoles in the arc dipoles. The contribution to $\Delta\beta/\beta$ from insertion magnets dominates that from the arc dipoles. The contribution from the Q1-Q3 triplet is nearly twice and the contribution from insertion dipoles is nearly equal that from all the arc dipoles before they are shuffled. Independent tuning of each insertion quadrupole should remove its contribution to $\Delta\beta/\beta$ and $\Delta X_p/\sqrt{\beta_x}$. The contributions from the insertion dipoles then become the principal source of $\Delta\beta/\beta$. These dipoles may need local $b_1$ correctors. Finally, one mradian rms errors in the orientation of the fields of quadrupoles Q1 to Q3 make a modest contribution to $Y_p/\sqrt{\beta_y}$ but should be an important source of coupling. Coupling has not been considered in the present study, however, special skew correctors will be located in the insertions for its correction.

## References


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