Application of Collins' Distortion Functions†

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Abstract

The interactive beam program, LATTICE, has been extended to calculate, optimize, and plot the *distortion functions* of T. Collins for sextupole magnets. King Ng has shown that these functions can be rigorously derived from a Hamiltonian, subject only to the thin lens approximation. The functions are calculated from stored phases and β values for each sextupole, each bending magnet with a sextupole component, and at other arbitrary *marker* locations. Very general fitting capabilities are provided, the only restrictions being that changes are not allowed that would alter the linear beam properties. The values of any of the five functions may be specified at any location in the lattice, and the coefficients of tune shift with amplitude may be specified. Plots of the phase space projections on the lateral and vertical planes are provided as online graphics with optional offline Tektronix and also PostScript plots. The tune shift coefficients have been compared with those generated by a Fourier transform of tracking data and with the values calculated by the MARYLIE code, yielding reasonable agreement. The phase space plots have been compared with those obtained by tracking a few thousand turns. In doing this latter comparison, it is important to use equivalent starting conditions as the linear ellipse is displaced, distorted, and shifted in phase.

Distortion Functions

Tom Collins[1] has shown that all of the important effects of nonlinear fields can be derived from a series of periodic functions that he calls distortion functions. These functions provide the same kind of information about the non-resonant distortion to orbits due to nonlinearities that the Courant and Snyder functions provide for the linear behavior. With the exception of resonant extraction and injection regimes, where this approach is not valid, these functions should suffice in fully describing the effects of nonlinearities in a properly designed machine (one where the working point is not too close to a driven resonance). The machine nonlinearities perturb the betatron frequencies and distort the beam envelopes. Of course, although distorted, the beam envelope is still periodic. The distortion functions are sufficient for describing all of these effects. As is the case for the Courant and Snyder functions, the distortion functions are independent of particle amplitude and phase.

For normal sextupole magnets, there are five such functions. These distortion functions depend only on the linear lattice properties (Courant-Snyder β and α functions and phase advance) and the strengths of the nonlinearities. The only approximation made is the nonlinearities are assumed to occur just at discrete points. It is well known that one obtains a completely canonical set of equations when he uses linear transformations combined with nonlinear "kicks". King-Yuen Ng[2] has shown that Collins' formulae can be precisely derived from the proper Hamiltonian and that the results for sextupole-induced tune shifts are in complete agreement with the results of Ohnuma.[3] Nikolitsa Merminga and Ng have compiled all of the expressions for distortion functions for the following types of nonlinearities[4]: sextupole, skew sextupole, octupole, skew octupole, and skew quadrupole (which, of course, is not really a nonlinear magnet). Probably the most important numerical result that one gets from this approach is the shifts in the tunes with increasing amplitudes of oscillation.

Calculation Method

The code first goes through the entire lattice to set up an array containing the Courant-Snyder functions at the middle of all bending magnets and sextupole magnets and at all marker locations "S" ("slit"). The array also contains pointers to the lattice so the code can obtain the magnet parameters. The five sextupole distortion functions are then calculated at each of these locations with the results stored in the same array. Finally, the three independent coefficients of tune shift with amplitude are calculated as well as the chromaticities. A table is printed containing these results. Included in this table is a breakdown of the contributing terms to the tune shifts, allowing possible resonant phenomena to be identified. If offline output has been specified, a similar table is prepared, but in this case, the cumulative length is appended in order that the functions may be plotted. At each location where the functions are calculated a line is printed containing the values of the functions; this line is followed by a line containing the derivatives of the functions. One may abbreviate this output by restricting printing to, for example, sextupoles; in this case the derivatives are not printed.

Plotting of Distortion Functions

A new command **pd** (for "plot distortions") has been added. This causes the plotting of the projections onto the two lateral planes of the phase space figure for the emittance given on the beam card. The plot is available at the center of any sextupole or bending magnet and at any other location that has been designated as a marker.

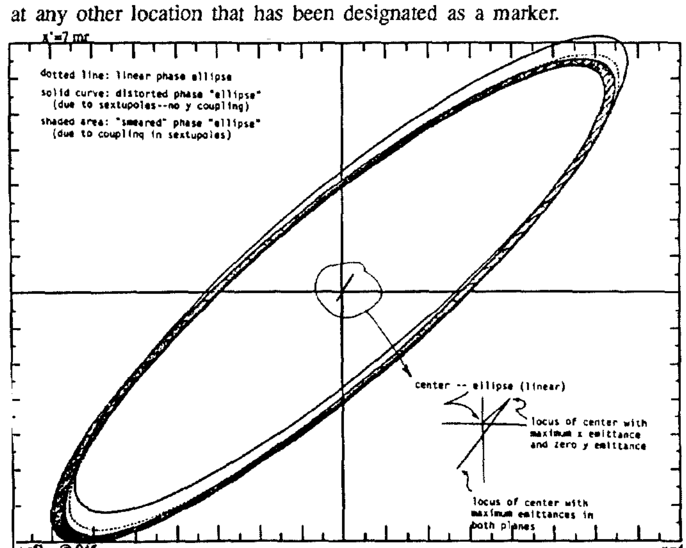

Figure 1 Distortion plot in the radial plane.

For the radial plane, the figure contains four parts. First there is the linear ellipse (no sextupoles); this ellipse is shown as a dotted curve on the offline PostScript plot and as a cyan curve on the online EGA display. Next, the distorted curve is plotted in the absence of coupling (e.g. for zero emittance in the vertical plane); this is shown as a solid curve on the PostScript plot and as a blue curve on the EGA plot. Thirdly, the area swept by with coupling is shown as a shaded area on the PostScript plot and as an array of green curves on the online plot. The fourth component is an array of line segments superimposed on the shaded area that show the phase shift due to the coupling.

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For each value of the radial phase variable, the values of the extremes in amplitude as a function of the vertical phase are calculated and plotted. These are connected by a line, so one can also see the shift in phase caused by the coupling. Finally, there are two line segments that show the shift in the effective center of the oscillation. The first line segment extends from the origin (which is the center in the absence of the nonlinearities) to the center of the distorted curve with zero vertical emittance but the given radial emittance. The second line runs from this point to the effective center for the coupled motion using the given emittances in each plane. From these two line segments, one may see the locus of the center for any other pair of emittances. The center is shifted because the sextupoles induce an effective asymmetric dipole component.

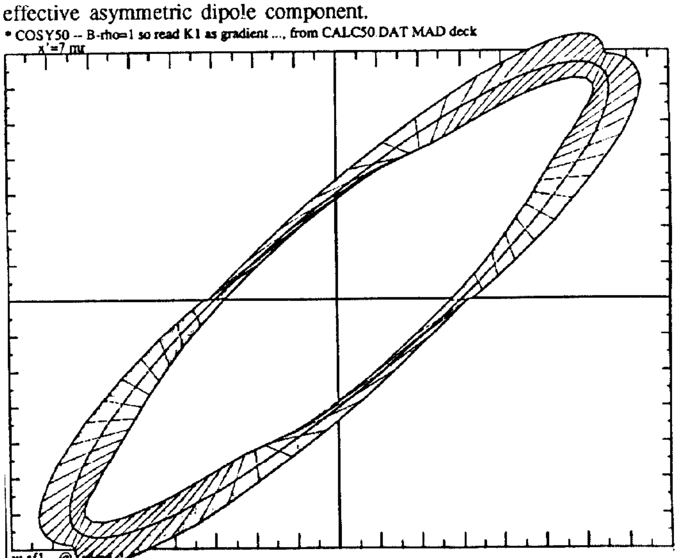

Figure 2 Distortion plot in the vertical plane.

The figures plotted for the vertical plane are similar, but there is no figure for uncoupled distorted motion, because such motion does not exist, nor is there any locus shown for the center, because the center is always at the origin.

Optional plots are provided where points are obtained by running through the set of radial and vertical phases. These plots are seen to map out the same region of phase space as the above distortion plots.

Fitting of Distortion Functions

The LATTICE fitting algorithms have been extended to include all of these values: the coefficients for the tune shifts and the values of the five distortion functions and their derivatives at any of the locations where they are calculated (sextupoles, bending magnets, and "slits"). It is also possible to specify the chromaticities to, for example, zero the chromaticities while minimizing some set of distortion functions or the tune shifts with amplitude. However, the user is not permitted to mix fitting specifications involving distortion functions with those involving the linear lattice. Moreover, he must avoid using variables that would cause any of the Courant-Snyder functions to change. This means that one may vary the field of sextupole magnets. It would be possible to eliminate this restriction with a massive rewrite of the code. The labelling of variables is exactly the same as for fitting lattice parameters, as is the execution of the fitting problem. When the iterate (i) command is given, the code examines the fitting specification and performs a fit of the lattice parameters or a fit of the distortion functions, depending upon the fit conditions. It does nothing if it discovers that the fit conditions include both types of conditions. A number of fitting algorithms may be chosen, including a very robust, albeit slow, method due to Powell and Brent that minimizes a global function without evaluating derivatives. When using this method, the number of variables need not equal the number of conditions.

Comparisons

Tune Shift.

Comparisons were made in collaboration with John Staples on three different lattices. The tune shifts with amplitude were calculated using the distortion functions and then compared with those derived by the FFT of the tracking data and also with the *Anharmonicities* calculated by MARYLIE. This was done for the LBL Advanced Light Source (ALS) lattice, for the Bevatron Upgrade Lattice (BUG), and for a modified BUG lattice where MARYLIE could calculate the same effects. The comparisons are generally good with the exception of the MARYLIE numbers for the first two lattices; this is because MARYLIE cannot decouple certain higher order effects in rectangular magnets and also cannot properly do this calculation when the radial and vertical tunes are similar. It is necessary, when comparing the tracking data, to use relatively modest amplitudes in order to get good agreement. We assume this to be due to fourth and higher order effects that arise from products of second order kicks.

Thus the third lattice was introduced with only sector magnets, no fringing fields, with vertical and horizontal tunes separated by about one (3.8, 2.65), and with the chromaticities zeroed by sextupoles. The results for the modified BUG lattice are as follows:

term	Distortion	FFT	MARYLIE
dQ_x/dW_x	25.344	24.2	30.74
dQ_y/dW_y	13.195	13.3	12.18
$dQ_x/dW_y=dQ_y/dW_x$	-10.150	-11	-11.40

This is quite satisfactory for the calculation of these second derivatives by completely different methods.

BESSY II.

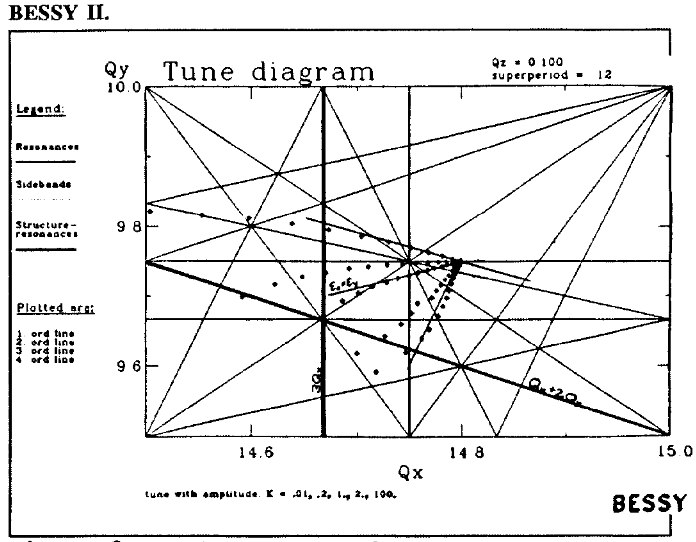

Figure 3 BESSY II Tune Plot.

Bettina Simon of BESSY has prepared a tune diagram showing tunes in a BESSY II lattice for increasing emittances. There are five groups, and within each the ratio of the vertical to horizontal emittances was kept constant. Tunes were obtained by Fourier transforms of the motion. This is shown as Figure 3. Godehard Wüstefeld has calculated the coefficients of tune shift with amplitude using LATTICE and has indicated these by adding line segments to this tune diagram. Each is seen to be tangent to the series of tune points, providing an excellent agreement where 4^{th} and higher order effects (clearly seen in the curvature of the loci of tune points) are not important.

Distortion Coefficients for a SSC lattice.

Both Ng and Collins have included an example of a possible SSC lattice containing ten superconducting dipoles per 80 degree symmetric cell; each dipole contains a systematic sextupole at low field. The question is whether sextupoles added to the quadrupoles are sufficient to compensate for the nonlinearities induced by the systematic sextupole fields. LATTICE yields the same values within about 10% for the five distortion functions at each of the magnets as compared to those listed in Ng's Table II. Moreover, the tune shifts agree exactly to the two-significant-figure results in Ng's paper.

Tracking.

John Staples[5,6] has added tracking to his version of LATTICE, together with the calculation of effective tune through a fast Fourier transform of the displacements at the beginning of the lattice for each turn. I have used this routine and then modified it to first calculate a proper starting point for the given emittances by using the distortion functions. It provides offline output and either online output or online phase space plots of the tracking data. Because the nonlinearities shift both the amplitudes and the phases, it is important to properly calculate the initial values for the two displacements and the two slopes if one is to get the correct value for the amplitude-dependent tune and also the correct locus of the projected phase plot.

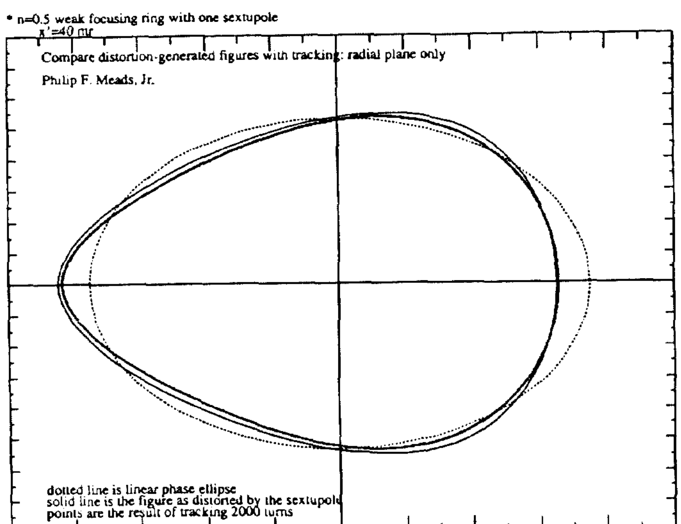

Figure 4 Compare radial phase space distortion figure with tracking points.

In Figure 4, we show the radial phase space plane for a simple ring comprising n= magnets with a single thin sextupole and no vertical motion (W_y=0). The plot is taken at the opposite side of the ring from the sextupole. The dotted line is the linear phase ellipse, the solid line is the figure plotted from the distortion functions (zero width because of the absence of coupling), and the points are the loci of 2000 turns. It is seen that the distortion figure and the actual tracking data are very similar and that the distortion figure predicts well the actual motion.

The comparison for coupled motion is not as good as is seen in Figure 5 (radial plane) and Figure 6 (vertical plane). In these two figures, tracking data for 999 turns have been superimposed over the distortion figures. It may be that the tracking data are too influenced by the nearby sum resonance (Q_x+Q_y=8) or the nonlinear resonance $2Q_x$+$2Q_y$=16, or there may be an error in calculating the distortion-matched starting point for the tracking.

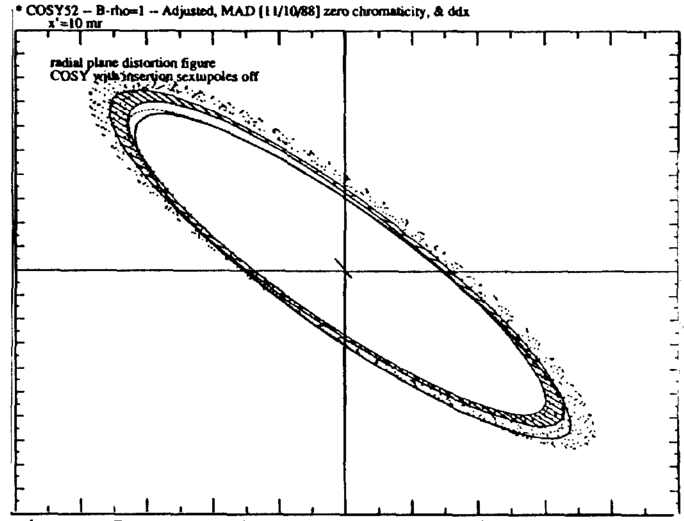

Figure 5 Comparing coupled motion: tracking and distortion figure--radial plane.

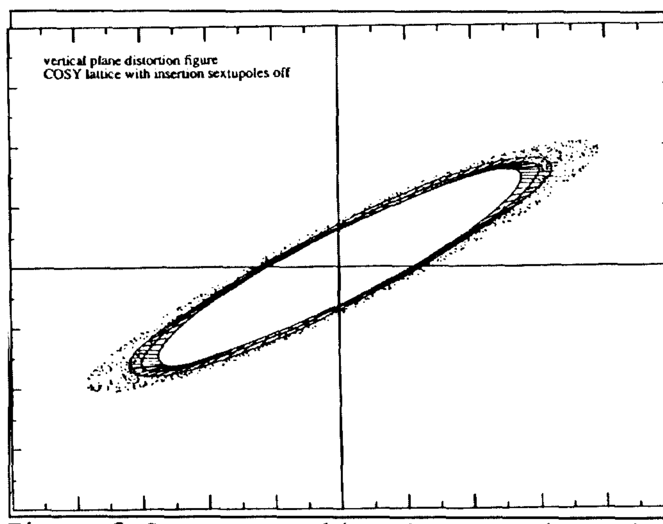

Figure 6 Compare tracking data to distortion figure—vertical plane.

References

1. T. L. Collins, **Distortion Functions**, Fermilab Internal Report 84/114 (1984).

2. K Y Ng, **Distortion Functions**, KEK Report 87-11 (September 1987), also reprinted as Fermilab Report TM-1281.

3. S. Ohnuma, Proceedings of the Interactions between Particle and Nuclear Physics, Steamboat Springs, Colorado (1984)

4. Merminga and Ng, **Hamiltonian Approach to Distortion Functions**, Fermilab Report FN-493 (August, 1988).

5. J. Staples, **LATTICE, An Interactive Beam Optics Design Code**, informal LBL report (1981)

6. J. Staples, **Supplement to LATTICE and QUICKTRAN Writeups**, informal LBL report (August 10, 1988)