Chasman-Green (CG), Triple Bend Achromat (TBA) and Quadruple Bend Achromat (QBA) lattice have been designed and the dynamic characteristics of these three lattices have been studied for the lattice selection of the 8 GeV low emittance synchrotron radiation source. The results showed that each lattice had both merits and demerits and the absolute superiority or inferiority between the lattices was not recognized so far as the dynamic characteristics were concerned. These results lead us to the conclusion that CG lattice which has the simplest magnet arrangement was suited for the 8 GeV synchrotron radiation source.

Introduction

RIKEN and JAERI, which are supervised by Science and Technology Agency of Japanese Government (STA), are planning to construct the large synchrotron radiation facility [1][2]. The emittance of the storage ring is planned to be less than 10^{-8} m.rad.

Generally, electron beams in a low emittance storage ring are strongly focused to achieve low emittance and this fact generates many problems which are not principal difficulty for a high emittance storage ring: The dynamic aperture of such low emittance storage ring is too small and the sensitivity against the field errors is very high. We then need to design the storage ring which overcomes these problems.

Chasman-Green lattice [3] which has two bending magnets in a cell and Triple Bend Achromat lattice [4] which has three bending magnets in a cell are well known as a low emittance lattice for synchrotron radiation source. In addition to these lattices, we designed a Quadruple Bend Achromat lattice (QBA) which had four bending magnets in a cell and the dynamic characteristics of these three lattices were studied. On the basis of these studies, we compared the dynamic characteristics to select the suitable lattice for the 8 GeV low emittance storage ring for synchrotron radiation source.

Table 1 Ring parameters and number of magnets

<table>
<thead>
<tr>
<th></th>
<th>CG</th>
<th>TBA</th>
<th>QBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emittance (nm.rad)</td>
<td>4.9</td>
<td>5.8</td>
<td>5.0</td>
</tr>
<tr>
<td>Tune νx</td>
<td>51.75</td>
<td>48.76</td>
<td>49.6</td>
</tr>
<tr>
<td>νy</td>
<td>19.82</td>
<td>26.39</td>
<td>19.60</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>1478.4</td>
<td>1477.8</td>
<td>1476.5</td>
</tr>
<tr>
<td>No. of cells</td>
<td>48</td>
<td>36</td>
<td>32</td>
</tr>
<tr>
<td>Dipole magnet (per cell)</td>
<td>96(2)</td>
<td>108(3)</td>
<td>128(4)</td>
</tr>
<tr>
<td>Quadrupole magnet</td>
<td>480(10)</td>
<td>432(12)</td>
<td>480(15)</td>
</tr>
<tr>
<td>Sextupole magnet</td>
<td>32(2)</td>
<td>216(6)</td>
<td>288(9)</td>
</tr>
</tbody>
</table>

Dynamic Aperture

The dynamic apertures of each storage ring are shown by square mark in Fig. 2. These small dynamic apertures are mainly dominated by the first and third order resonances which are induced by the strong sextupole fields of chromaticity correction sextupole magnets. To avoid these resonances, the amplitude dependent tune shift should be suppressed to small values. According to the perturbation theory, the amplitude dependent tune shifts in a superperiod are given by

\[ \Delta \nu_{xc} = C_{11} 2J_x + C_{12} 2J_y \]
\[ \Delta \nu_{yc} = C_{21} 2J_x + C_{22} 2J_y \]

where

\[ x = J_x \sin \phi_x \cos \psi_x, \quad y = J_y \sin \phi_y \cos \psi_y \]

and \( J_x(J_y) \) and \( \phi_x(\phi_y) \) denote the action and angle variable, respectively. Coefficients \( C_{ij} \) are expressed by eqs.(3),(4),(5).

\[ C_{11} = -18 \pi (A_3 m^2/3) (v_{xc} - m) + 3 (v_{xc} - m) \]
\[ C_{12} = 36 \pi (2B_1 m A_1 m (v_{xc} - m) - B_2 m^2 (v_{xc} - 2v_{yc} - m)) \]
\[ C_{22} = -18 \pi (4B_1 m) (2B_1 m (v_{xc} - m) - B_2 m^2 (v_{xc} - 2v_{yc} - m)) \]

where \( v_{xc} \) and \( v_{yc} \) are the tune for one superperiod.

Fig 1 Lattice functions of one superperiod
Fig. 2 Dynamic aperture (without harmonic sextupoles, \( A_{1m}, A_{3m}, B_{1m} \), and \( B_{2} \) are expressed by eqs. (6), (7), (8), (9).

\[
A_{1m} = \frac{S_{k}}{(48 \pi^{2})} \beta_{t} \frac{1}{2} \cos \left( \int \frac{1}{\beta_{t}} ds \cdot \nu_{x} \theta + m \theta \right) 
\]

\[
A_{3m} = \frac{S_{k}}{(48 \pi^{2})} \beta_{t} \frac{1}{2} \cos \left( 3 \int \frac{1}{\beta_{t}} ds \cdot \nu_{x} \theta + m \theta \right) 
\]

\[
B_{1m} = \frac{S_{k}}{(48 \pi^{2})} \beta_{t} \frac{1}{2} \cos \left( \int \frac{1}{\beta_{t}} ds \cdot \nu_{x} \theta + m \theta \right) 
\]

\[
B_{3m} = \frac{S_{k}}{(48 \pi^{2})} \beta_{t} \frac{1}{2} \cos \left( \int \frac{1}{\beta_{t}} ds \cdot \nu_{x} \theta + m \theta \right) 
\]

where \( S_{k} \) is the strength of sextupole magnets. (9)

As we can see in eqs. (3), (4), (5), the first and the third order resonances driven by the sextupole fields are,

\[
v_{x} \theta = N \cdot 3 \nu_{x} \theta = N \cdot (10)
\]

In CG lattice, \( \nu_{x} \theta \) is 2.16. This means that \( m=6 \) or 7 in the first term in eq. (3) \( (A_{16} \) or \( A_{37} \) term) and \( m=2 \) in the second term of eq. (3) and in the first term of eq. (4) \( (A_{12} \) or \( B_{12} \) term) are the most harmful. Harmonic expansion spectrums of \( C_{ij} \) and the dynamic aperture prediction in the single resonance approximation by CATS [5] showed that \( A_{12} \) and \( A_{37} \) term gave the limitation of dynamic aperture. Therefore, \( A_{12} \) and \( A_{37} \) term were suppressed by the additional sextupole magnets (harmonic sextupoles). The strength of the harmonic sextupoles were also optimized by CATS. Similarly, we knew that the major term which gave the limitation of dynamic aperture in TBA lattice were \( A_{12} \) and \( A_{37} \). However, we failed simultaneous suppression of \( A_{12} \) and \( A_{37} \) term. This is obvious from Fig. 3. In case of CG lattice, the position dependence of \( A_{12} \) and \( A_{37} \) which we denote \( A'_{12} \) and \( A'_{37} \) show the same behavior. This means that \( A_{12} \) and \( A_{37} \) only with chromaticity correction sextupoles have the same sign. Therefore, we can suppress \( A_{12} \) and \( A_{37} \) by setting the harmonic sextupoles in the non-dispersive section where \( A'_{12} \) and \( A'_{37} \) have the same sign. In case of TBA lattice, the signs of \( A'_{12} \) and \( A'_{37} \) in the dispersive section are different. Then \( A_{12} \) and \( A_{37} \) only with chromaticity correction sextupoles have the different sign. While, the signs of \( A'_{12} \) and \( A'_{37} \) in non-dispersive section are the same. This means that \( A_{12} \) and \( A_{37} \) can not be suppressed simultaneously by the harmonic sextupoles which are set in non-dispersive section. In case of QBA lattice, the harmful harmonics were \( A_{13} \) and \( A_{39} \). The signs of \( A'_{13} \) and \( A'_{39} \) in dispersive section are different but they are different in non-dispersive low beta section. We can then suppress \( A_{13} \) and \( A_{39} \) simultaneously. However, if we suppress \( A_{13} \) and \( A_{39} \) to the extent, where \( A_{13} \) and \( A_{39} \) are not the main source of dynamic aperture limitation, the other harmonics grow and give the dynamic aperture limitation. Consequently, the dynamic aperture can not be enlarged by the harmonic sextupoles. This situation that if we suppress the particular harmonics, the other harmonics grow and give the dynamic aperture limitation is more or less true for CG lattice. But in case of TBA and QBA lattice, the situation is more serious and complex than CG lattice. Solid lines in Fig. 2 show the dynamic apertures enlarged by this method.

Lattice Comparison

Lattice characteristics greatly depend on the degree of optimization and it is very difficult to extract a universal trend. Nevertheless, we compared the dynamic characteristics of the lattice to select the suitable lattice for the 8 GeV storage ring. Compared items are the dynamic aperture size and the sensitivity against the errors.

Dynamic Aperture

We evaluated not only the dynamic aperture for the ideal lattice but also for the realistic lattice which have various kinds of errors. Particle trackings with errors were done by RACETRACK [6]. If the magnets have various errors, the closed orbit distortions(COD) occur. Therefore, we evaluated the dynamic aperture size as a function of COD, which can be varied by the degree of COD correction. Table 2 shows the error conditions. Tracking results for the ideal machine and for the machine with errors are shown in Fig. 2 and Fig. 4, respectively. In machines have enough dynamic aperture if the errors are not taken into account. However, the dynamic aperture size reduces to less than 2/3 even if the COD is corrected completely. For TBA and QBA lattice, aperture size is almost the same size which is needed for injection.
Sextupole magnet Misalignment | Ax, Ay | 0.2 mm
Quadrupole magnet Misalignment | Ax,l | Ay | 0.2 mm

Sensitivity

It is very important for users of synchrotron radiation that the electron beams keep the same condition sufficiently long time. However, if the machine has high sensitivity against the errors, beam condition will easily change. From the standpoint of machine designer, such sensitive machine is not favorable. Therefore, we investigated the sensitivity against the errors. The investigated items are the followings.

- Closed orbit distortions Xco and Yco arising from the alignment errors of quadrupole magnets.
- Beta value distortions ∆β arising from field gradient errors of quadrupole magnets ∆k/k and COD in sextupole magnets.
- Spurious dispersions AD arising from the field gradient errors ∆k/k and tilted errors ∆θ of quadrupole magnets and COD in quadrupole and sextupole magnets.

The evaluation point is the high beta straight section. For the calculation of spurious dispersions, the following errors are supposed; ∆k/k = 10⁻⁴, Xco = Yco = 1 mm, ∆θ = 1 m rad.

Figure 5(a) shows the sensitivity against the alignment errors of quadrupole magnets. From Fig. 5(a), we know that the sensitivity against the quadrupole magnet alignment errors for CG lattice was the highest of the three lattices and 0.1 mm alignment errors produced 8 mm COD. Figure 5(b) shows that the beta value distortions arising from the gradient errors of quadrupole magnets are almost the same between the three lattices and ∆β is about 4 % - 6 % for the 0.1 % gradient errors. The beta value distortions arising from COD in sextupole magnets are from 2 %, which is the case for horizontal distortions for TBA lattice, to 10 %, which is vertical distortions for QBA lattice. Vertical beta distortions for QBA lattice is exceptional because the sextupole arrangement for chromaticity correction is somewhat different from the other two lattices to enlarge the dynamic aperture. Figure 5(c) shows that the spurious dispersions for CG lattice is the largest in the three lattices. If we assume the 0.1 % gradient errors, 0.1 mm COD and 0.5 m rad tilt errors, horizontal and vertical spurious dispersions are 44 mm and 30 mm, respectively.

Summary of Comparison

We knew that the each lattice fulfilled the irreducible minimum requirements as a low emittance storage ring. To compare the dynamic characteristics totally, we summarized the characteristics as shown in Table 3. Table 3 shows us that each lattice has its own merits and demerits and absolute superiority or inferiority does not exist. We then could not have definite reason to determine the lattice type as far as the dynamic characteristics were concerned.

If we apart from the stand point of dynamic characteristics, we can easily know that the lattice which have simplest magnet arrangement and shortest cell length is CG lattice. These results lead us to the conclusion that the most suitable lattice for the 8 GeV low emittance storage ring is CG lattice.

Fig. 4 Horizontal dynamic aperture as a function of closed orbit distortions.

Sensitivity

Chasman-Green lattice, Triple Bend Achromat lattice and Quadruple Bend Achromat lattice have been designed and the dynamic characteristics have been studied. We could not recognize the absolute superiority of the specific lattice between the three lattices in dynamic characteristics. However, Chasman-Green lattice has the simplest magnet arrangement and the shortest cell length. Accordingly, we determined to choose Chasman-Green lattice as the 8 GeV low emittance storage ring lattice.

Table 3 Comparison of dynamic characteristics of CG, TBA, and QBA lattice.

<table>
<thead>
<tr>
<th>Item</th>
<th>CG</th>
<th>TBA</th>
<th>QBA</th>
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<tbody>
<tr>
<td>Dynamic</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>COD</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>∆θ</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Conclusion

We knew that each lattice fulfilled the irreducible minimum requirements as a low emittance storage ring. To compare the dynamic characteristics totally, we summarized the characteristics as shown in Table 3. Table 3 shows us that each lattice has its own merits and demerits and absolute superiority or inferiority does not exist. We then could not have definite reason to determine the lattice type as far as the dynamic characteristics were concerned.

If we apart from the stand point of dynamic characteristics, we can easily know that the lattice which have simplest magnet arrangement and shortest cell length is CG lattice. These results lead us to the conclusion that the most suitable lattice for the 8 GeV low emittance storage ring is CG lattice.

References