SYMMETRY CORRECTED SECOND ORDER ACHROMAT*

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Abstract

A second order achromat[1] is a magnetic system in which all second order geometric and chromatic aberrations (except T566 vanish). When it is used with other optical elements in a ring, sextupoles in the achromat can be adjusted to control some chromatic properties of the ring. Geometric aberrations will remain cancelled if the sextupoles are properly placed. However, some chromatic aberrations such as T166 and T266 may be left uncorrected. Using symmetry of the first order orbits it is possible to cancel these and some other chromatic terms independent of the chromaticity

Introduction

The Bates Linear Accelerator Center is in the process of constructing an electron storage ring[2]. This ring will be used for internal target experiments and as a pulse stretcher to increase the duty factor of the beam for energies up through 1 GeV. One of the properties which characterizes the Bates beam quality is its good emittance. For example, it is .01mm-mr at 500 MeV. It is advantageous to maintain this good emittance both for internal target experiments and for the extracted beam. For this reason all reasonable precautions in the design are being taken; from the optical design to the effects of wakefields. The optical design incorporates the use of a symmetry corrected second order achromat to minimize the centroid shifting energy dependent aberrations as well as others. These aberrations have the effect of increasing the effective beam emittance through their coupling to the beam energy spread.

Symmetry Achromat Principles

Two main principles are central to this design. The first is that of the second order achromat[1]. For this a set of at least four repetitive cells is used to form a unit with phase advance that is a multiple of 2π. This will cause the geometric aberrations to vanish.

Sextupole components can be adjusted to make all second order aberrations vanish (except T566). In this type of lattice, the sextupoles in a particular family are separated by a π phase advance and as many families as needed (or can be squeezed in) are located at the proper optical positions.

Secondly, the rules of symmetry in first order optics are used[3]. In particular, an aberration in an optical system can be represented by the following equation:

$$q = S(s)\int_0^s f(\tau)S(\tau)d\tau - C(s)\int_0^s f(\tau)C(\tau)d\tau$$

Where S and C are the sinelike and cosinelike orbits respectively, and f is the driving function for the aberration q. In the case of the achromat, the phase advance is a multiple of $2\ \pi$ and therefore the sinelike function is odd and zero at the end, and the cosinelike function is even and is 1 at the end and $q = -\int_0^s f(\tau)s(\tau)d\tau$

The following list illustrates the dependencies of some aberrations on the first order optics symmetry and shows how some symmetries affect the aberrations for an even bending radius function (and odd sinelike trajectory).

Aberrations	Function	dependence
T166 = 0	if d	˜even
T116 = 0	if s_x	˜odd
T126 --	if c_x	˜even
T266 --	if d'	˜odd
T216 --	if s_x'	˜even
T226 = 0	if c_x'	˜odd

Finally, it is useful to write the expression for chromaticity in terms of other optical quantities[4].

$$\chi = \text{Const}*\{(2T_{111}+T_{212})D+T_{116}+T_{226}\}$$

Not much can be said about tne value of an aberration if the only condition invoked is the requirement that the chromaticity be of a certain value. Furthermore, the chromaticity is independent of the centroid shifting aberrations T166 and T266.

Application in Rings

There have been applications of the second order achromat principle in rings and other systems[5]. This has been useful especially to control the geometric aberrations particularly for resonant extraction and final focus systems. In general, a ring may have straight sections, or cells apart from the repetitive achromat cells. The elements in these cells may introduce chromatic aberrations in addition to those arising from the achromat elements. These can be controlled to some degree by using the sextupoles in the achromats. For example, if one can fit four families of sextupoles then one could correct the chromaticity χ_x and χ_y and $d\beta x/d\delta$ and $d\beta y/d\delta$. This adjustability is important for ring performance. However, it is interesting to examine what happens to the aberrations and their effect on the beam.

First consider the case of an achromat which does not incorporate symmetry principles. The typical four cell fodo structure achromat is shown schematically below.

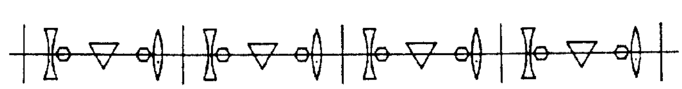

Figure 1. Four cell FODO achromat.

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For the example above, a system with 8 cells forming 180 degrees of bend is chosen. A straight section made up of a continuation of the fodo structure is added to form a ring with a circumference comparable to the examples which follow. The bend plane aberrations with and without straight sections and for different chromaticities are listed in Table 1. The second order optical quantities are calculated with DIMAD[6] either in the middle of a straight section or at the end of the 180° bend as applicable.

Table 1. Bend plane aberrations for FODO achromat and straight section.

	180° Uncorrected	180° Corrected	Full Ring Uncorrected	Full Ring Corrected	Full Ring Corrected
chrom	-	0	-7.04	0	-10
$d\beta/d\delta$	-	0	4.42	0	0
T166	-5.16	0	26.51	-13.7	43.8
T116	51.82	0	133.4	3.1	191.2
T126	-95.68	0	176.6	0	-255.8
T266	-5.9	0	19.2	-10.1	31.5
T216	30.14	0	59.2	3.4	84.4
T226	-51.59	0	-63.8	-3.1	-92.4

The bends may be achromatic to second order when sextupole corrected, however when the sextupoles are used to compensate for the straight sections in this system, aberrations such as (T166) and (T266) resurface.

A lattice such as the one shown below, can be used to illustrate the benefits of symmetry.

Figure 2. Symmetric four cell achromat.

The resulting aberrations are calculated and shown in Table 2. The straight sections used are similar to those being designed for the Bates SHR. (However, the overall circumference of this ring is similar to that of the fodo example above). These straight section regions are designed for injection, extraction, and internal target experiments.

Table 2. Aberrations for symmetric four cell achromat.

	180° Uncorrected	180° Corrected	Full Ring Uncorrected	Full Ring Corrected	Full Ring Corrected
chrom	-	0	-11.3	0	-10
$d\beta/d\delta$	-	0	-.18(4)	0	0
T166	0	0	101.8	-42.7	122.5
T116	0	0	-96	.95	-118.5
T126	-269.1	0	1620	0	1825
T266	-2.6	0	8.5	-3.6	10.3
T216	1.4	0	-10.8	.14	-11.9
T226	0	0	131.6	-.95	149.8

Table 2 shows that each four cell unit symmetry corrects T166 and some other aberrations. However, T266 is not corrected. When a straight section is added to the achromat, the non zero T266 allows T166 to propagate. The symmetry which causes T266 to vanish is not allowed by an achromat which demands a **+I** matrix.

The Bates South Hall Ring (SHR)

For the Bates SHR, we have added a small straight section between two second order achromats in order to add a phase advance of π. This performs the following function. Already in each achromat T166 was symmetry corrected as shown above. The overall symmetry of the 180° bend now causes T266 to be corrected. The overall symmetry also causes T166 to double; however, twice zero is still zero. A sketch of the layout of the ring is shown in Figure 4. The aberrations of this system are shown in Table 3. Note that T166 and T266 are always zero.

Table 3. Aberrations for symmetric achromat with -I transform.

	180° Uncorrected	180° Corrected	Full Ring Uncorrected	Full Ring Corrected	Full Ring Corrected
chrom	0	0	-13.7	0	-10
$d\beta/d\delta$	-	0	-463	0	0
T166	0	0	0	0	0
T116	0	0	-153	.95	-118
T126	286	0	2382	0	1825
T266	0	0	0	0	0
T216	-2.4	0	-15.6	.14	-11.9
T226	0	0	196	-.95	149.8

Minimization of Sextupole Elements

In some cases a large number of sextupoles is required if the constraints of repetitiveness and symmetry are imposed. It is possible to minimize the sextupole element in this system. In particular, the sextupoles in regions of small or no dispersion are not particularly effective, although they were located there originally for the second order achromat. If instead one uses the π transform, and positions the sextupoles so that members of a family are 3π apart (as opposed to π) then one can eliminate half the sextupoles while maintaining mechanical symmetry. This system is illustrated in the sketch below.

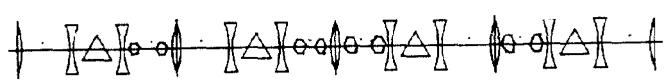

Figure 3. Sextupole placement in symmetric four cell achromat, second four cells after π transform are mirror symmetric.

The aberrations resulting from this system are identical to those listed in Table 3.

Note that the system described in Table 2 (without the **-I** transform) could not utilize this scheme while maintaining symmetry. In fact, for minimum sextupole count, the sextupole positions in the second 90° of bend would be reversed in each cell compared to the first 90°. However, since the overall symmetry is not used in that case it is not important.

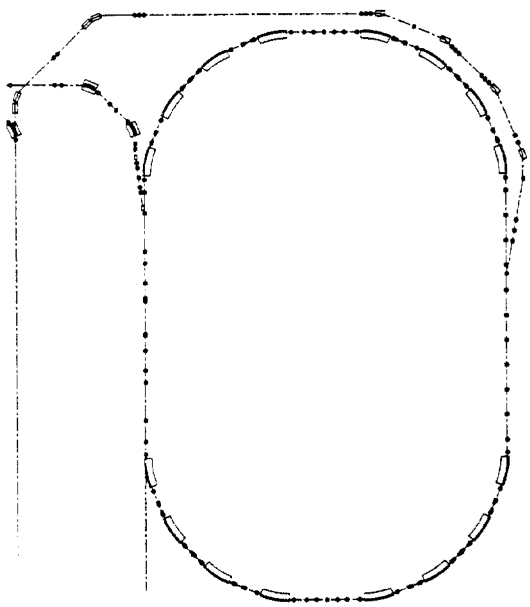

Figure 4. Element Layout of the Bates SHR

Summary

In some systems, depending upon beam emittance and energy spread, it is important to control centroid shifting aberrations, as well as second order effects. A second order achromat provides a good starting point for the control of geometric and chromatic aberrations. However when used in a ring with sections outside the achromat, these elements add chromatic aberrations not fully controllable with sextupoles inside the second order achromat. By taking advantage of the appropriate first order orbit symmetry it is possible to eliminate the centroid shifting chromatic aberrations, while still having full control over the second order machine functions such as chromaticity. Furthermore, it is possible to minimize the number of sextupoles required for chromatic correction with this scheme.

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