PREDICTION OF LONG-TERM BEAM STABILITY USING HIGH ORDER TORROIDAL INVARIANTS

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Introduction

In light sources, damping rings, and in the SSC understanding non-linear aperture limits for successful cost-effective designs is becoming ever more important. SSC injection requires 10^7 turns, and beams collide for 10^8 turns. Ideally one would like to know the dynamic aperture for this number of turns, how it depends on tune modulations, and how sensitive it could be to "unknown or omitted physics" or choice of model for the lattice.

The major obstacle to finding long-term stability limits is the fact that, tracking 1 to 64 particles through the SSC lattice for 10^7 turns requires 500 hours of CPU time on the CRAY XMP. We have been pursuing a two pronged approach to overcoming this obstacle:

i) looking for "early warning" signs that a particle is long-term unstable, and
ii) creating maps that reproduce element-by-element tracking with sufficient accuracy and can be evaluated faster.

The goal in i) above is to detect potential long-term instability in 10^4 turns, and the goal in ii) is to gain a factor of 30 in tracking speed. Together this would allow determination of long-term stability for 64 particles in 1 CRAY CPU minute. In this paper we discuss approach i).

There are two ways that have been suggested to look for signs of long-term instability. They both rest on the belief that if a particle is long-term unstable, it will experience a significant and discernible amount of chaotic motion. This chaotic motion may be detected by

a) looking for small departure from regular motion such as a slow change of an action invariant, or
b) tracking two neighboring particles, and looking for an exponential growth of their phase space separation.

This paper describes efforts of type a) above, and presents some recent comparisons with procedures of type b).

The rms beam radius in the SSC at injection is about 0.6 mm measured where $\beta_x = \beta_y = 370$ m. The dynamic aperture[1] for 400 turn particle loss occurs at approximately $x = y = 8$ mm, and the so-called linear aperture, where rms x-smear = y − smear = 6.4%, occurs at about $x = y = 6$ mm (initial $P_x = P_y = 0$ always).

In Figure 1 we show the linear Courant-Snyder invariant for 400 turns at $x = y = 6$ mm. The action $J = (\text{amplitude})^2/2$ plotted here exhibits an rms variation of about 12%. Using normal form techniques and differential algebra methods described in reference 2, we can find higher order invariants. In Figure 2 we show a plot of the sixth order invariant for this same case. The rms variation of this invariant is about a factor of 10 smaller than the linear invariant.

Figure 3 shows this same 6th order invariant plotted for 10,000 turns. On this time scale the invariant exhibits large periodic swings, and we suspect the presence of a nearby synchrobetatron resonance. Figure 4, a phase plot of amplitude versus phase angle ($\varphi_x - \varphi_y - 3\varphi_{c\tau}$) for several initial amplitudes, confirms the presence of a large fifth-order resonance. Changing the cavity voltage so as to move off this resonance, we arrive at the results plotted in Figure 5.

*Operated by the Universities Research Association Inc., for the Department of Energy.

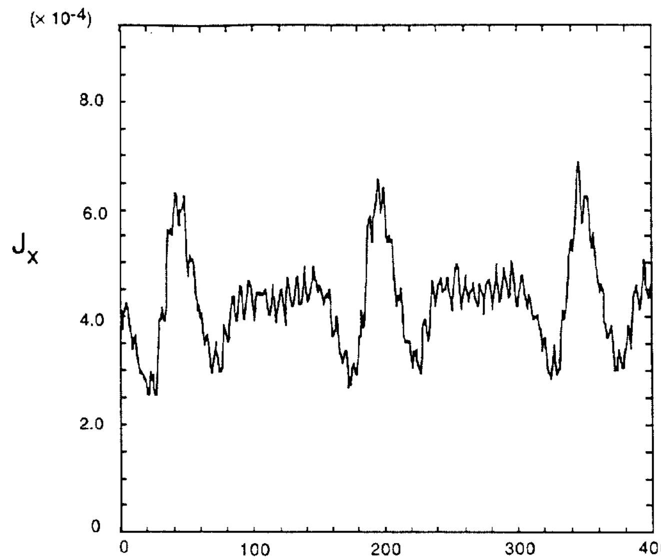

Figure 1. Second order (Courant-Snyder) invariant for 400 turns at x = y = 5.4 mm.

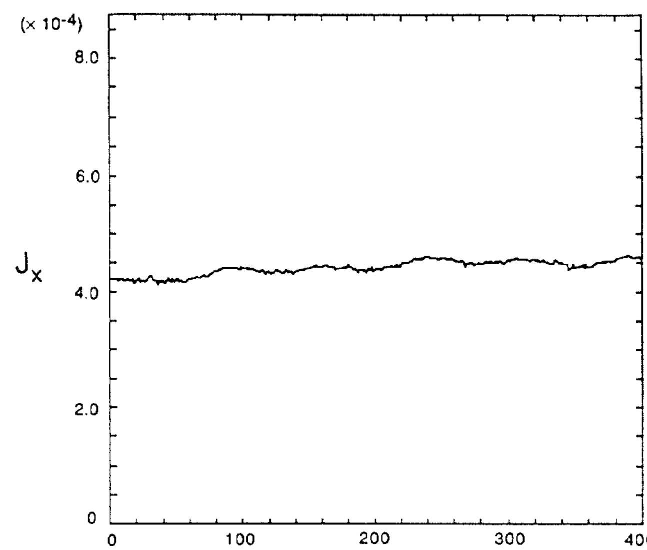

Figure 2. Sixth order invariant for 400 turns at x = y = 5.4 mm.

In Figure 5 there still remains a significant amount of regular, high-frequency excursion from our approximate invariant. These high-frequency departures are uninteresting; they merely indicate that a true invariant would contain higher order terms. We can rid ourselves of them by "filtering." We take a fast fourier transform of the data of Figure 5, remove the high order terms, and exhibit a plot of the low order terms in Figure 6.

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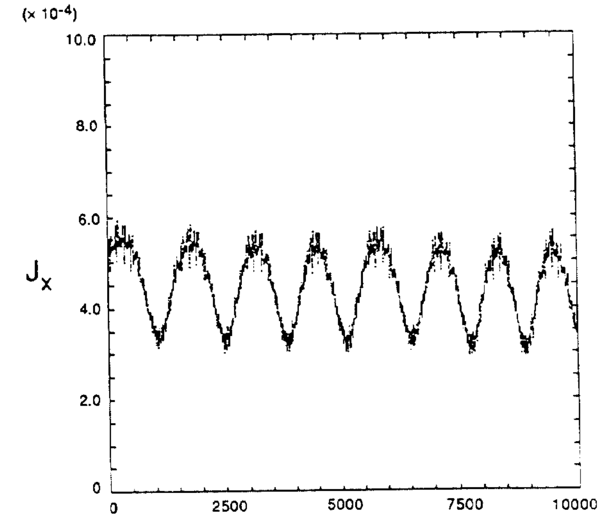

Figure 3. Sixth order invariant for 10,000 turns at x = y = 6.0 mm.

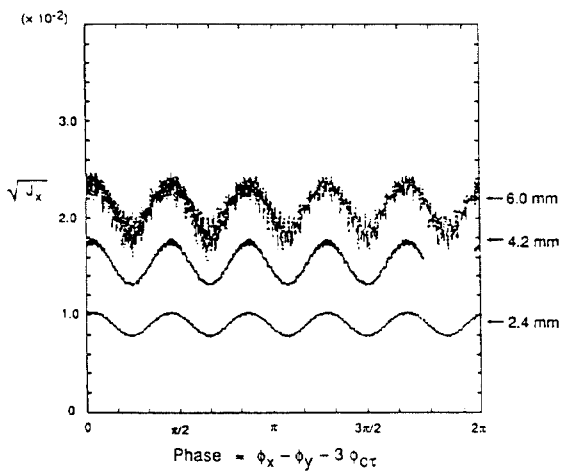

Figure 4. Phase diagram. Sixth order invariant for 10,000 turns for x = y = 2.4, 4.2, and 6.0 mm.

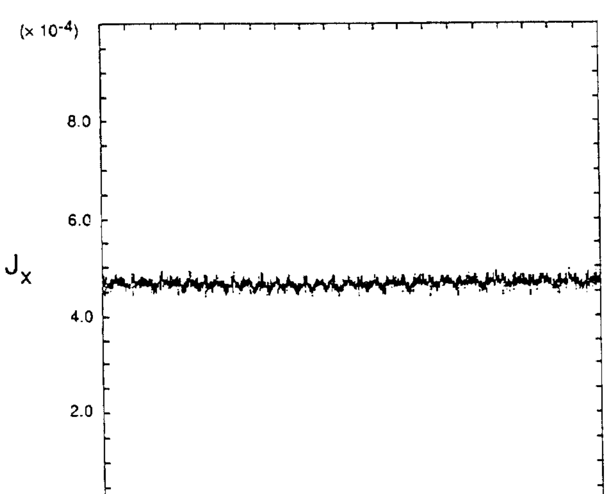

Figure 5. New Synchrotron Tune. Sixth order Invariant for 10,000 turns at x = y = 6.0 mm.

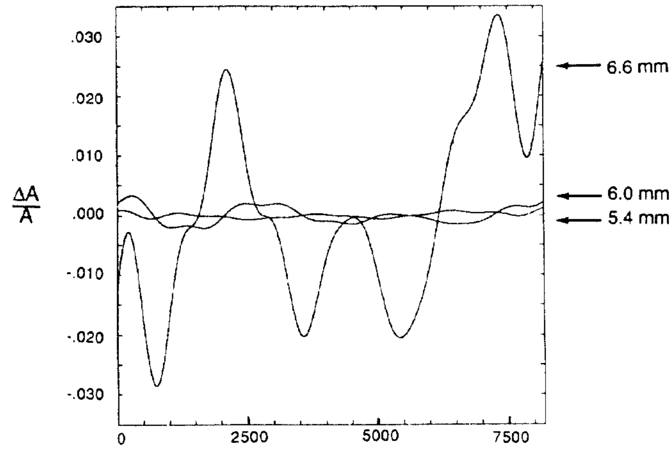

Figure 6. Filtered sixth order invariant for 10^4 turns for three initial amplitudes.

What change of amplitude is acceptable on a 10^4 turn time scale? It is not possible to answer this question in an absolute way, but the following argument suggests an order of magnitude guideline. If as a result of chaos, there is a random component to the change in amplitude of $\Delta a(1)$ for each turn, then after N turns the expected change in amplitude would be $\Delta a(N) \approx \sqrt{N}\, \Delta a(1)$, and it follows in particular that

$$\Delta a(10^8) \approx 10^2\, \Delta a(10^4)$$

If we require (arbitrarily) that $\Delta a(10^8) \leq$ the rms beam radius, then we find

$$\frac{\Delta a\,(10^4)}{a} \leq 10^{-3} \tag{1}$$

for amplitudes "a" that are about ten beam radii. As a result of these considerations we suggest using criteria (1) as a working definition of a long-term stability aperture. We have called it previously[3] the "diffusive dynamic aperture." Here a particle is within the "diffusive dynamic aperture (DDA)" if it, and all particles of lesser amplitude, satisfy equation (1).

Returning to Figure 6 we see that the particle at amplitudes ≤ 5.4 are within the DDA, particles at amplitudes ≥ 6.0 are outside of this aperture. In Figure 7 we show these results extended for 10^5 turns.

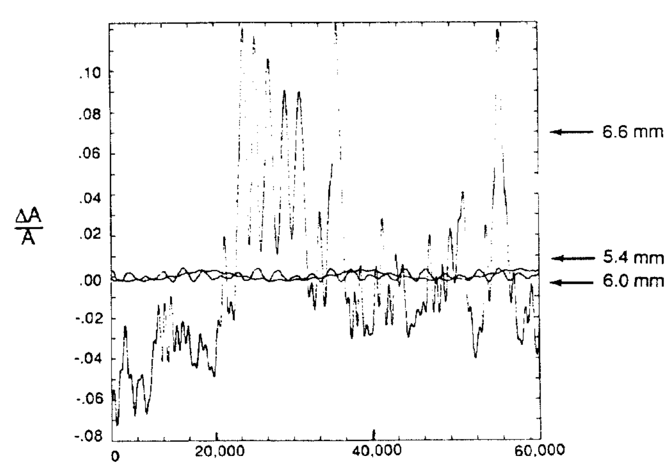

Figure 7. Filtered sixth order invariant for 10^5 turns for three initial amplitudes.

We are aware that the long term changes in amplitude can arise from sources other than chaotic motion. However it is our opinion that it provides a margin of safety to reject such motion as unacceptable, even if its source is not from chaos. It is our opinion that significant long-term motion, on a time scale of 10^4 turns, is potentially hazardous, and should be avoided.

In Figures 8, 9, and 10 we show a plot of phase space separation for two particles started at the same amplitude, separated only by one part in 10^6. F. Schmidt has studied the behavior of such particle pairs in reference (4). According to his experience and criteria, particles at 5.4 and 6.0 mm are stable, the particle at 6.6 mm is unstable. This agrees with our determination.

We wish to acknowledge the support and direction of Alex Chao, and helpful conversations with E. Forest, M. Berz, J. Peterson, and F. Schmidt.

References

1. The lattice for all cases presented here is a full SSC injection lattice with random multipole errors in all dipoles according to specifications provided by the Parameter Working Group [July 6, 1988]. The random error b_2 is assumed to be binned, and have the corrected value of 0.4 units.
2. Forest, E., Berz, M., Irwin, J., SSC-166 (also to be published in Particle Accelerators)
3. Irwin, J., Diffusive losses Due to Long Range Beam-Beam Interactions, APS Bulletin 33, p. 909 (1988).
4. Schmidt, F., Tracking Results with Sixtrack for the 1988 Dynamical Aperture Equipment, CERN SPS/88-49 (AMS).

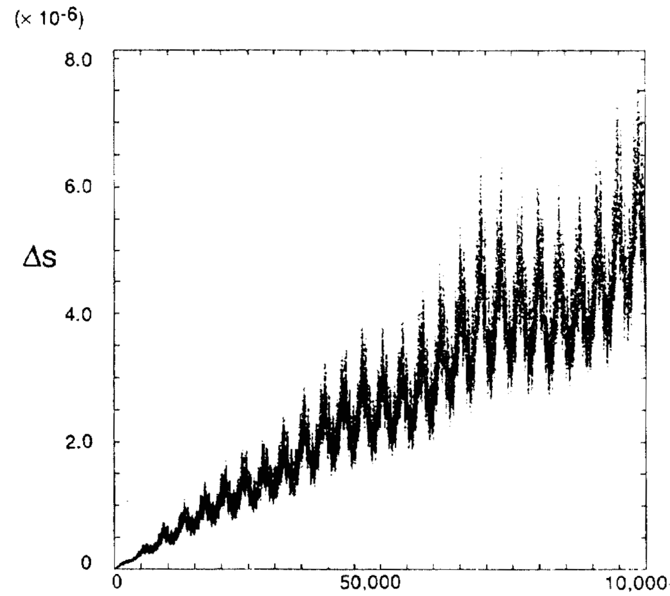

Figure 9. Phase space separation for two particles at initial x = y = 6.0 mm.

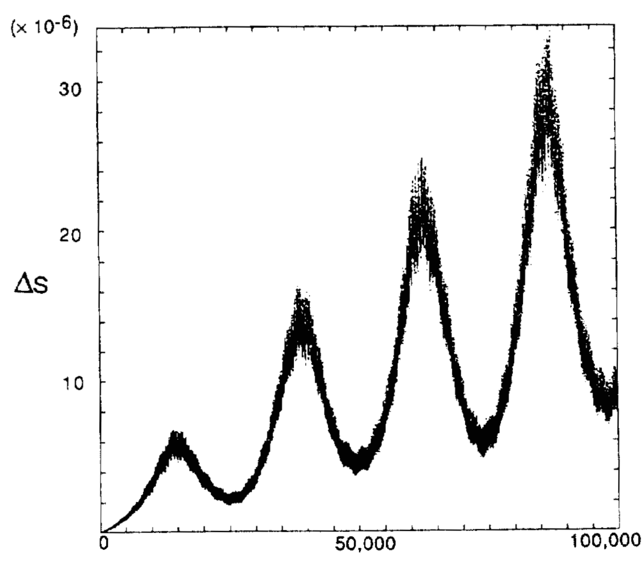

Figure 8. Phase space separation for two particles at initial x = y = 5.4 mm.

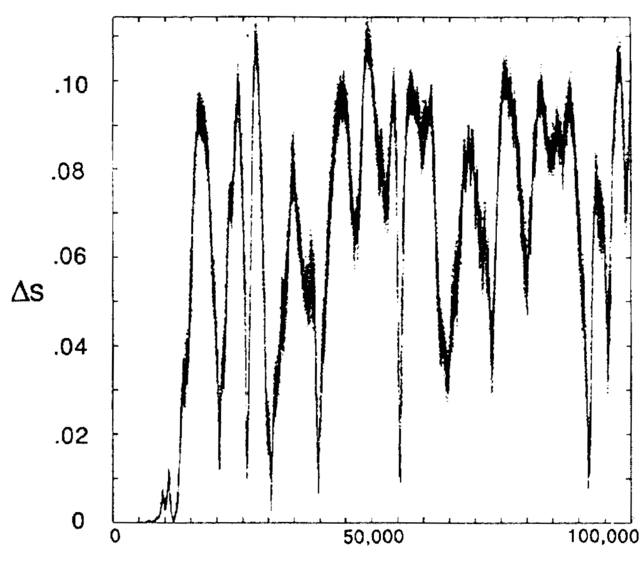

Figure 10. Phase space separation for two particles at initial x = y = 6.6 mm.