# BEAM DYNAMICS IN WIGGLERS: TRACKING IN PEP WITH DAMPING WIGGLERS AND MEASUREMENTS AT SPEAR*

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## Abstract

Future synchrotron radiation sources and damping rings will have a large fraction of their circumferences filled with wigglers and undulators. Thus investigations are needed of both linear and nonlinear effects of wigglers on electron beam dynamics. The nonlinear wiggler fields have been implemented in PATRICIA [1]. Results of tracking with PATRICIA for PEP with damping wigglers will be presented. Experimental measurements of tune shift with amplitude in a SPEAR wiggler are also included.

## Equation of motion

The equation of motion of an electron in a wiggler is given by [2]

$$\frac{d^2y}{dz^2} = \frac{-1}{2k\rho_w^2}\,\sinh ky\,\cosh ky = \frac{-y}{2\rho_w^2} + \frac{-k^2}{3\rho_w^2}\,y^3 . \qquad (1)$$

Here $\rho_w$ is the wiggler bending radius, $z$ is the distance along the wiggler, and $k$ is $2\pi$ over the wiggler period, $\lambda_w$.

The first term gives the linear focusing, while the second term is the strongest nonlinear effect. The linear term is the most damaging in rings with a small number of strong sextupoles like the Advanced Light Source [3]. The focusing breaks the periodicity of the phase advance between sextupoles, and thus reduces the dynamic aperture. The nonlinear term is the most damaging in damping wigglers and in rings with many weak sextupoles such as ELETTRA [4] in Trieste. Because damping wiggler strengths need not vary, the linear focusing can be included in the lattice design. It need not break the symmetry between sextupoles. Therefore, in PEP with damping wigglers it is wiggler nonlinearities that reduce the dynamic aperture.

## PEP with Damping Wigglers

The PEP storage ring has tremendous potential as a synchrotron radiation source. One way to help realize its full potential is to decrease the electron beam emittance. PEP has already been run in a configuration with stronger focusing in the horizontal quadrupoles with an emittance reduction of a factor of five from the colliding beams lattice [5]. With the addition of 200 meters of damping wigglers the emittance could be further reduced by a factor of eight to 6 Angstrom-radians at 6 GeV [6].

In a lattice with 200 meters of damping wiggler, it is important to carefully choose the wiggler period, the wiggler strength, and the beta functions at the wigglers in order to maximize emittance damping without destroying the dynamic aperture. The emittance reduction produced by damping wigglers when the dispersion is matched to zero at the entrance to each wiggler can be expressed as

$$\varepsilon_x = \varepsilon_{x0}\,\frac{1+Q}{1+D} ,$$

$$D = \frac{L_w\,\rho_0^2}{2\,L_0\,\rho_w^2} \quad ; \qquad Q \propto \beta_x\,L_w\,\lambda_w^2\,B_w^5 . \qquad (2)$$

Where $\varepsilon_x$ and $\varepsilon_{x0}$ are the horizontal emittances with and without wigglers, $\rho_0$ is the radius of curvature in the ring bending magnets, $L_w$ is the total length of damping wiggler, $L_0 = 2\pi\rho_0$, $\beta_x$ is beta in the wigglers, $\lambda_w$ is the wiggler period, and $B_w$ is the peak wiggler field.

The factor D results from radiation damping in the wigglers, and Q results from the quantum excitation due to the small oscillating dispersion created by the bending of the electron orbit in the wiggler fields. Effective damping wigglers have high fields or large D and short periods for small Q.

Unfortunately, the high-field wigglers with short period length needed to damp the emittance have strong nonlinear fields that reduce the dynamic aperture. The second term on the right in equation 1 gives a tune shift quadratic in betatron amplitude:

$$\Delta\nu_y = \frac{\pi}{4}\,\frac{L_w\,\beta_y}{\lambda_w^2\,\rho_w^2}\,y_0^2 . \qquad (3)$$

Where $y_0$ is the peak vertical betatron amplitude at the wiggler and $\beta_y$ is the beta function at the wiggler.

Together equations 2 and 3 show that the choice of $\lambda_w$ is a compromise between decreasing emittance and increasing dynamic aperture. When designing a lattice, the two equations can be used as guides with tracking and matching programs for choosing the beta functions and the wiggler parameters.

Tracking Results: The nonlinear fields of wigglers were added to the tracking program PATRICIA. After some iteration, the wigglers were chosen with a 10 cm period and $B_w$ = 11 kG. A seventeen meter wiggler was put in each half period for a total of 204 meters of damping wiggler. At 6 GeV this gives a D of 8, a Q of 0.10, and a tune shift with amplitude of $\Delta\nu_y/y^2$ = 0.0012 1/mm$^2$. PEP has a periodicity of 6 and each period is symmetric about its center. HARMON and

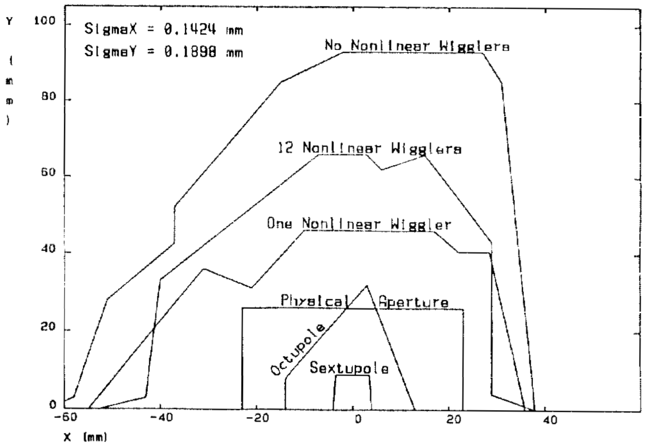

Figure 1. Dynamic apertures from tracking with damping wigglers for PEP. Tracking is done for artificial wigglers with no nonlinearities, eleven artificial wigglers and one wiggler with nonlinearities intrinsic in idealized fields, twelve wigglers with nonlinearities from idealized fields, twelve wigglers with construction-tolerance octupole nonlinearities scaled from measurements at SPEAR, and twelve wigglers with similar rotated sextupoles scaled from SPEAR.

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PATRICIA were used to maximize the dynamic aperture without wiggler nonlinearities. Figure 1 shows the dynamic aperture with and without the nonlinear fields of the damping wigglers. In both cases the linear focusing of the wigglers is included, so the dynamic aperture reduction comes only from the wiggler nonlinearities, not from a change in phase advance between sextupoles. Also included in figure 1 is the dynamic aperture if nonlinearities are included in only one of the twelve wigglers. This is of interest because the damping wigglers would be added to PEP in stages, not all at once. One might expect an increase in the dynamic aperture when the total wiggler nonlinearity is reduced by a factor of twelve, but this is not seen. The problem arises from breaking the periodicity of the wigglers. The sixfold periodicity of the sextupoles was maintained, but the periodicity of the wiggler distribution was eliminated.

The dramatic dependence of the dynamic aperture on the periodicity of the wiggler nonlinearities leads to concern with how magnet strength and alignment errors will affect the dynamic aperture. Wiggler magnets will be implemented in PATPET to study such effects in the near future.

## Experimental Measurements at SPEAR

Method: The vertical tune shift with amplitude was measured in the 15-period wiggler at SPEAR. The wiggler has a period length of 12.85 cm, a peak field of 14.5 kG, and a hybrid design using Nd-Fe-B magnet material combined with Vandium Permandur poles. The measurement was simplified by making a vertical beam bump in the wiggler, rather than exciting a vertical betatron oscillation. The resolution of small changes in tune needed for this experiment could not have been achieved with a spectrum analyzer, because the tune had a .002 modulation from 60 Hz ripple in the magnet power supplies as well as smaller ripple at 360 and 3 Hz. To overcome this problem the tune was measured using a phase-lock loop (PLL) circuit designed and built by J. Sebek at SSRL. The PLL input was from a strip-line, and the output of the voltage-controlled oscillator (VCO) was used to excite the electron beam using a different strip-line. With the VCO output counted on a frequency counter for ten seconds, the ripple was filtered out and tune changes could be measured repeatably to within an rms deviation of only .00008. Such high accuracy resulted in the smooth curves of tune vs beam-bump amplitude seen in figure 2.

Data: Figure 2 shows the measurements of the tune shift for 14.5 kG and electron energies of 1.62 and 2.35 GeV along with best fit parabolas. The best fit curvature in both cases agrees with theory within experimental accuracy (figure 3). Also shown in figure 3 are the measured curvatures at 2, 4, 5, and 10 kG. These are consistently somewhat greater than predicted by theory. Figure 4 shows the slope of the measured tune shift with amplitude. According to theory, such a linear tune shift should not occur in a wiggler with mid-plane vertical symmetry.

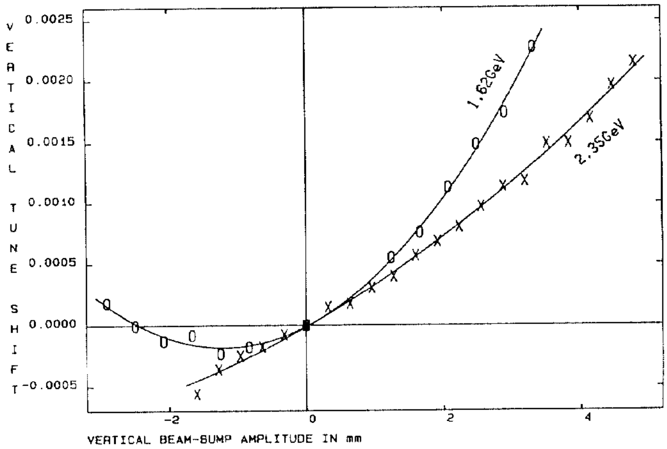

Figure 2. Tune shift with beam-bump amplitude measurements for the 15 period wiggler at 14.5 kG and electron energies of 1.62 and 2.35 GeV. The solid lines are best fit quadratics.

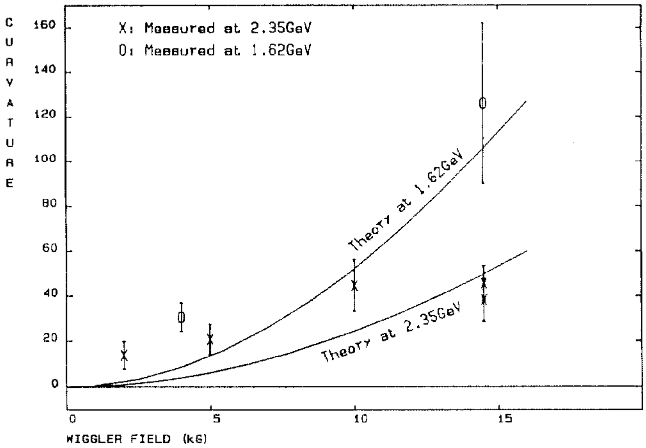

Figure 3. Best fit curvatures ($1/m^2$) to tune shift with beam-bump amplitude. The solid lines are theoretical curves assuming a perfect wiggler.

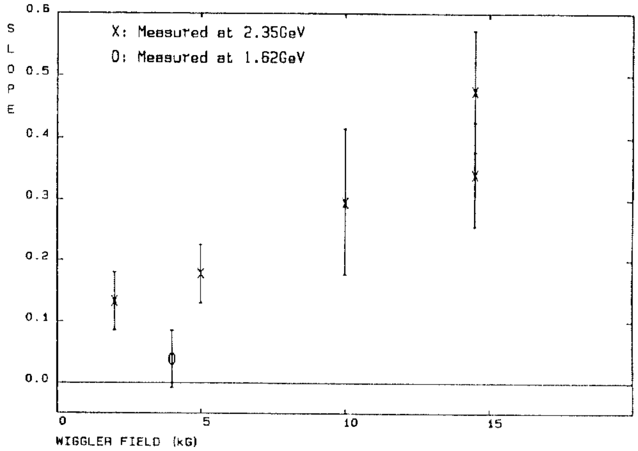

Figure 4. Best fit slopes (1/m) to tune shift with amplitude. In a perfect wiggler the slopes would be zero.

Multipoles from construction tolerances: The discrepancies between theory and measurement could be attributed to multipole errors in the wiggler due to construction tolerances. Magnetic measurements made prior to implementing the wiggler in SPEAR showed an integrated horizontal field along the center-line of 220 G*cm at 14.5 kG peak vertical field, 150 G*cm at 5 kG, and 90 G*cm at 1.2 kG. This nonzero integral gives an order of magnitude of the effects of construction tolerances. The octupole strength necessary to produce the difference between measured and theoretical quadratic tune shifts with amplitude seen at 2.35 GeV and 10 kG, has a wiggler pole to center-line change in integrated horizontal field of 570 G*cm. The rotated sextupole strength required for the slope seen at 2.35 GeV and 10 kG has a wiggler pole to center-line change of 490 G*cm. The field errors from mechanical tolerances are expected to be larger close to the magnet poles. Thus the octupole and sextupole errors do not appear unreasonable when compared to the integrated field error on the center-line.

Multipole fields from construction tolerances have been observed in other wigglers. When the first wiggler, a $SmCo_5$, 2.3 kG wiggler with thirty 6.1 cm periods and a 3 cm gap, was installed in SPEAR, it produced a rotated octupole resonance. The integrated octupole strength was 2030 G*cm at the pole tip [7]. This is nearly four times larger than the octupole field needed to explain the measured tune shift curvature in the 15 period wiggler.

Error Analysis: In order to gain further confidence that the discrepancies between measurement and theory are really due to wiggler field errors, possible sources of systematic and statistical error were investigated. Errors due to vertical beta function uncertainty, frequency measurement uncertainty, and beam-bump amplitude uncertainty were included in the error bars in figures 3 and 4. Tune shift from the following effects were analyzed ...

1) The rotated sextupole in the vertical correctors: Quadrupoles powered as dipole correctors have a substantial sextupole field [8]. The theoretical prediction of the sextupole strength agrees to within a few percent of measurements. Therefore, this contribution to the tune shift can be calculated with some accuracy and the result subtracted from measurement. The rotated sextupole gives a tune shift with amplitude that can be written as

$$\Delta\nu_y = \frac{\beta_y \theta y}{8 a^2} \quad ; \quad \theta = \theta_0 + \theta' y \qquad (4)$$

Here y is the amplitude of the beam bump at the corrector, a is the quad bore radius, $\theta_0$ is the kick angle of the corrector without the bump, and $\theta' y$ is the angle for a given bump amplitude. Both the systematic curvature and slope of the tune shift with bump amplitude due to correctors was included when calculating the tune shift with amplitude from the wiggler. Figures 3 and 4 already include this correction. The contribution to the slope is about -0.02 1/m (a 5% correction at 2.35GeV and 14.5kG), and to the curvature about -20 1/m**2 (a 20% correction at 1.62GeV and 14.5kG).

2) Nonzero initial closed orbit in the wiggler: Because the beam position monitors on either side of the wiggler were not working, the closed orbit in the wiggler without the beam bump could not be measured. This introduced an uncertainty in the slope of the tune shift in the center of the wiggler of $c*\sigma_y$, where c is the curvature of the tune shift and $\sigma_y$ is the rms (properly beta weighted) closed orbit. For the measurements at 1.62GeV and 14.5kG, where the curvature was largest, this error completely washed out the measurement of the slope. (That is why this data point is not included in figure 4.) For the other measurements, this error is included in the error bars of figure 4.

3) The main ring sextupoles: The small horizontal orbit distortion in the main ring sextupoles occurred when the vertical beam bump was made. This caused negligible tune shift.

4) Tune shift due to path length change: Because the rf-frequency is constant, the increase in closed orbit length from the vertical bump must be compensated by a decrease in horizontal orbit. To achieve this decrease in horizontal orbit requires a decrease in energy and a tune shift proportional to chromaticity. This tune shift is also negligible compared to that measured in the experiment.

5) Edge fields of the ring bending magnets: The edge fields have a $y^3$ term in the vertical equation of motion analogous to that found in wigglers. The strength of this term was calculated by numerically integrating an approximation [9] to the end fields. This tune shift gave no more than a slight contribution to the error bars in figure 3.

6) Octupole and rotated sextupole in the ring quadrupoles: Measurements of the individual quadrupole magnet multipole errors were never made at SPEAR. The multipole errors were only measured in the prototype. Using the prototype as a guide, a rough estimate of the slope and curvature of the tune shift with amplitude from multipoles produces ±.04 1/m (10% of the slope at 2.35GeV and 1.45kG) and ±.5 $1/m^2$ (a negligible error).

All of the above sources of error in the measurement are included in figures 3 and 4, and still there are substantial variations between the tune shift from an ideal wiggler and the measurements. These I attribute to construction-tolerance multipoles. In one respect, that I feel I must mention, the data appears contradictory. The slope of the tune shift with amplitude should increase as the electron energy decreases for a fixed wiggler field. One would expect the slope to be greater than observed for the data taken at 1.62 GeV and 4 kG. If there is a chance in the future, I will retake the data for this point.

Tracking in PEP with Construction Tolerance Errors: When the rotated sextupole and octupole fields are scaled in length for the proposed 17 m damping wigglers in PEP, tracking simulations show that they are more damaging to dynamic aperture than the nonlinearities of an ideal wiggler (figure 1). The dynamic apertures in figure 1 were calculated assuming all the damping wigglers had the same octupole and rotated sextupole field errors. If the field errors change randomly from one wiggler to the next, the sixfold symmetry is broken and the dynamic aperture is reduced even further.

## Conclusion

More work is needed studying the effects of construction-tolerance field errors on beam dynamics both for damping wigglers and for wigglers designed to produce synchrotron radiation in the next generation of storage rings. Measurements should be made on existing wigglers to determine their multipole content. Further experiments should be done at rings with wigglers presently installed. Such experiments should include measurements of tune vs amplitude, of resonances from wigglers, and of dynamic aperture reduction from wigglers. Tracking with PATRICIA for SPEAR with the rotated sextupole and octupole components included in the 15 period wiggler indicate that the dynamic aperture may be inside the physical aperture. Thus such a dynamic aperture experiment could be possible at SPEAR.

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