THE EFFECT OF MAGNETIC FIELD ERRORS ON THE RADIATION SPECTRUM OF ELETTRA UNDULATORS

B. Diviacco and R.P.Walker

Sincrotrone Trieste, Padriciano 99, 34012 Trieste, Italy

Abstract

The effect of magnetic field errors on the brightness of the radiation produced by undulators in ELETTRA has been investigated by direct numerical calculation. The model has allowed the statistical variation in the intensity and position of spectral peaks to be studied as well as the effect of segmenting the undulator into a number of individually compensated sections. A comparison with an existing analytic theory is also made.

Introduction

Permanent magnet undulators will be the most important means of producing high brightness radiation in the next generation of low emittance storage rings such as ELETTRA [1]. It is well known that the performance of such devices is limited, particularly for higher harmonic operation, by the achievable field quality [2,3,4]. Field errors, arising from magnetisation errors in the permanent magnet blocks as well as dimensional and constructional errors, introduce electron trajectory deviations and hence a variation in the phase of the emitted radiation that results in reduced brightness.

In the following we examine the effect of errors in one of the proposed undulators for ELETTRA [5] using a simple field model and a direct spectral calculation. Undulator U2 has a period of 56 mm and at minimum gap a K value of 3.4. At 2 GeV the device will produce radiation with photon energy tunable from 100 eV to 1 keV using the first and third harmonics. In these calculations a total length (L_{tot}) of 5 m has been assumed coresponding to 90 periods. The object of the study was to determine a specification for the required field quality and also to investigate the possibility of using higher harmonics.

Random Error Model and Spectrum Calculations

In the following the real undulator field distribution is approximated by a series of half-sinusoids representing individual poles, with nominal strength B_o and a random error assumed Gaussian with a standard deviation $\sigma_B B_o$. Half amplitude poles are included at either end in order to produce no net change in angle or position of the electron beam in the case of no field errors. Such a model clearly involves several approximations. Only field errors in the direction of the main field component are considered since errors in the orthogonal plane are generally much smaller, particularly in the hybrid structure. The field errors are assumed to be sinusoidal, localized to individual poles and uncorrelated. For the hybrid, measurements of the residual errors in one such device indicate that this is a good approximation [6], however data is lacking for the pure permanent magnet case. Finally, the field at the end of the magnet is represented in only a simple way - however calculations show that provided the number of magnet periods is reasonably large (N > 20) the resulting radiation spectrum is not sensitive to the detailed end field distribution.

The electron trajectory in such a field lies in the horizontal x-z plane, and is given to a good approximation by :

$$x'(z) = -\frac{e}{\gamma m c}\int_{-\infty}^{z} B_y(z)\,dz \qquad x(z) = \int_{-\infty}^{z} x'(z)\,dz$$

The radiation quantity of most interest is the on-axis spectral brightness (flux per unit solid angle per unit bandwidth), which can be calculated by :

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} x'(z)\, e^{i\Phi(z)}\,dz\right|^2$$

where Φ is a phase function given by :

$$\Phi(z) = \frac{\omega}{2c}\left(\frac{z}{\gamma^2} + \int_{-\infty}^{z} x'^2(z)\,dz\right)$$

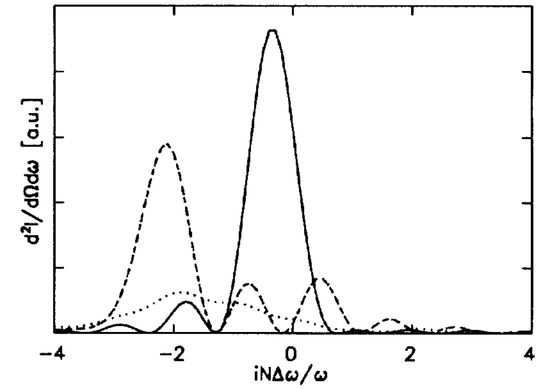

Fig. 1 Three sample spectra with the same rms field amplitude error

Figure 1 shows the first harmonic spectra generated by three sample electron orbits in the U2 undulator, corresponding to different sets of random errors with the same standard deviation (1%). Note the large variation in performance: for a given magnetic field tolerance, the output is strongly influenced by the exact error distribution. A single case cannot therefore be representative of the general behaviour and a statistical analysis is needed. We decided to characterize each individual spectrum by the relative shift in frequency and normalized intensity of the peak. The statistical behaviour is then shown by histograms of these two quantities for an ensemble of undulators each

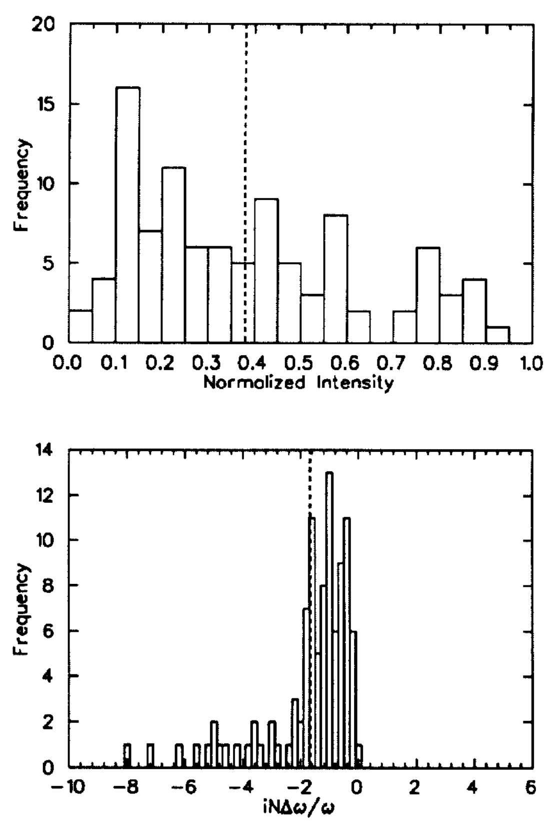

Fig. 2 Distribution of intensity and shift in frequency of first harmonic peak for an ensemble of 100 undulators, uncompensated case, first harmonic (σ_B = 1%)

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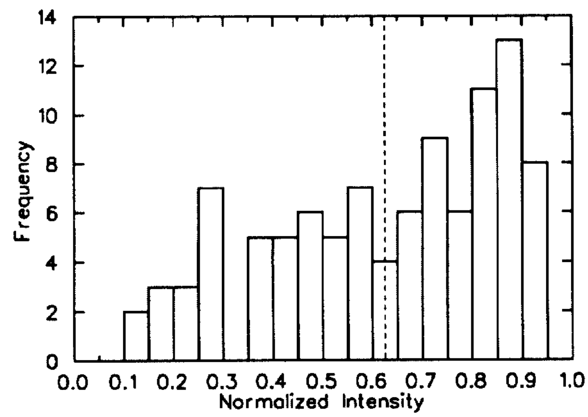

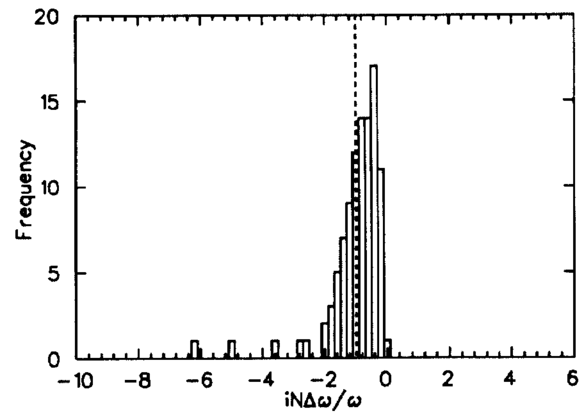

Fig. 3 As fig. 2, compensated case

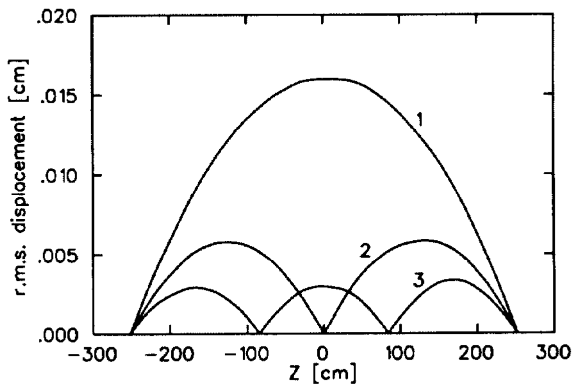

Fig. 4 Average trajectory deviation in an ensemble of undulators compensated in 1, 2 or 3 sections ($\sigma_B = 1\%$)

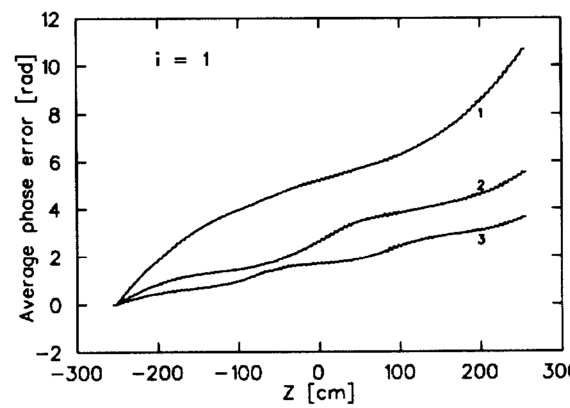

Fig. 5 Average phase error for the same case as fig. 4

with a different set of errors. An example is shown in fig. 2. Although an average intensity and shift can be associated with a given error σ_B, it is clear that large deviations from the average are possible and that a nearly ideal spectrum could be obtained by a suitable arrangement of errors. Such variations are ignored in the analytic theory of ref. 2 which derives only average results. The net shift in the frequency of the peak results from the effective finite observation angle of the radiation.

To compensate for both the final position and angle error steering correction is required at both ends of the magnet. The compensation scheme can be easily included in the simulation by appropriate adjustment of the end pole amplitudes. Figure 3 shows the statistical distributions for the compensated case for the same ensemble of undulators as in fig. 2. It can be seen that the whole distribution shifts to higher intensities with an increase of the average intensity by a factor 1.7. The position of the peak is also more well defined and closer to the ideal case since the average angular deviation of the trajectory is reduced. Thus compensation, which is necessary for the storage ring operation if a high degree of beam stability is required, also improves the emission properties of the device.

Segmentation

The results above suggest the possibility of segmenting the undulator into shorter individually compensated sections. This can be included in the model simply by overlapping the half-amplitude poles of adjacent compensated sections. Figure 4 shows the calculated rms displacement from the central axis for an ensemble of U2 undulators divided into n = 1, 2 and 3 sections. It can be seen that the average amplitude of the random walk is strongly reduced by segmentation, varying as expected as $L_s^{3/2}$ where $L_s=L_{tot}/n$. The accumulated phase error also reduces (fig. 5), being proportional to L_s^2 at the end of each section and hence L_{tot}^2/n for the whole device. The resulting improvement in performance for the 3 section case is shown in fig. 6. There is an increase in average spectral brightness by a further 30% compared to the 1 section case and a narrowing of the distribution with fewer samples of low intensity, showing the interest of this technique as a practical way of reducing the effect of field errors.

In the above no separation was assumed between sections. A separation introduces an additional phase error which depends on the exact fringe field distribution and hence varies with both the separation distance and the field amplitude (vertical gap) [7]. In the worst case of a phase difference of π (for 2 or 3 sections) the lineshape becomes double-peaked and the spectral brightness is reduced by about a factor of 2, as shown in fig. 7 (for n = 3). Also shown is the effect of errors in this case and it can be seen that the spectrum remains dominated by the phase error. This hase been verified for an ensemble of undulators also. Thus in cases where the brightness is important and is not dominated by electron beam emittance effects a separation between sections should be avoided.

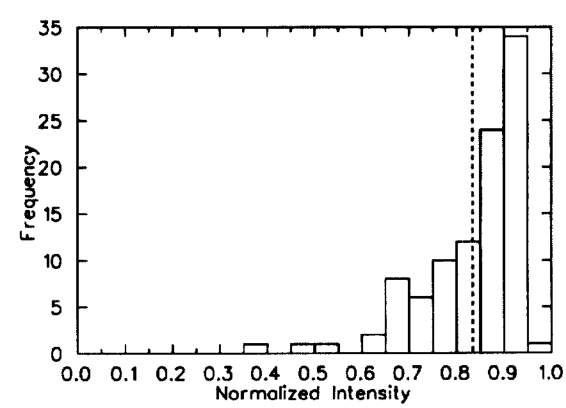

Fig. 6 As fig. 2, compensated in 3 sections

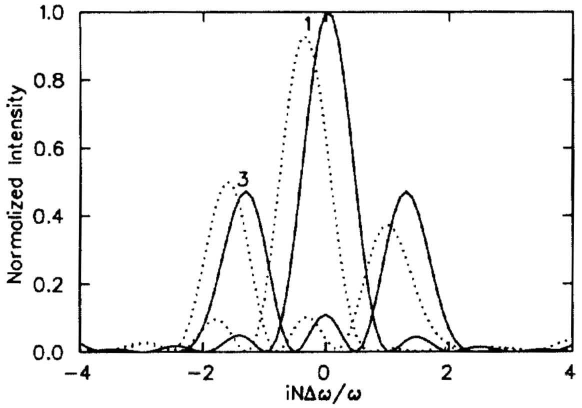

Fig. 7 Solid lines - ideal spectra of N period undulator in one section and in three sections with π phase error. Dotted lines - effect of field errors for one sample case (σ_B=1%)

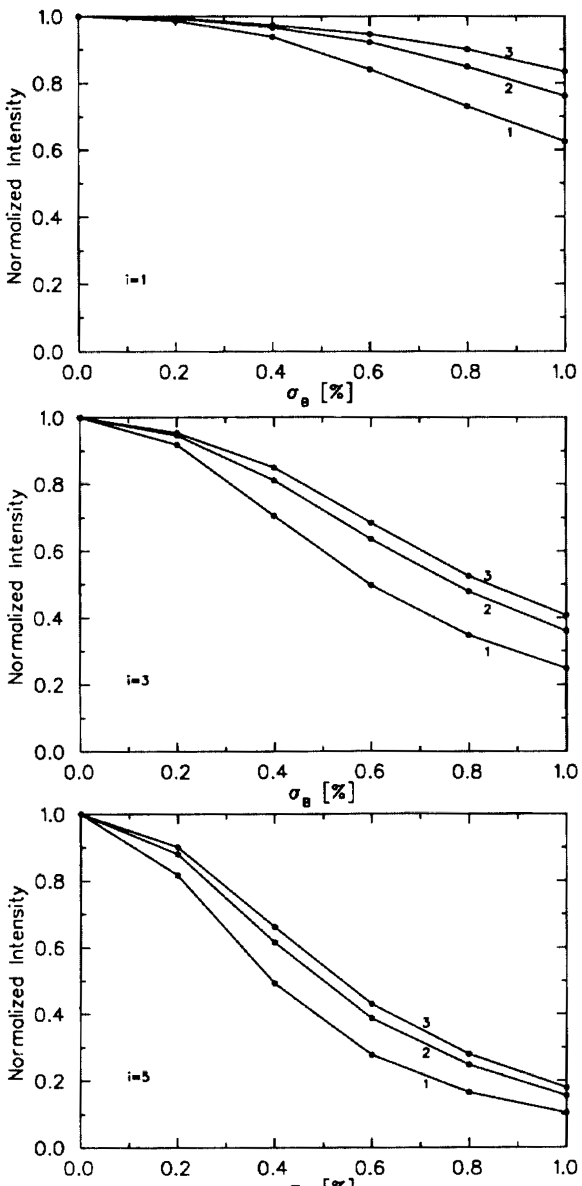

Fig. 8 Variation of average intensity with field error for different harmonics (i) and number of sections

Dependence of Harmonic Intensities with Field Error

The average relative brightness of the first, third and fifth harmonics of a U2 undulator as a function of the field error is shown in fig. 8, with the undulator compensated in n = 1, 2 and 3 sections (with zero separation). For each data point an ensemble of 100 undulators has been considered. For the first harmonic the intensities for the different numbers of sections can be seen to depend on the quantity (σ_B^2/n) in agreement with the magnitude of final phase error. For the higher harmonics however the improvement due to segmentation is smaller.

As predicted [2], for a given error higher harmonics are reduced in intensity by a larger factor than the first. Figure 9 shows a comparison of the results for 1 section with the prediction of the analytic theory of ref. 2. The agreement appears to be quite good for the first harmonic, but the intensities of the third and fifth harmonic seem to be overstimated by the theory, a result that is not presently understood.

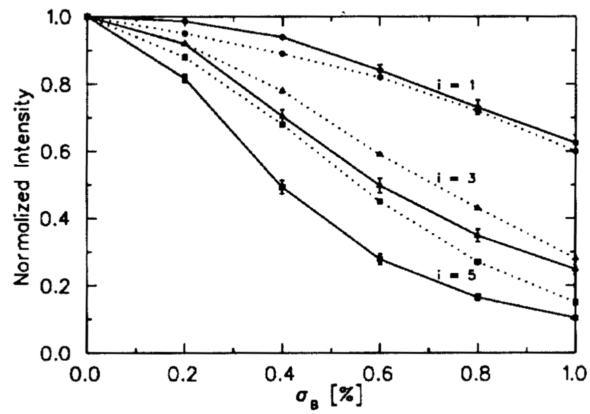

Fig. 9 Comparison of calculated average intensity (solid lines) with theory [2] (dotted lines) for different harmonics (i)

Conclusion

The large statistical variation in peak brightness occuring for a given rms field error, that is ignored in the existing analytic theory, has been highlighted. Some discrepancies between theory and the present results for higher harmonics have also been noted. Introducing additional steering elements in an undulator to reduce trajectory deviations can improve the spectral brightness, reducing the effective field errors by about a factor $1/\sqrt{n}$, where n is the number of sections. For the ELETTRA undulator studied, assuming separation into 3 sections, an rms field error of 0.5 % can be specified which on average gives a brightness reduction of 25 % for the third harmonic and 50 % for the fifth harmonic.

References

[1] A. Wrulich, this Conference.

[2] B.M. Kincaid, J. Opt. Soc. Am. B2 (1985) 1294.

[3] R. Tatchyn et. al., Int. Conf. Insertion Devices for Synchrotron Sources, Proc. SPIE 582 (1986) 47.

[4] E.E. Alp and P.J. Viccaro, Nucl. Instr. Meth. Phys. Res., A266 (1988) 116.

[5] R. Rosei and R.P. Walker, Proc. Synchrotron Radiation Instrumentation Conf., Aug. 1988, Tsukuba, Japan, to be published.

[6] K.E. Robinson et. al., IEEE J. Quantum Electron. QE-23 (1987) 1497.

[7] B. Diviacco and R.P. Walker, Sincrotrone Trieste Internal Report ST/M-88/9, May 1988.