COMPUTER SIMULATION OF FEL SIDEBANDS IN A STRONGLY DISPERSIVE WAVEGUIDE

E.J. Sternbach*
Lawrence Berkeley Laboratory
1 Cyclotron Rd.
Berkeley, CA 94720

Abstract

With the development of high power FEL's, it is necessary to consider the development of sidebands in the radiation spectrum. These sidebands can considerably widen the bandwidth of the FEL output and if sufficient power is developed, this can lead to the detrapping of electrons. It has been noted previously that the effects of waveguide dispersion can affect the sideband gain and location[1][2]. In this report the author describes a set of coupled equations suitable for the study of the radiation spectrum of an FEL. These equations include all the effects of waveguide dispersion. A computer simulation that numerically integrates these equations is used to study sideband growth and saturation in the regime of strong dispersion.

Introduction

Many instances of high power FEL's have been demonstrated in recent years. In the regime where powers are high and electron bunching is strong, the development of sidebands to the fundamental FEL frequency is likely. These sidebands broaden the radiation spectrum and under certain conditions can cause detrapping of the electrons from the FEL bucket. It has been noted previously that in a waveguide FEL, the dispersion caused by the waveguide will have a significant effect on the sideband physics. Of particular interest is the regime where the group velocity of the radiation becomes equal to the parallel velocity of the electrons[1]. These conditions cannot be properly described by the usual FEL equations since the major assumption, that of slowly varying amplitude and phase, breaks down. In this paper, a formalism previously developed to deal with waveguide FEL's is briefly described. A computer simulation based on this formalism is used to illustrate some aspects of general sideband physics, particularly the regime of strong dispersion.

Basic Equations

In this section the equations used in the computer simulation are described. The derivation of these equations are described in great detail elsewhere[3][4].

If we assume that the radiation field consists of a finite number of discrete frequencies, then we can derive a pair of first order linear differential equations for each frequency. It is assumed that only forward traveling radiation is important and that nonlocal effects are negligible. The equations that result are

$$\frac{da_{sj}^{mn}}{dz} = \mathcal{J}_j^{mn} \sin\theta_{pj} \quad , \tag{1a}$$

$$\frac{d\phi_{sj}^{mn}}{dz} = \frac{\mathcal{J}_j^{mn}}{a_{sj}^{mn}} \cos\theta_{pj} \quad , \tag{1b}$$

with

$$\mathcal{J}_j^{mn} = \frac{4\pi e^2}{m_e cab} a_w \lambda_{0j} \frac{\cos\frac{m\pi}{2}\sin\frac{n\pi}{2}}{(1+\delta_{m0})} \quad , \tag{2}$$

Here a_s and ϕ_s are respectively the amplitude and slowly varying phase of the radiation. The superscripts mn refer to the transverse waveguide mode and the subscript j refers to the particular frequency being followed. The quantity θ_p represents the difference in phase between the peak of the current component at that frequency and the bottom of the pondermotive well defined by radiation at that frequency. The quantity $\mathcal{J}_j^{mn}$ depends on the FEL geometry and is given here for a linear wiggler in a rectangular waveguide. The terms a and b represent respectively the x and y dimensions of the waveguide. The term λ_0 is the Fourier amplitude of a quantity that goes like the number line density of electrons divided by γ, the relativistic energy factor. The term a_w is the wiggler vector potential normalized by e/m_ec^2.

The evolution of the current, and therefore the factors θ_p and λ_0, can be obtained by following the motion of individual particles. In terms of γ and t, the particle motion in a linear wiggler is given by

$$\frac{dt}{dz} = \frac{1}{v_z} \quad , \tag{3a}$$

and

$$\frac{d\gamma}{dz} = \frac{\omega}{c} v_x \sum_{\substack{m,n\\j}} a_{xj}^{mn}(z,t) \quad , \tag{3b}$$

where a_x is the normalized vector potential of the radiation field. Each term in the sum of (3b) will move at a different speed in a waveguide. Since the vector potential of the wiggler field is much larger than that of the radiation field it is a quite good approximation to assume that v_x and v_z do not depend on the magnitude of the radiation field. Thus they are

$$v_x = \frac{a_w(z)}{\gamma} \quad , \tag{4}$$

and

$$v_z = c\sqrt{1 - \frac{1 + a_w^2(z)}{\gamma^2}} \quad . \tag{5}$$

The Computer Simulation

The computer simulations described in this paper were performed on a CRAY 2 supercomputer. Eqns. (1) and (3) were integrated using a version of the GEAR integration package. Since following an entire electron pulse would require too much memory, a length of the electron beam is followed with periodic boundary conditions. The radiation equations follow a particular Fourier component so the longer this length is, the more resolution one can achieve in the simulation. For the simulations done here the length of beam in the simulation was 32 times the bucket length of the fundamental frequency. This gives 3.1% resolution in the radiation spectrum calculations.

The sidebands in these simulations are assumed to start from noise. This noise is assumed to have a flat power spectrum and the total noise power is an input parameter. The simulations in this paper have an input noise power of 5 watts.

Basic Sideband Physics

The sideband instability is seen only when the FEL approaches

* Work supported by the U.S. Department of Energy, at the Lawrence Berkeley Laboratory under contract #DE-AC03-76SF00098.

Table I. Simulation parameters

Relativistic energy γ	7.1	
Wiggler peak field	3.80	kG
Wiggler wavelength	9.8	cm
Radiation frequency	34.6	GHz
Beam current	800	amps
Waveguide size	9.8 × 2.9	cm
Equilibrium radiation power(TE^{01})	200	MW

saturation and the electron beam is tightly bunched. Thus for this section we will use a simulation where there is one macroparticle per bucket to study sideband gain. This is good model as long as sideband power is small compared to the power in the fundamental. This will allow us to understand some aspects of sideband physics without being confused by complexities due to the electron distribution.

The simulations of this section use the parameters of Table I. These are similar to the parameters of the ELF experiment at Livermore. The plots in Fig. 1 were started from an equilibrium where the particle was started in the center of the bucket. Thus the equilibrium exhibits no synchrotron oscillations. Normally both the upper and lower sidebands grow. Contrary to expectations from the simple theory to date, the lower sideband has significantly higher gain than the upper sideband. When only the lower sideband is allowed to grow, it exhibits a spectrum nearly the same as the simulation with the full spectrum. However, if only the upper sideband is allowed to grow, there is no amplification at frequencies higher than the fundamental.

The observed features can be explained if one considers the creation of FEL sidebands as stimulated scattering. The lower sideband is the Stokes wave and the upper sideband is the anti-Stokes wave. The scattering medium is the tightly bunched electrons which

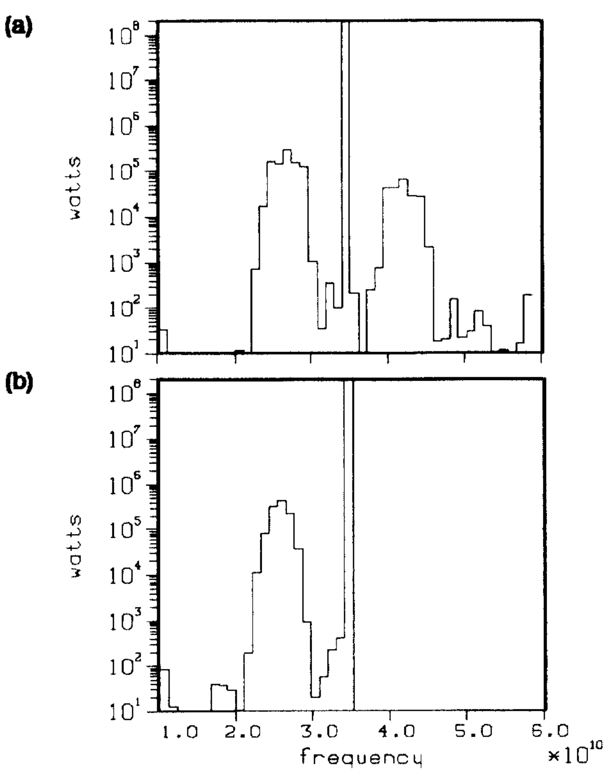

Figure 1 Spectrum at 3 meters.
(a) Full radiation spectrum. **(b)** Only lower sideband is allowed to grow. Not shown is a case where only the upper sideband was allowed to grow. No amplification of the upper sideband was seen at all in that case.

naturally oscillate at the synchrotron frequency. If one starts with an equilibrium where there is initially no synchrotron oscillation, then the upper sideband cannot grow in a two wave process since it must grow at the expense of the synchrotron oscillation. Fig. 2 illustrates that if one starts from an equilibrium with a large synchrotron oscillation, and if only the upper sideband is allowed to grow, then one does observe amplification at the upper sideband frequency and the synchrotron oscillation decays in amplitude.

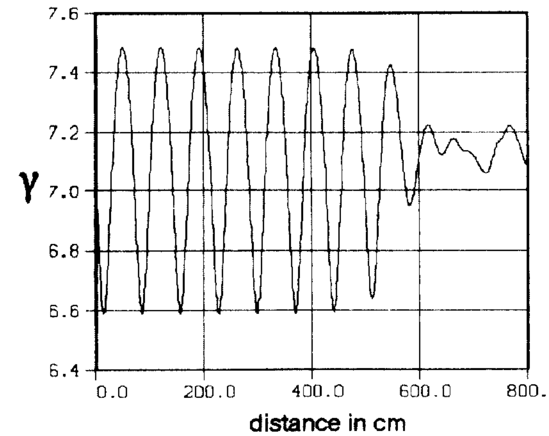

Figure 2 When only the upper sideband is allowed in the simulation, it must grow at the expense of the synchrotron oscillation.

Effect of Dispersion on Location and Gain of Sidebands

In this section we examine some one particle per bucket simulations of FEL sidebands for different levels of dispersion. The parameters used are basically that of Table I except that for different waveguide sizes the wiggler field needed for resonance is changed. For each of these simulations, only a TE^{01} mode is followed. The dispersion then depends only on b, the y dimension of the waveguide. The x dimension, a, in these simulations are adjusted to give the same cross sectional area for the waveguide in each simulation.

Fig. 1a and Figs. 3a through 3c represent simulations in successively narrower waveguides. This represents increasing the amount of dispersion. The final plot in Fig. 3c represents a case where the group velocity of the fundamental frequency in the waveguide is equal to the parallel velocity of the electrons. This "no-slip" condition can be written most simply as

$$(\gamma_t^{mn})^2 = k_w k_s \quad , \tag{6}$$

where for each waveguide mode

$$\omega^2 = (\gamma_t^{mn})^2 + k_s^2 \quad . \tag{7}$$

Here k_w is the wiggler wave number and k_s is the radiation wave number. Since k_s depends on ω through γ_t, the three dependent quantities are the radiation frequency, the wiggler wavelength, and the waveguide size.

The effect of dispersion on sidebands can be summarized as follows. Dispersion causes the sidebands to occur at a greater distance from the fundamental frequency. Even for the simulation in Fig. 1a where dispersion is relatively small, this is a significant effect. An analysis ignoring dispersion would predict a separation of the sideband from the fundamental for Fig. 1a to be about 3.5 GHz. It can be seen that the actual separation is closer to 7GHz. This separation can be seen to increase for smaller waveguide heights in Fig. 3. Also the gain can be seen to decrease for smaller waveguide heights. When the "no-slip" condition is satisfied as in Fig. 3c, the

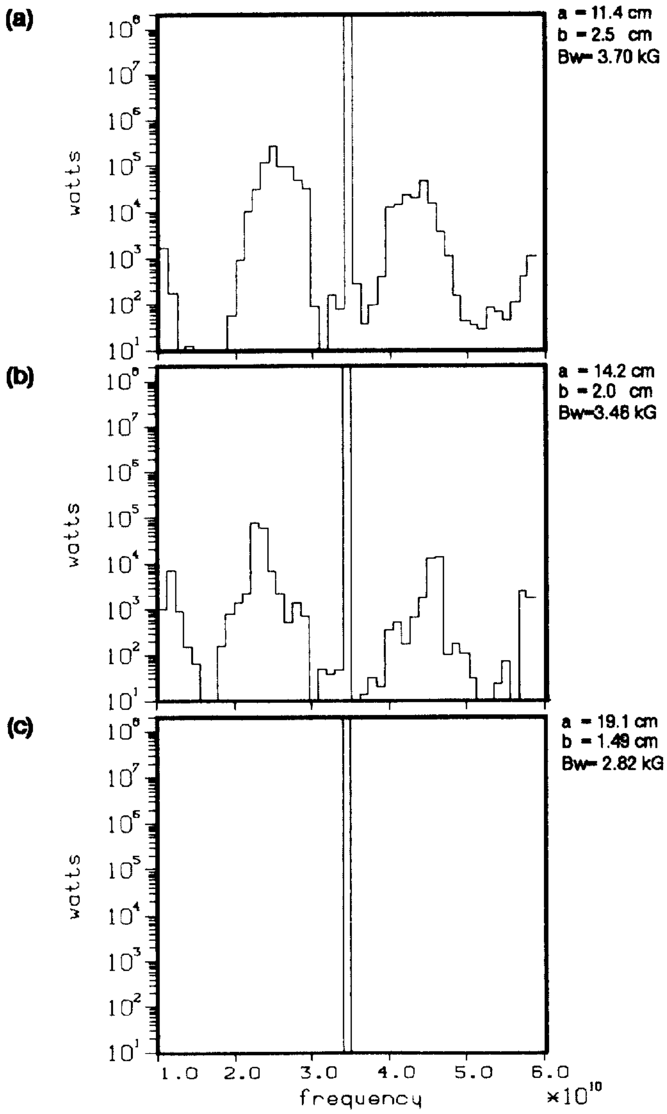

Figure 3 Radiation spectra at 3 meters distance.

sideband gain can be seen to be suppressed entirely. For a sideband frequency to grow, it must modulate the current over a region of many FEL buckets. If the radiation group velocity and the electron parallel velocity are identical, then no information can travel along the electron beam and therefore sidebands cannot grow.

Nonlinear Sideband Simulations

In this section we examine a full many particle sideband simulation where the sideband power is allowed to grow until it is of the same order of magnitude as the fundamental frequency. In this regime, the one particle per bucket simulation is no longer relevant. The parameters used are once more those of Table I with the exception that the FEL is started from an input signal of 60kW and the peak wiggler field is 3.59kG to start the FEL at the peak of the gain curve.

The radiation spectrum in Fig. 4 is taken at 6 meters where there is sufficient sideband power for the system to be highly nonlinear. Four upper sideband peaks are clearly visible. The first lower sideband peak is clear and is nearly as high as the fundamental. Other sideband peaks exist below this, but as waveguide dispersion becomes more prominent the spectrum becomes more complicated. As the buckets for the lower sidebands become large enough for these

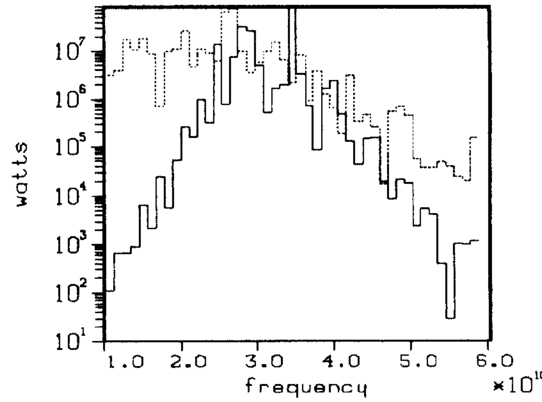

Figure 4 Radiation spectrum at 6 meters for full FEL simulation. Dotted line represents the results of a one particle per bucket simulation and can be seen to be quite far from the many particle simulation in the nonlinear regime.

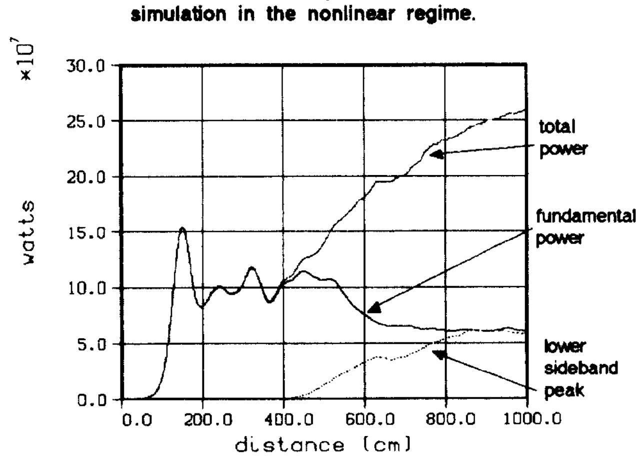

Figure 5 Power vs. distance for full FEL simulation

frequencies to become self- amplifying, the electrons "leak" out of the fundamental frequency bucket to interact with the lower sideband bucket and therefore are decelerated. Thus if an untapered FEL is long enough, power will continue to be produced by electrons cascading into lower buckets and this process will continue to widen the FEL spectrum into lower frequencies.

Conclusion

Computer simulations can give many insights into the behavior of FEL sidebands. They seem to confirm that the sideband instability is a process similar to stimulated scattering. Simulations performed in highly dispersive waveguides show that dispersion can have a large effect on the location and gain of FEL sidebands. In the case where the electron velocity and the radiation group velocity become equal, the sideband instability is completely suppressed.

References

[1] S.S Yu et al., *Nuclear Instruments and Methods*, **A259**, 219(1987).

[2] J. Masud et al., *Phys. Rev. Lett.*, **58**, 763(1987).

[3] E.J.Sternbach, *Nuclear Instruments and Methods*, **A272**,323(1988).

[4] E.J. Sternbach, "Formalism for the Study of Multiple Frequency Effects in a Waveguide FEL," Lawrence Livermore National Laboratory Research Memo #88-27.