AN ULTRA-LOW EMITTANCE DAMPING RING LATTICE AND ITS DYNAMIC APERTURE

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Abstract

A few of the problems in the design of an ultra-low emittance damping ring were considered. Once the goals of low emittance and fast damping times are acheived in the linear lattice design, there remain the serious problem of the nonlinear beam dynamics. Low emittance lattices usually have small dynamic aperture due to the particularly strong sextupoles necessitated by the small average disperion function. The dynamic aperture solution is to abandon the conventional interleaved sextupole scheme and to place sextupoles in pairs 180 degrees apart in betatron phase with no other sextupoles in between. The deleterious effect of the sextupole thickness can be partially corrected by octupolar fields.

Linear Lattice

The ring is made of twelve arc sections and twelve straight sections for wigglers.

Arcs

The large bending radius and short FODO cells in the arcs fulfill the requirement that they produce little quantum excitation in the beam. At the same time, a large circumference will allow the damping of many bunches at a time, thus decreasing the effective damping time.

The optimization of the phase advance per cell depends on whether the dynamic aperture or the emittance contribution of the arcs is to be compromised. For the best possible dynamic aperture, the sextupoles, which are needed to correct the chromaticity, are inserted in the lattice in pairs minus unit matrix apart in both planes (see section on sextupole arrangements). In this arrangement, the thin-lens sextupoles are invisible to the lattice (no aberrations to any order), but the choice of phase advance per cell is restricted to 180(2n+1)/m degrees where n,m are integers [1]. This is a strong restriction, as the compensation disappears quickly when the value of the phase advance departs from the optimal value. In any case, the most space efficient phase advance per cell is 90 degrees in both planes where a sextupole can be inserted every two cells (m=2).

If the compensation of the sextupoles geometrical aberrations is secondary in importance to the emittance, the optimal phase advance per cell is around 145 degrees for the horizontal plane. There is no restriction on the value of phase advance in the vertical plane. At this stage one can reduce the effect of the sextupoles' geometric aberrations by inserting one near each appropriate quadrupole because the strengths of the individual sextupoles are minimized. In practice, the high chromaticity per cell which makes the sextupole strengths large induces one to choose a cell with a smaller horizontal phase advance and chromaticity. The emittance does not vary much in the neighbourhood below the optimal value of phase advance per cell.

An emittance of about 3×10^{-10} m-rad at 4 GeV for the FODO cell was obtained with a cell length of 7.2 m. An extra factor of fifteen in emittance reduction is obtained by the strong damping of the wigglers.

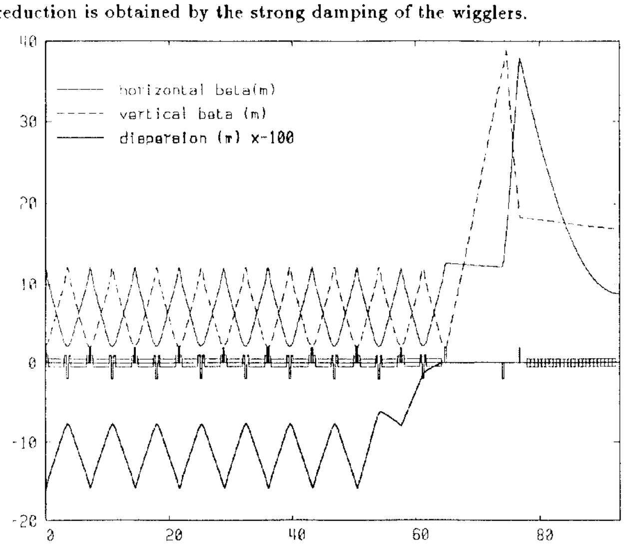

Figure 1: Machine functions for one half of a superperiod. The wiggler is represented as a series of small boxes on the right hand side.

Table 1: Main parameters of ring

Energy (GeV)	4
Circumference (m)	2229.
Emittance at 4 GeV (m-rad)	2.5×10^{-11}
Horizontal damping time at 4 GeV (msec)	13.5
Horizontal and vertical tunes	61.74/62.84
Number of superperiods	12
Energy loss per turn at 4GeV (MeV)	4.4
Energy spread at 4GeV (%)	.09
Momentum compaction factor	.00029

Table 2: Arc parameters.

Number of FODO cells	192
Length of each cell (m)	7.2
Number of bending magnets	432
Length of bending magnet (m)	2.5
Bending radius (m)	152.8
Bending field (kG)	0.87
Total number of quadrupoles in arc	420
Total number of matching quadrupoles	72
Length of arc quadrupole (m)	0.3
Strength of arc quadrupole (m^{-2})	±1.34
Field gradient in arc quadrupole (kG/cm)	±1.78
Number of focusing sextupoles (SF)	96
Number of defocusing sextupoles (SD)	192
Length of sextupole (m)	.3
Integrated strength of sext. (SF/SD)(m^{-2})	-8.2/8.0
Second derivative of field in sext (kG/cm^2)	±1.36

Wigglers

The high fields of the wigglers (the hybrid Samarium-Cobalt type is assumed) provide almost all of the damping in the ring. Because the wiggler generates its own dispersion function, the wiggler period must be short to ensure a small enough quantum excitation, though it may greater than that of the arcs.

The wiggler field is modeled as a sine function with the maximum on-axis field related to the gap between the poles and the period length. The vertical focusing of each wiggler pole is taken into account in the machine function matching.

The emittance written in the following way and exhibits the wiggler and the arcs contributions (see ref. [2]):

$$\epsilon[\mathrm{m-rad}] = 3.84\times10^{-13}\gamma^2 \frac{\left(\int \mathcal{H}\rho^{-3}ds\right)_{arcs} + \left(\int \mathcal{H}\rho^{-3}ds\right)_{wigglers}}{\left(\int \rho^{-2}ds\right)_{arcs} + \left(\int \rho^{-2}ds\right)_{wigglers}} \qquad (1)$$

where the quantum excitation term $\mathcal{H}$ is the one found in reference [3] by Sands and the ρ's are the bending radii.

From table 4 , the wiggler damping term is thirty times that of the arcs, while the quantum excitation terms are about equal.

The alternative to using wigglers for damping is to use high field bending magnets in the arcs which forces the magnet to be short enough to prevent the dispersion from growing. The result is many extremely short cells and a smaller circumference, basically determined by the maximum magnetic field acheivable by iron. Eventually one runs out of room for the quadrupoles which must become stronger at the same time to keep up with the phase advance per cell.

Table 3: Wiggler parameters.

Total length	360. m
Number	12
Maximum field for 4 GeV operation	10.7 kG
Minimum bending radius	12.5 m
Wiggler period	120.0 mm
Full gap width	25.6 mm

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Table 4: Emittance-related quantities in equ. (1).

arc quantum excitation term (m^{-2})	5.6×10^{-7}
arc damping term (m^2)	3.8×10^{-2}
wiggler quantum excitation term (m^{-2})	7.4×10^{-7}
wiggler damping term (m^{-2})	1.2×10^{0}
emittance w/o wigglers at 4GeV (m-rad)	3.4×10^{-10}
emittance w/ wigglers at 4GeV (m-rad)	2.5×10^{-11}
emittance w/o arc at 4GeV (m-rad)	1.5×10^{-11}

Sextupole Configurations and Dynamic Aperture

Tracking

The stability of the tranverse particle motion in the damping ring is simulated by numerically tracking the trajectory through many turns of the ring. A particle's motion is considered unstable when its amplitude reaches one meter within five hundred turns. Though this number of turns represents only one third of a damping time constant, the dynamic apertures for the different situations can be compared on an equal footing. The tracking program used was PATPET[4], a combination tracking and orbit program derived from PATRICIA[5] and PETROS[6].

Two Families Interleaved: If sextupoles are placed at every quadrupole then their strengths are at a minimum. This approach was the first one used, with the result that the dynamic aperture is very small (see fig. 2). The term "interleaved" refers to any sextupole arrangement that is not self-compensating to all orders. In this case the term applies to the alternation of positive and negative sextupoles. That the interleaved sextupoles scheme has a very small dynamic aperture relative to the other arrangements is very striking in view of the fact that the strengths of the individual sextupoles in the largest dynamic aperture in figure 2 are at least twice as much as those in the interleaved arrangement. Sextupoles configurations that cancel second order geometrical aberrations was examined for dynamic aperture improvement. This was done by using more than two families of sextupoles (in special cases, two families are sufficient) but it did not improve the dynamic aperture. These efforts are to be included in my doctoral thesis [7].

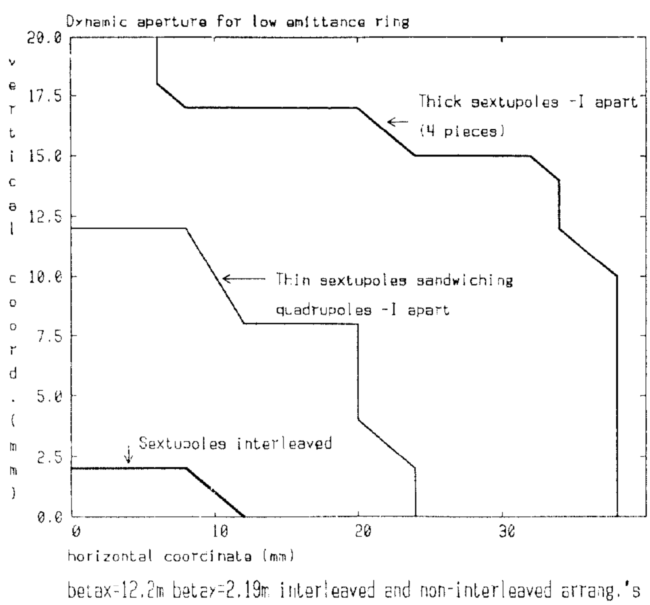

Figure 2: Dynamic aperture of the error-free low emittance ring with various arrangements of sextupoles. See figure 3 for a diagram of the thin-sextupoles-sandwiching-quadrupoles arrangement.

Two Families Non-interleaved: If sextupoles are placed exactly minus identity matrix apart, or 180 degrees apart in phase if the beta functions are equal at the sextupoles, then there is no lasting nonlinear kick, and the dynamic aperture is in principle infinite. Of course quadrupole strength error or a deviation in particle energy will make the phase advance between sextupoles not exactly 180 degrees and the dynamic aperture will be finite. In other words, the self-compensation is sensitive to the phase advance between thin-lens sextupoles. For instance, when a gaussian-distributed relative strength error of standard deviation of .2% was put in the quadrupoles, the dynamic acceptance decreased from practically infinity to 3×10^{-5} m-rad in the horizontal plane and to 1×10^{-4} m-rad in the vertical plane.

In reality, the sextupoles are thick magnets which can be modeled as a series of thin-lens sextupoles separated by drift spaces. Since this is a form of interleaving there will be some degree of dynamic aperture reduction. The cases where the sextupole were split in two and in four gave similar dynamic acceptances of about 1.5×10^{-4} m-rad in both planes.

A comment should be made on the invariant ellipse in between the sextupole kicks. Suppose a 25 mm by 2 mrad "ellipse" of particles in the horizontal plane (near dynamic aperture) is injected at the start of an error-free superperiod. Then the maximum kick given by the SF's is 5 mrad, which is much greater than 2 mrad, the maximum slope of the unperturbed ellipse at the SF's positions. Thus the ellipse is greatly deformed in between sextupoles kicks, but is more or less restored outside of the sextupole self-compensation unit.

Aberrations due to Sextupole Length

Consider the trajectory through two 90 degree FODO cells with thick sextupoles at both ends for self-compensation. These two cells can be regarded as an independent beamline whose aberrations are to be reduced locally. The two cells associated with the focusing sextupole SF, say, can be diagrammed as follows:

|———————— 180 degrees ————————|
SF-QF-SF-B-D-QD-D-B-D-QF-D-B-D-QD-D-B-SF-QF-SF
|——one 90 deg. cell——||——one 90 deg. cell——|

Figure 3: Sextupole placement within two FODO cells.

where D is a drift space the same length as the sextupoles. The cells containing the defocusing sextupoles are adjacent and are equivalent. Notice that two sextupoles instead of one is placed next to the end quadrupoles. This is done to obtain a simple and symmetric analysis. Also for simplicity no space is provided between the quadrupole and sextupole. This arrangement is thought to be close enough to the perfect compensation scheme that the separation of the two sextupoles near one quadrupole may be considered as a perturbation. Separating the sextupoles in this way unnecessarily decreases the dynamic aperture, but this conservative symmetric arrangement allows an easier observation of the nonlinearities emerging from near-perfect compensation.

The trajectory at any element can be written in terms of the coordinates at the beginning of the beamline, i.e. the entrance of the first sextupole. The calculation of the trajectory through a quadrupole is simplified to that through a thin-lens quadrupole with drift spaces an each side of appropriate length. Presumably, this approximation does not greatly modify the general form of the sextupole aberration. The trajectory through the sextupole is calculated by sectioning the magnet into a series of thin-lens sextupoles separated by drift spaces. The approximation improves when the number of thin-lenses increases.

A program using the REDUCE [8] algebraic manipulation language available on the SLAC VM was written which expresses in analytical form the aberration due to the length or separation of the sextupoles. The number of pieces into which the sextupole was split can be entered as a parameter.

The expressions for the transverse coordinates x, x', y and y' at the exit of the last sextupole is a huge polynomial in various powers of all the input quantities, in particular, the initial coordinates at the first sextupole. Only terms of order three or lower in the initial coordinates are extracted from the program. To make the analysis more convenient, the initial and final coordinates are linearly transformed to the middle of the closest quadrupole. Since the transformations are linear, no aberration information is lost.

Below is part of the result for the simplest case, when the thick sextupole is treated as one thin-lens sextupole centered in the original sextupole's position in the beamline. The full expression correct to third order in x_0, x_0', y_0 and y_0' are very long and cannot be published here due to lack of space. Therefore we set $y_0 = y_0' = 0$. In the following expressions, M is the integrated strength of each sextupole, L_S is the length of the sextupole, L_Q is the length of the quadrupole and L_C is the length of the cell:

$$\begin{aligned} x \;=\; & -x_0 + x_0^2 x_0' \left(-\frac{1}{4} M^2 (L_S + L_Q)^3 + \frac{3\sqrt{2}}{8L_C} M^2 (L_S + L_Q)^4 \right. \\ & \left. - \frac{3}{8L_C^2} M^2 (L_S + L_Q)^5 + \frac{\sqrt{2}}{16 L_C^3} M^2 (L_S + L_Q)^6 \right) \\ & + x_0'^3 \left(\frac{1}{16} M^2 (L_S + L_Q)^5 - \frac{\sqrt{2}}{32 L_C} M^2 (L_S + L_Q)^6 \right) \qquad (2) \end{aligned}$$

$$\begin{aligned} x' = & -x_0' + x_0^3 \left(-M^2(L_S+L_Q) + \frac{5\sqrt{2}}{2L_C} M^2 (L_S+L_Q)^2 \right. \\ & - \frac{5}{L_C^2} M^2 (L_S+L_Q)^3 + \frac{5\sqrt{2}}{2L_C^3} M^2 (L_S+L_Q)^4 \\ & \left. - \frac{5}{4L_C^4} M^2 (L_S+L_Q)^5 + \frac{\sqrt{2}}{8L_C^5} M^2 (L_S+L_Q)^6 \right) \\ & + x_0 x_0'^2 \left(\frac{1}{4} M^2 (L_S-L_Q)^3 - \frac{3\sqrt{2}}{8L_C} M^2 (L_S+L_Q)^4 \right. \\ & \left. + \frac{3}{8L_C^2} M^2 (L_S+L_Q)^5 - \frac{\sqrt{2}}{16L_C^3} M^2 (L_S+L_Q)^6 \right) \end{aligned} \tag{3}$$

Note that without the sextupoles, the final coordinates are just minus the initial coordinates. Each final coordinate has two non-linear terms in x_0 and x_0'. Splitting the sextupoles in more pieces yields the same terms but with more complicated polynomials in the quadrupole length (L_Q) and the sextupole length (L_S).

Using present design values (L_C=7.2 m, L_Q=.3 m, L_S=.4 m, the space between the sextupoles and their neighboring magnets is taken up by the sextupoles) the quantity $(L_S+L_Q)/L_C$, approximately the separation of the sextupoles in units of the cell length, turns out to be a small value, 0.097, which is regarded as a perturbative quantity. The value used for the SF integrated strength is about -8 m^{-2}. The final coordinates are then, with $\beta = \frac{2+\sqrt{2}}{2} L_C$:

$$x = -x_0 - (3.6\text{x}10^{-1}\text{m}^{-2}) x_0^2 (x_0'\beta) + (3.4\text{x}10^{-4}\text{m}^{-2}) (x_0'\beta)^3 \tag{4}$$

and

$$(x_0'\beta) = -(x_0'\beta) - (3.9\text{x}10^2\text{m}^{-2}) x_0^3 + (3.6\text{x}10^{-1}\text{m}^{-2}) x_0 (x_0'\beta)^2. \tag{5}$$

The strong "left-over" nonlinear perturbation term -3.9x10^2 x_0^3 in equ. (5) will exceed the linear coordinate $(x_0'\beta)$ when x_0 equals about .05 m. Roughly, the dynamic aperture will be of that order of magnitude. Our goal is to somehow eliminate these third order term.

Octupole effect: An octupole produces a nonlinear kick proportional to a third degree polynomial in the position coordinates:

$$\Delta x' = \frac{1}{6} O (x^3 - 3xy^2) \tag{6}$$

and

$$\Delta y' = \frac{1}{6} O (y^3 - 3x^2 y) \tag{7}$$

where O is the integrated octupolar field strength. Since the effect of the octopole length is fourth order in coordinates, one can treat the octupoles as thin lenses. If identical octopoles of strength O were superposed on the fields of the QF's at the extremities of the beamline as in figure 4 (they contribute addititively due to the polarity of the octupolar fields, see ref. [1]) then their contributions to the aberrations are:

$$x_{\text{oct.}} = 0 \quad \text{and} \quad x'_{\text{oct.}} = \tfrac{1}{3} O x_0^3$$

This octupole contribution cancel only one of the third order sextupole terms, albeit the strongest one (in equ. (5)), as the other two have two extra powers of $(L_S + L_Q)/L_C$, which is a small quantity. To second order in that factor, the required value of the octopole is

$$O = -3M^2 (L_S + L_Q)(1 - \frac{5}{\sqrt{2}} \frac{(L_S + L_Q)}{L_C}). \tag{8}$$

The same results hold for correcting the aberrations of SD in the vertical plane. A more complicated scheme can be pursued with extra octupolar fields centered at the other quadrupoles within the two FODO cells as shown in figure 4. These can be made to cancel the other third order terms in the horizontal plane.

SF-(QF/O)-SF-B-D-(QD/O2)-D-B-D-(QF/O3)
-D-B-D-(QD/O4)-D-B-SF-(QF/O)-SF

Figure 4: Octupole placement in compensated FODO cell. Octupolar fields would be "shaped" onto the quadrupole magnet poles.

Space does not permit me to mention the terms involving y_0 and y_0', except for the fact that the largest term among these cancel when the largest horizontal aberration term is cancelled. Obviously, terms of higher order in $(L_Q+L_S)/L_C$ cannot all be cancelled, but they could be selectevily minimized by optimizing the strengths of the O2, O3 and O4 octupoles.

Tracking Results

Tracking was done using the single octupole correction of the sextupole third order aberration. Octupoles of integrated strength -94 m^{-3} were placed in the center of the quadrupoles that are sandwiched between sextupoles. Because the SF's and the SD's are powered almost equally (there are twice as many SD's as SF's), the strength of the octupoles required on the QD's and the octupoles required on the QF's are the same. The sign of the octupoles strength must be negative for both.

The inclusion of octupoles causes a definite increase in the dynamic aperture of 20 or 50% along both planes, as shown in figure 5. Also, the dynamic aperture worsens for larger octupole strengths, more or less verifying that the calculated octopole strength was optimal.

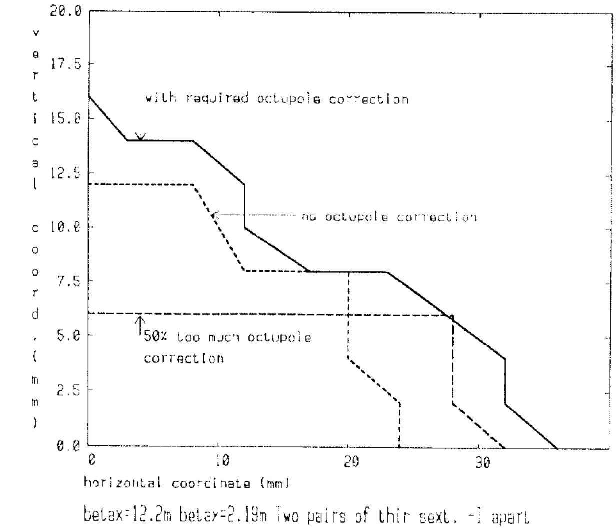

Figure 5: Dynamic aperture with octupoles correcting the effect of the separation of sextupoles sandwiching quadrupoles minus unit matrix apart.

Conclusion

A ring with very low emittance was demonstrated to have a surprisingly large dynamic aperture. The use of the basic non-interleaved sextupole scheme proved to be a great improvement over the interleaved sextupole dynamic aperture. Some further correction was achieved using octupoles to cancel third order terms.

An analysis of the natural wiggler multipole component effect on the dynamic aperture should be conducted now that a large starting dynamic aperture is achieved.

Further optimization such as that of the cell length in order to decrease the emittance in "exchange" for some deterioration of the dynamic aperture may be considered after all the nonlinear beam dynamic studies have been completed.

Acknowledgements

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References

[1] K.Brown and R.Servranckx, "First- and Second-Order Charged Particle Optics" SLAC-PUB-3381, p.68

[2] H.Wiedemann, "Future development for Synchrotron Radiation Sources at Stanford" IEEE PAC 1987 Proceedings P.395

[3] M.Sands, "The physics of Electron Storage Rings", SLAC-121, p.133

[4] PATPET, tracking program by L.Emery and H.Wiedemann, SSRL, SSRL ACD-Note #36/revised, June 1988

[5] PATRICIA, tracking program by H.Wiedemann, SSRL, SLAC PEP-note 220, Sept. 1976

[6] PETROS, orbit correction program by J.Kewisch and K.G.Steffen, DESY, DESY PET-76/09

[7] L.Emery, doctoral thesis in progress, Applied Physics Department, Stanford University

[8] REDUCE, by A.C.Hearns, The Rand Corporation