Z-PINCH WIGGLER

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Abstract

There have been several proposals for wigglers making use of the intense electromagnetic fields available with high density plasmas.[1] A novel approach, the use of a hollow z-pinch generated plasma, allows for the exploitation of the high fields generated by plasma oscillations without the electron beam passing through the plasma. The approach is also inherently flexible in allowing different wiggler designs including the ability to do tapering. Several z-pinch wiggler variations will be presented and discussed.

Introduction

Plasmas have been proposed for both acceleration of particles[2] and deceleration of particles, in the guise of wigglers for free electron lasers (FEL's).[1,3] The strongest argument for using such devices is the theoretically extremely high gradients achievable by various perturbations of a plasma from uniformity. The major drawbacks include plasma instabilities and scattering of particles of the beam by the plasma. Regarding the latter it is interesting to note that most accelerators and wigglers operate in a pressure regime of 10^{-6} torr or less while typical plasma pressures are many orders of magnitude greater. While plasma wiggler of a few periods, or plasma accelerators which are the equivalent of a few rf cavities in length have been demonstrated in proof of principle experiments it is necessary for really useful devices to overcome this vast difference in operating regimes otherwise these devices will remain impractical due to their inefficiencies. A z-pinch[4,5] is a plasma column produced by a large axial current. The azimuthal magnetic field surrounding the plasma acts to confine the plasma. If the current is initiated in a hollow cylinder then the plasma produced is hollow as well and collapses on itself, a plasma focus.[5] During the collapse there exists a hollow plasma tube, the hollow z-pinch.

The proposed hollow z-pinch wiggler has as its strongest selling point the elimination of plasma on axis, so that beam-plasma scattering is also eliminated. Thus, it maintains the advantages of other plasma devices and improves on them by returning the beam to its preferred high vacuum environment. The remainder of this paper will look at z-pinches in section 2, a specific application for a wiggler in section 3, and conclude in section 4.

Section 2

The z-pinch is a special case of the general screw pinch.[4] The general screw pinch is a plasma confined radially with some combination of axial and azimuthal magnetic fields. Much research has been performed in this area because of direct interest in these concepts for fusion and because of their applicability to the more complicated toroidal configurations. The study of instabilities both experimentally and theoretically is well advanced. It is well known that z-pinches are unstable to various exchange modes, due mainly to high plasma pressure on axis. It is also well known that z-pinches have achieved some of the highest plasma densities, due mainly to the ease of achieving mega-ampere currents running axially through the plasma. One concept taking advantage of the latter strength and reducing the former weakness is the plasma focus, in which a hollow z-pinch is initiated and allowed to ballistically collapse on itself, trading radial momentum gained during acceleration towards the axis into on-axis pressure. At full collapse the plasma can achieve very high densities in this manner with an inertial term adding to the magnetic confinement term.

Examination of the plasma z-pinch can begin with the equilibrium radial pressure balance equation[4]:

$$\frac{d}{dr}\left[p + \frac{B_\theta^2}{2} + \frac{B_z^2}{2}\right] + \frac{B_\theta^2}{r} = 0$$

where r is the radius, Bθ and Br are the azimuthal and radial magnetic fields, respectively. For a pure z-pinch Bz is identically zero. The classic solution was provided by Bennett[6], but is not relevant to this paper because Bθ peaks at the plasma radius while p peaks on axis, exactly the conditions that a plasma wiggler would do best avoiding. It is important to note that, short of imposing a confinement wall or interior fields, an equilibrium solution with p, Bθ, and Bz all equal to zero on axis doesn't exist.

This observation leads to two different hollow pinches of interest. First, it is interesting to consider the case for which Bz does have a finite value on axis because of the greatly enhanced stability of such a configuration, although this involves consideration of the more complicated general screw pinch. An example, to be discussed again in section 3, is the solution for zero plasma pressure, with uniform interior Bz for an annular plasma of inner radius a, and outer radius b. One solution for the fields for r < a, a < r < b, and b < r is:

$$B_z = \begin{cases} B_{z0} & r < a \\ B(r)_z & a < r < b \\ 0 & r > b \end{cases}$$

where

$$B(r)_z = \sqrt{2} \cdot \frac{B_{\theta 0}}{b - a} \cdot \sqrt{\frac{3}{2} \cdot \left[b^2 - r^2\right] - 4 \cdot a \cdot (b - r) + a^2 \cdot \ln \frac{b}{r}}$$

$$B_{z0} = B(a)_z$$

and Bθ is given by:

$$B_\theta = \begin{cases} 0 & r < a \\ B_{\theta 0} \cdot \frac{r - a}{b - a} & a < r < b \\ B_{\theta 0} \cdot \frac{b}{r} & r > b \end{cases}$$

Simply put, the idea here is to solve for the equilibrium condition of the hollow z-pinch. The assumption made is that the initial z-pinch radius is much larger (a factor of 3-10 times) and the initial

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Bz produced by a solenoid much smaller. Ramping the plasma current up to the appropriate value pinches the plasma down and, under the assumption of high conductivity, boosts the trapped magnetic field due to conservation of flux. This model allows order of magnitude estimates for the fields and currents needed for achieving plasmas of interest as wigglers. It emphasizes a point made by Joshi, et. al.[1], as well, which is that plasma density does not need to be high for a plasma to be of interest as a wiggler. In particular, for b = 3 mm and a = 2 mm I find that $Bz(a) = 1.5\ B\theta 0$. If one wishes a compressed axial magnetic field $Bz0 = 100$ Tesla then the required current is 1 MA, while the $B\theta$ component is 67 Tesla.

The other case of interest is the ballistically collapsing plasma tube. This plasma is governed by the two equations:

$$\frac{d}{dr}\, p = j_z \cdot B_\theta - \rho \cdot \frac{\partial}{\partial t}\, v_r$$

$$j_z = \frac{1}{4\cdot\pi\cdot r} \cdot \left(\frac{d}{dr}\ r \cdot B_\theta\right)$$

for which the time derivative is a partial derivative. Again, for the zero plasma pressure case the LHS of the first equation drops out, and substituting for jz:

$$-\frac{B_\theta}{4\cdot\pi\cdot r} \cdot \frac{d}{dr}\left(r \cdot B_\theta\right) = \rho \cdot \frac{\partial}{\partial t}\, v_r$$

Further investigation of plasma wigglers based on these equations will be pursued at another time. The problem of optimizing a plasma as a wiggler during ballistic collapse depends on programming the current, jz, and is a time dependent problem outside the scope of this paper.

Section 3

For calculating wiggler parameters of the example developed in the previous section I select the mode of operation to be that of a cyclotron maser and make use of the theory as discussed by V. L. Bratman, et. al.[7] Accordingly, the efficiency is given by:

$$n = \frac{1}{2\cdot\pi\cdot\mu\cdot\Gamma\cdot N} \cdot \frac{f}{1 - 1/\gamma}$$

Where f is the normalized average energy of the electrons, fmax = 5.6, and typically f is O(1); γ is the initial normalized electron energy (total energy divided by the rest mass energy); $\mu = 1-1/p$, with p = the square of the phase velocity of light inside the plasma tube; Γ is the doppler frequency gain factor; and N is the number of periods of the wiggler. For the values $N = 100$, $\gamma = 10$, $f = 1$, μ .054, $\Gamma = 20$ (because $\Gamma = 2\gamma$ for cyclotron masers as opposed to $2\gamma^2$ for FEL's) the efficiency is:

$$n = .0015$$

and the output light has a wavelength, λ, of 5.35 um, based on the equation:

$$\lambda = \lambda h/2\gamma$$

with $\lambda h = 2\pi mc^2/eB$, m = electron mass, c = speed of light, e = charge, B = magnetic field.

This example is based on values found in [7], and no attempt has been made to optimize the efficiency, n. Still, for $\gamma = 10$ (~ 5 MeV) and a beam current of 1 kA the optical output should be 7.5 MW (peak) for this single pass device.

Section 4

Plasma wigglers are of interest because of the intense electromagnetic interactions possible. Field strengths on the order of 10-100 Tesla are necessary for very compact short wavelength FEL's. This paper makes the point that simple compression of an axial magnetic field by a hollow plasma tube can achieve significant field strengths with moderate currents, thus providing a simple mechanism, the cyclotron maser, for achieving generation of coherent radiation. Experiments in developing this idea could be very fruitful in the context of pushing FEL and advanced accelerator technology further along the path of greater compactness.

Higher beam energies and greater plasma currents for the generation of sub-micron light are the natural paths in further development. But, the parameters used here are quite possible to achieve in a small laboratory, and thus amenable to quick development and investigation. Characterization of higher order instabilities than the ones naturally suppressed by the hollow z-pinch, not addressed at all in this paper, will certainly affect electron beam propagation and need to be studied to ascertain their influence on FEL performance.

In conclusion, this paper has presented a simple model of a hollow z-pinch plasma, compressing a trapped axial magnetic field. This axial field in turn is unused for generation of micron wavelength light as a cyclotron maser. Other z-pinch wigglers are possible but have not been examined in this paper. The key point being made is that very simple plasma configurations can produce interesting FEL's and that bench top experiments can be performed to learn quite a lot about a subject of great potential use, strong electromagnetic beam-plasma interactions with the elimination of scattering problems.

References

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