THE BACKWARD MODE IN AN ELECTROMAGNETICALLY PUMPED FREE ELECTRON LASERS*

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Abstract

In this paper, a detailed analysis on the backward mode of an electromagnetic standing wave wiggler for FELs is conducted. The analysis results in a clear understanding of the backward mode. The mode, for its absolute instability, should be suppressed in experiments for forward mode radiation to avoid beam disrupt. On the other hand, it can be used to be a remarkable, self-excited high power source with high efficiency in low-frequency region, and can saturate on a much higher level than that of the conventional forward mode.

Introduction

There exist two kind modes in FELs, forward propagating short-wavelength mode and backward propagating long-wavelength mode, as has been shown in a number of previous investigations. Studies in Raman regime[1]-[5] show that the forward mode ($k \simeq 2\gamma_b^2 k_o$) is convective unstable (i.e., it grows exponentially in space as it propagates along the beam), while the backward mode ($k \simeq -\frac{1}{2} k_o$, for $\gamma \gg 1$) is absolute unstable (i.e., it grows exponentially in time). For its absolute instability in nature, the backward mode will grow to such a size that the beam is disrupted and the laser cannot produce the desired short-wavelength radiation. Studies in Compton regime[6]-[7] shows very similiar results. For systems long compared with the growth length of the forward mode, this mode dominates initially because it is faster growing than the backward mode. However, it saturates at a much lower lever than the backward mode and thus for late times, the backward mode dominates. Which mode dominates deponds on the system length and on the time of interaction.

If a new experiment aims at obtaining forward radiation, solutions of (removing) restraining the backward mode are needed, such as designing the system to be long enough to produce the desired short-wavelength radiation, but to be short enough to avoid the backward mode. However,if a high efficient source of backward mode is desired as to be a microwave undulator, the backward mode is a remarkable candidate. All these previous results suggest the importance of studying the backward mode.

Recently, more and more interests focus on the electromagnetically pumped FELs[8]-[12]. Current millimeter and sub-millimeter sources based on cyclotron resonance interactions are capable of producing high power of several MW. To increase the pump wave amptitude a_ω(and consequently improve the FEL gain). The wiggler field can be stored in a high Q resonator[11] acting as a standing wave. Research on the forward mode in the standing wave wiggler has been completed in MIT[12]. In this paper, detailed study on the backward mode in the standing wave pump FELs is carried out.

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Kinetic Description of E-beam in the CPSW Wiggler

Defining the CPSW wiggler field

$$A_x + iA_y = -\frac{mc^2}{e} a_\omega \left[e^{i(k_\parallel z + \omega t)} + e^{-i(k_\parallel z - \omega t)} \right] \tag{1}$$

where $k = \omega/c = \sqrt{k_\parallel^2 + k_\perp^2}$ is the wave number of the pump, $k_\parallel$ is the longitudinal wave number, $a_\omega = \rho A(z)/mc^2$ is the wiggler amplitude and is assumed to be independent of the transverse coordinates while A(z) is the amplitude of vector potential of the wiggler field.

By using single-particle theory[13], one could get a full kinetic description. For a single electron in the wiggler field, its kinetic behavious on the transverse plane could be approximately described in the following equations:

$$\gamma\beta_x + i\gamma\beta_y = -a_\omega \left[e^{i(k_\parallel z + \omega t)} + e^{-i(k_\parallel z - \omega t)} \right] \tag{2}$$

$$\gamma^2 \beta_\perp^2 = 2a_\omega^2 (1 + \cos 2k_\parallel z) \tag{3}$$

On the other hand, the electron longitudinal motion could be given approximately by

$$1/\beta_\parallel(z) = \frac{d}{dz} ct = 1 + \frac{1 + 2a_\omega^2}{2\gamma^2} + \frac{a_\omega^2}{\gamma^2} \cos 2k_\parallel z \tag{4}$$

$$1/\overline{\beta_\parallel} = 1 + (1 + 2a_\omega^2)/2\gamma^2 \tag{5}$$

$$ct(z) = \left(1 + \frac{1 + 2a_\omega^2}{2\gamma^2}\right) z + \frac{a_\omega^2}{2\gamma^2} \frac{\sin 2k_\parallel z}{k_\parallel} \tag{6}$$

Assuming the signal field of backward mode has the following form

$$\vec{A}_s = A_{sx} + iA_{sy} = \frac{mc^2}{e} a_s e^{-i(k_s z + \omega t)} \tag{7}$$

the Hamiltanian of the system[13]

$$\mathcal{H}_1 = -\left\{ (E^2/c^2 - m^2c^2) - \left(\frac{e}{c}\right)^2 (A_x^2 + A_y^2) \right\}^{1/2} \tag{8}$$

substituding (1) and (7) into (8), one obtains

$$\mathcal{H}_1 = -\{mc\}\{\gamma^2 - \mu^2 + 2a_{\omega 1} a_s \cos[k_s z + (\omega_s + \omega)t]\}^{1/2} \tag{9}$$

where

$$\mu^2(z) = 1 + a_s^2 + a_{\omega 1}^2 = 1 + a_s^2 + 4a_\omega^2 \cos^2 k_\parallel z \tag{10a}$$

$$a_{\omega 1} = 2a_\omega \cos k_\parallel z \tag{10b}$$

$$\psi(z,t) = k_s z + (\omega_s + \omega)t \tag{10c}$$

From the canonical equations

$$\frac{\partial \mathcal{H}}{\partial(-E)} = t' \tag{11a}$$

$$\frac{\partial \mathcal{H}}{\partial t} = -(-E)' \tag{11b}$$

the equations of motion could be obtained:

$$\psi'(z,t) = k_s - \frac{\gamma(k_s + k)}{\{\gamma^2 - \mu^2 + 2a_{\omega 1} a_s \cos\psi\}^{1/2}} \tag{12a}$$

$$\gamma'(z,t) = \frac{-(k_s + k) a_{\omega 1} a_s \sin\psi}{\{\gamma^2 - \mu^2 + 2a_{\omega 1} a_s \cos\psi\}^{1/2}} \tag{12b}$$

There are the equations of motion under the perturbation of the backward mode. By assuming $\gamma^2 \gg \mu^2$ and $\gamma^2 \gg a_s a_{\omega 1}$, above equations could be simplified as

$$\psi'(z,t) = -k - \frac{(k_s+k)}{\gamma^2}\left\{\frac{\mu^2}{2} - a_{\omega 1} a_s \cos\psi\right\} \tag{13a}$$

$$\gamma'(z,t) = -\frac{(k_s+k)}{\gamma} a_{\omega 1} a_s \sin\psi \tag{13b}$$

These simplified equations could be used to further the detailed information about electron kinetics in a computer simulation code, which is beyond of this paper.

Spontaneous Spectrum of Backward Wave Radiation

By using the method of the Lienard-Wiechert potentials, the spontaneous emission by a single electron undulating close to the wiggler axis can be expressed as[14]

$$\frac{d^2 I}{d\Omega\, d\omega_s} = \frac{\rho^2 \omega_s^2}{4\pi^2 c}\left|\int_0^L (\beta_x + i\beta_y) e^{i(\omega/c)\int_0^z [1-\hat{n}\cdot\vec{\beta}(z)dz_1}dz\right|^2 \tag{14}$$

where $\hat{n}$ is the direction of observation, L is the wiggler length, $\vec{\beta}(z)$ is the electron velocity at position z in units of the light velocity in vacuum c, and ω_s is the emission frequency.

Following the same method in ref.[12], i.e., using the electron trajectory specified by eqs.(2),(4) and (6) and the Bessel relation

$$e^{\pm i\delta\sin\theta} = \sum_{n=-\infty}^{\infty} J_{\pm n}(\delta) e^{in\theta} \tag{15}$$

to eliminate the time t, the integration in eq.(14) can be performed in a straight forward manner

$$\frac{d^2 I}{d\Omega\, d\omega_s} = \frac{(\rho k_s L)^2}{4\pi^2/c}\left(\frac{a_\omega}{\gamma}\right)^2 \left|\sum_n f_n \frac{e^{-i2\nu_n}-1}{2\nu_n}\right|^2 \tag{16}$$

where the coupling coefficient f_n and the resonance parameter ν_n are defined according to

$$f_n \equiv J_{-n}(Q) + J_{-(n+1)}(Q), \quad Q \equiv \frac{a_\omega^2}{2\gamma^2}\frac{k+k_s}{k_{\parallel}} \tag{17a}$$

$$\nu_n \equiv \frac{L}{2}\left[(2n+1)k_{\parallel} - (k+2k_s) - (k+k_s)\frac{1+2a_\omega^2}{2\gamma^2}\right] \tag{17b}$$

The intensity distribution function of spontaneous emission, which is specified by eq.(16), has the exact same form as that of forward mode except factors Q and ν_n, It is those factors that make the difference between the backward mode and forward mode, especially the resonance condition.

By letting ν_n be zero, the resonance condition can be derived as

$$k_s(1+\bar{\beta}_{\parallel}) = (2n+1)k_{\parallel}\bar{\beta}_{\parallel} - k \tag{18}$$

This is the frequency relation between radiation field and wiggler field. Also, the relativistic factor of resonance electron can be defined from $\nu_n = 0$:

$$\gamma_r^2 = \frac{(1+2a_\omega^2)(k+k_s)}{2[(2n+1)k_{\parallel} - (k+2k_s)]} \tag{19}$$

then the resonance parameter ν_n satisfies

$$\nu_n = \frac{L}{2}[(2n+1)k_{\parallel} - (k+2k_s)]\left[1-\frac{\gamma_r^2}{\gamma^2}\right] \tag{20}$$

Note that the squared sum in eq.(16) can be written as:

$$\sum_{n=-\infty}^{\infty} f_n^2 \frac{\sin^2\nu_n}{\nu_n^2} + \sum_{n=-\infty}^{\infty}\sum_{m\neq n} C_{m,n}$$

where the cross terms $C_{m,n}$ which can be written as:

$$C_{m,n} = \frac{f_n f_m}{4\nu_n \gamma_n}[1+\cos 2(\nu_n - \nu_m) - \cos 2\nu_n - \cos 2\nu_m]$$

describe the interference between the n^{th} resonance and its neighbouring resonance. They can be neglected when the resonance pecks given by eq.(20) are well separated, i.e. when

$$|\nu_n - \nu_{n\pm 1}| > 2\pi \Longrightarrow L k_{\parallel} > 2\pi \tag{21}$$

In that case, in evaluating each individual term of the sum, the argument of the Bessel function Q can be approximated by its value at the resonance

$$\begin{aligned} f_n &\simeq J_{-n}(Q_n) + J_{-(n+1)}(Q_n) \\ Q &\simeq Q_n = \frac{a_\omega^2}{4\gamma^2}\left\{(2n+1) + \frac{k}{k_{\parallel}}\right\} \end{aligned} \tag{22}$$

In eq.(18), the harmonic number n is chosen so as to maintain $k_s > 0$, Because of $k_{\parallel} \lesssim k, \bar{\beta}_{\parallel} \simeq 1$, $n \geq 1$. For the fundamental mode (n=1), the frequency relation can be written as

$$k_s(1+\bar{\beta}_{\parallel}) = (3\bar{\beta}_{\parallel} - 1)k \tag{23}$$

which can be expressed more approximately as

$$k_s \simeq k \qquad (k_s \lesssim k) \tag{24}$$

In this case, the resonance parameter ν_n can be written as

$$\begin{aligned} \nu &\doteq \frac{1-\bar{\beta}_{\parallel}}{1+\bar{\beta}_{\parallel}} kL\left[1-\frac{\gamma_r^2}{\gamma^2}\right] \\ &= \frac{1+2a_\omega^2}{4\gamma_r^2} kL\left[1-\frac{\gamma_r^2}{\gamma^2}\right] \end{aligned} \tag{25}$$

For The forward mode of CPSW wiggler FEL, the frequency relation for the fundamental mode (n=0) yields[12]

$$\nu_0' = kL'\left[1-\frac{\gamma_r^2}{\gamma^2}\right] \tag{26}$$

As the cold beam gain of FEL takes its maximum value when its corresponding resonance parameter equals to a fixed value (say $\nu = 1.3$), eqs.(25) and (26) can result in the interation region length relation between forward mode and backward mode, which says

$$L' = \frac{1+2a_\omega^2}{4\gamma^2} L \tag{27}$$

where L, L' are the lengthes of interation region for the backward mode and forward mode in CPSW pump FEL respectively.

The relation specified in eq.(27) is exact the same as that in magnetostatically pump FEL[6],[7], If the short-wavelength radiation is disired, the length of interation can be designed between L' and L , i.e. longer than L' but shorter than L. On the other hand, there is another way of obtaining same resonance parameters. By fixing the interaction region, different $\left[1-\frac{\gamma_r^2}{\gamma^2}\right]$ can result in same resonance parameters. This solution requires large energy spread for the backward mode, which means the backward mode would disrupt the electron beam in a desired short-wavelength radiation if it is not suppressed, or means the backward mode has a much low requirement on electron beam quality in a desired long-wavelength radiation.

The spontaneous spectrum of backward mode has the exact same form as that of forward mode. However, the differences between the two modes can be reflected from the two parameters ν_n and Q_n. For the backward mode

$$Q_1 = \frac{a_\omega^2}{4\gamma^2}\left[3 + \frac{k}{k_{\parallel}}\right] \tag{28}$$

while the corresponding fundamental Q_0' for the forward mode can be obtain from reference[12]

$$Q_0' = \frac{a_\omega^2}{1+2a_\omega^2}\left(1+\frac{k}{k_\parallel}\right) \tag{29}$$

The independent variable of the coupling parameter of forward mode, Q_0', can vary in a very large number region so that the harmonic number n can be 0, ±1, ±2, ... In contrast, Q_1 in backward mode is much smaller. Note the Bessel function for small argument ($|x|\ll 1$).

$$J_0(x) \simeq 1 - \frac{x^2}{4!} \qquad J_n(x) \simeq \frac{1}{n!}\left(\frac{x}{2}\right)^n \quad (n = \pm 1, \pm 2, \cdots) \tag{30}$$

In order to have large spontaneous emission, f_n has to be chosen as large as possible. Since the arguments of coupling coefficient f_n for the backward mode are very small, the harmonic number n can only be chosen to be n=1. So as to avoid vanishing coupling parameter (and consequently avoid vanishing spontaneous emission). This result appeals that the spontaneous spectrum of backward mode is very simple.

Comparation of spontaneous emission between the backward mode and forward mode shows that the instantaneous spontaneous emission of backward mode is lower than that of forward mode. However, because the backward mode saturates at much late time than forward mode, the total emission power of backward mode might be greater than forward mode, which will be proved in the next section.

Gain Calculation for the Backward Mode

It is reasonable to calculate the gain for the fundamental harmonic mode only, since the spontaneous spectrum of backward modes is quite simple and only the fundamental harmonic is meanful. From the results of above section

$$f_1 = J_{-1}(Q_1) + J_{-2}(Q_1) \simeq -\frac{1}{2}Q_1 \tag{31a}$$

$$Q_1 = \frac{a_\omega^2}{4\gamma^2}\left\{3+\frac{k}{k_\parallel}\right\} \tag{31b}$$

$$\frac{d^2 I}{d\omega_s\, d\Omega} = \frac{(\rho k_s L)^2}{4\pi^2/c}\left(\frac{a_\omega}{\gamma}\right)^2\left[f_1^2\frac{\sin^2 \nu_1}{\nu_1^2}\right] \tag{31c}$$

and one has

$$G = -\frac{(a_\omega L\,\omega_p)^2}{2c^2}\frac{d}{d\gamma}\left[\left(\frac{f_1}{\gamma}\right)^2\frac{\sin^2\nu_1}{\nu_1^2}\right] \tag{32}$$

from eq.(20)

$$\frac{d\nu_1}{d\gamma} = \frac{L}{2}\left[3k_\parallel - (k+2k_s)\right]\frac{2\gamma_r^2}{\gamma^3} \tag{33}$$

substituting eq.(18) into (33), one has

$$\frac{d\nu_1}{d\gamma} \simeq \frac{1+2a_\omega^2}{4\gamma^3}(3k_\parallel + k)L \tag{34}$$

which yields after combining with eq.(32)

$$G = -\frac{\omega_p^2 L^3 a_\omega^2}{4c^2\gamma^3}(3k_\parallel + k)\frac{1+2a_\omega^2}{2\gamma^2}f_1^2\frac{d}{d\nu_1}\frac{\sin^2\nu_1}{\nu_1^2} \tag{35}$$

It is obvious that the gain of backward mode has the similiar form to that of forward mode except the extra factor $(1+2a_\omega^2)/2\gamma^2$. The factor makes the instantaneous gain of backward mode is much smaller than that of forward mode. This result can also be obtained from the comparasion of spontaneous spectrums. However, by comparing the maximum gains of the two modes, the comparasion of saturation level of the two modes can be revealed. The derivative of $-\sin^2\nu_n/\nu_n^2$ has its maximum value and it is independent of harmonic number. The main difference of gain between the two modes occurs on the difference of interation length and the extra factor.

Defining G_m' as the maximum gains of the fundamental harmonic forward mode. The radio of maximum gain of the two fundamental modes can be expressed approximately as follows

$$\frac{G_m}{G_m'} \doteq \frac{(3k_\parallel+k)}{k+k_\parallel}\left(\frac{L}{L'}\right)^3\frac{1+2a_\omega^2}{2\gamma^2}\left(\frac{f_1}{f_0'}\right)^2 \tag{36}$$

where f_0' is the coupling parameter of fundamental harmonic forward mode, $(3k_\parallel-k)/(k+k_\parallel) \simeq 2$ when $k_\parallel/k \approx 1$ and L/L' has been given in eq.(27), thus

$$\frac{G_m}{G_m'} = \left(\frac{4\gamma^2}{1+2a_\omega^2}\right)^2\left(\frac{2f_1}{f_0'}\right)^2 \tag{37}$$

when arguments of the coupling parameters are very small, eq.(37) can be simplified in terms of eqs.(30) and (31) as

$$\frac{G_m}{G_m'} \simeq \left(\frac{4a_\omega^2}{1+2a_\omega^2}\right)^2 \tag{38}$$

Along with improving the output power of electromagnetic sources which will be used as wigglers in FELs, the effects of backward mode will concern greater importance. For large arguments of coupling coefficients (i.e., $a_\omega > 1$), the equations (30) and (31) will no longer exist. Figs. 1 and 2 in ref. [12] shows that term $(2f_1/f_0')$ in eq.(37) has the possibility to be equal to 1, so eq. (37) can be simplified as

$$\frac{G_m}{G_m'} \simeq \left(\frac{4\gamma^2}{1+2a_\omega^2}\right)^2 \tag{39}$$

This relation is the same as that in magnetostatically pumped FEL[6],[7], which means the discussions in ref. [6] and [7] are also meanful for the CPSW pumped FELs.

All of our calculations and discussions are based on an amplifier model. If a particular FEL operates as an amplifier, controlling of modes can be achieved with ease. By chosing the length of interation region between L and L′ , the backward mode will be suppressed and the desired short-wavelength radiation (forward mode) will be obtained. However, the suppression of backward modes in a FEL oscillator is much more complicated because the backward modes are absolute unstable. Many methods of suppressing backward modes have been suggested, which will not be discussed in this paper. If a long-wavelength radiation based on FEL mechanism is required,however, the backward mode is a remarkable candidate.

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