STABILITY OF BEAMS HOLLOW IN LONGITUDINAL PHASE SPACE

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Abstract

Intense beams, $\sim 100\ \mu$A, may be accumulated in storage rings by means of charge exchange injection together with a sweep of the closed orbit and/or incoming beam parameters. Such "painting" schemes can result in 6-D charge distributions peaked at the synchronous orbit and stable under space charge forces. Some of these schemes, however, may produce, at some point during the procedure, charge distributions that are hollow in longitudinal phase space. Under these conditions tracking simulations incorporating longitudinal space charge effects suggest that a dipole instability may develop. The development time depends on the total charge and its distribution. We find that hollow beams may be practicable provided that the total charge is less than a critical value. Initial growth rates are given.

Introduction

Charge exchange injection is used both to increase the brightness of an accumulated beam and also to increase the overall stored intensity. In the latter case the closed orbit may be moved transversely during injection to increase the size of the filled volume to reduce transverse space charge forces and the number of foil traversals. It is also possible to "paint" longitudinally provided the incoming beam momentum spread is less than the momentum spread required for the accumulated beam. Techniques proposed include modulation of the energy of the incoming beam or altering the synchronous energy and bucket parameters of the ring.[1] The longitudinal emittance may be filled from small to large amplitude or vice-versa. The latter sense of fill means that some time is spent with a beam hollow in longitudinal phase space. This note presents results of simulations which show that such distributions are non-stationary; an example is shown in Fig. 1. The results have been confirmed by others.[2]

While the consequences are not catastrophic, they are undesirable. Methods to control them are awkward and in general it is preferable to populate small amplitudes first.

Simulations

The model machine is the 440 MeV accumulation ring proposed for the TRIUMF Kaon Factory. This is intended to cycle at 50 Hz and deliver 100 μA cw. It would operate below transition energy with a small amplitude synchrotron tune, Q_s, of 0.048, a revolution frequency 1.1 MHz, and harmonic number h of 45. The bucket population at the end of 20 ms accumulation would be 3×10^{11}. Two programs have been used, LONG1D[3,4] and ACCSIM.[5] The former concentrates on a precise modelling of longitudinal motion including space charge. The latter treats longitudinal motion more approximately but also incorporates transverse motion, foil interactions, etc. Tracking takes place in an electric potential made up of localized rf kicks and continuous space charge voltage. The longitudinal equations are symplectic to second order.[3]
Three rf cavities with $\hat{V}$ of 188 kV are placed equidistant around the ring. Ensemble co-ordinates are updated, the space charge potential evaluated and energy changes calculated 24 times per turn.

$$\Delta E = \left\{ - \left(\frac{e^2}{4\pi\epsilon_0} \right) \frac{g_0}{\gamma^2} \cdot \frac{2\pi h^2}{R_s} \cdot \frac{d\lambda}{d\phi} \right\} \Big/ 24 \ , \qquad (1a)$$

$$\Delta \phi = \left\{ - \left[(2\pi h \eta_s)/(\beta_s^2 E_s) \right] (E - E_s) \ \right\} /24, \qquad (1b)$$

where E is the total energy (MeV) and ϕ the phase of a macroparticle, the subscript s refers to synchronous values, g_0 is the usual beam pipe to beam radius geometric factor and $\lambda(\phi)$ the charge per unit rf phase (also termed "line density"). At the cavities an additional

$$\Delta E = e\hat{V}(\sin\phi - \sin\phi_s) \ , \qquad (1c)$$

is included. For this storage ring $\phi_s = 0$.

Initial Population

It had been observed early in the investigation that the onset of non-stationary behaviour was affected by both the details of the calculation and the filling of the initial distribution. To distinguish between numerical artifacts and beam dynamics three initial annular distributions were prepared. Each had a parabolic density distribution in section, starting at 0.34 and ending at 0.78 of the bucket height defined by V_{rf} in the absence of space charge. The associated phases lay between $2\pi/5$ and $\pi/2$. The first two distributions, termed high and low symmetry, were populated by a particle in cell method. The normalized phase space $(\phi, \dot{\phi}/\omega_s)$ was divided into rings whose width varied inversely with the local parabolic distribution. Each ring was divided into azimuthal elements. The high symmetry case was constrained to an even number of equal elements, the low symmetry case had a mixture of odd and even numbers in each ring. The latter method produced a small, but non-zero initial dipole moment. In the third method the same distribution was populated by a Monte Carlo process. Greater statistical noise led to a much higher initial multipole content.

Analysis

Simulations were summarised by plots of the amplitude of the dipole motion as a function of turn number, Fig. 2, 3. Macroparticle (E, ϕ) coordinates were transformed to $(\phi, \dot{\phi}/w_s)$ and the dipole amplitude defined as the distance, ρ, of the ensemble centre of gravity from the origin in the latter system; ρ has dimensions deg, the amplitude thus defined is independent of intensity. ω_s is the small amplitude value. The actual spread in synchrotron tune means that ρ(t) has a small modulation at twice the synchrotron frequency.

Calculations

Annular distributions in longitudinal phase space likely to occur in the TRIUMF Kaon painting process would contain about half the 3.12×10^{11} protons of a filled acceptance. Initial populations were tracked with constant charge equivalent to 6.24, 3.12, 1.56, 0.78 and 0.39×10^{11} protons together with one case where space charge was "switched off".

All initial distributions had the same aspect ratio while the bucket shape was charge dependent. The initial distributions become matched through a filamentation process. This takes about 10 synchrotron oscillations or 200 turns. Concurrently, and continuing for a few hundred turns, there is a swift redistribution of particles about (and between) the different momentum rings within the annulus. This derives from the non-linear map used for tracking (1a-1c) and is exacerbated by the additional non-linear terms from space-charge. A statistical artifact associated with the ripple in the computed space charge potential (Fig. 1a) also contributes to the effect. (Merely from the spread in synchrotron frequency, a de-coherence time of

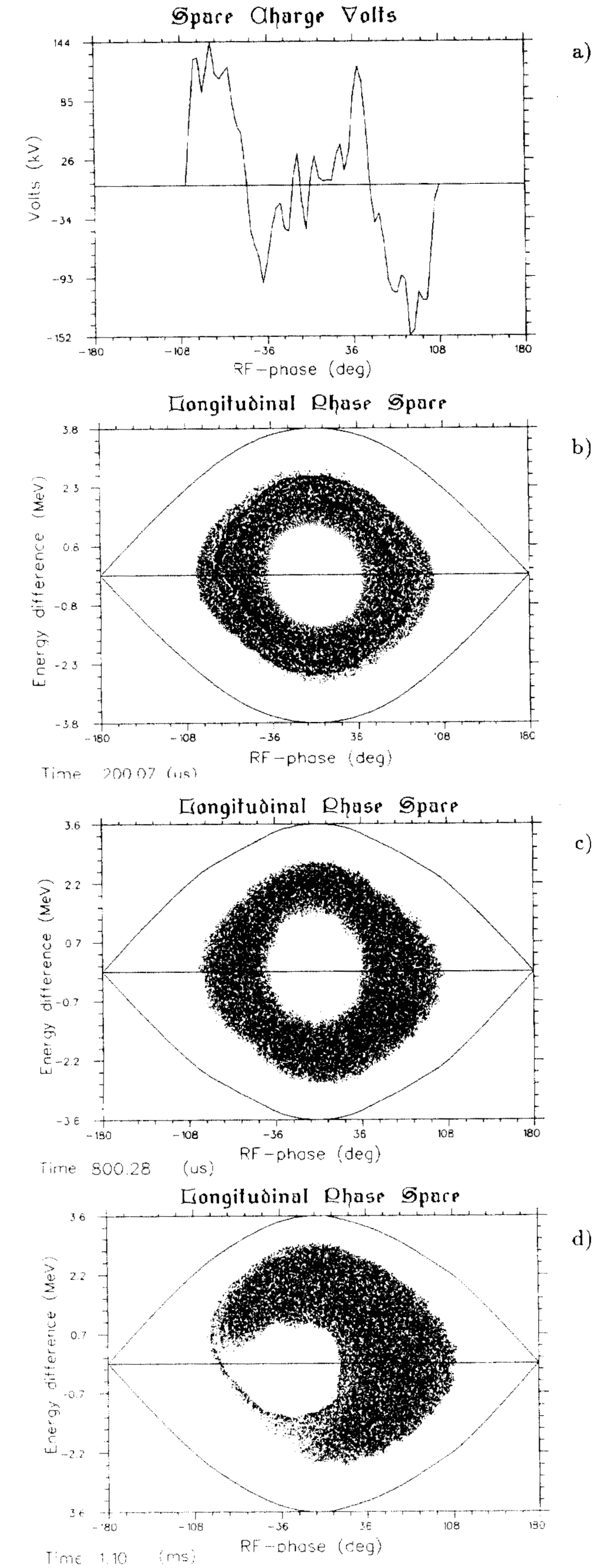

Fig. 1. Evolution of a dipole distribution in a bucket containing 3×10^{11} protons. $V_{rf} = 566$ kV for the ring. (1a) shows the space charge voltage associated with the matched ensemble (1b). The dipole distribution begins to be apparent by turn 820 (1c) and is well developed by turn 1127 (1d).

~ 4 synchrotron periods is expected for the annulus). The effect is to randomise the initial carefully ordered ensemble. It is expected that N, completely randomised, particles will have dipole moment $\approx \overline{\rho}/\sqrt{N} = 0.005$ rad for 60,000 macroparticles and average synchrotron amplitude $\overline{\rho} = 65°$.

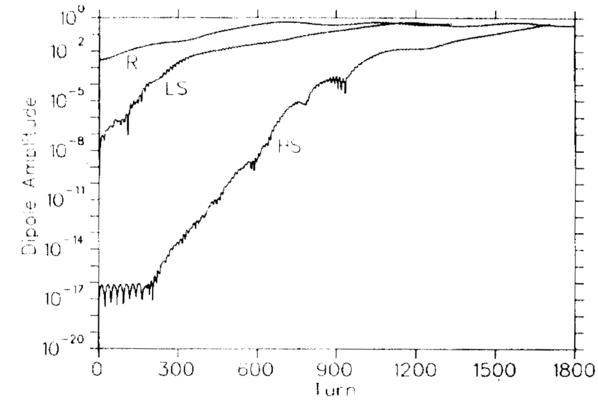

Fig. 2. Evolution of dipole moment for three initial distributions containing 3.12×10^{11} protons/bucket. R was randomly populated and the starting amplitude is the expected statistical level $0.005 \left(1/\sqrt{60000}\right)$.. HS had a very high degree of initial symmetry; the dipole moment grows toward $1/\sqrt{60000}$ as computer tracking randomises the distribution. LS starts with moderate symmetry. All three distributions exhibit the same rate of growth of dipole amplitude beyond the value expected statistically. All three have the same saturation level.

Runs with 3.1×10^{11} protons for the three starting populations are compared in Fig. 2. It can be seen that the more symmetrical distributions have "knees" around $\rho = 0.005$, which is the starting point for the random distribution. Growth in the dipole amplitude beyond this point is considered a real beam dynamic phenomenon. Evidently, one cannot do better than use a Monte Carlo populated initial ensemble for conducting "computer experiments".

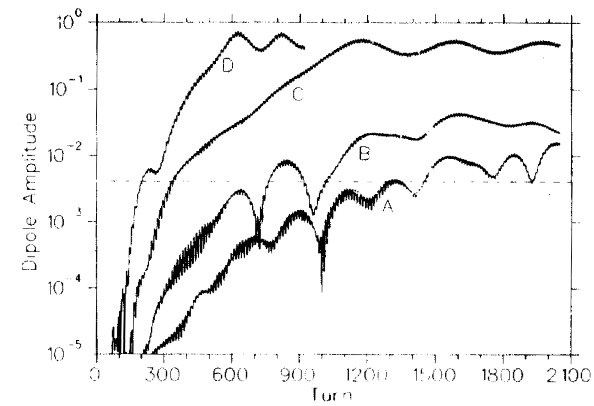

Fig. 3. Effect of charge per bucket on normalized dipole moment. A) 0.78×10^{11}, B) 1.56×10^{11}, C) 3.12×10^{11}, D) 6.24×10^{11} protons per bucket. The dashed line is the statistical amplitude associated with 60,000 macroparticles

Initial populations with low-order symmetry and four different intensities are compared in Fig. 3. The growth rate and the saturation dipole amplitude are plotted in Fig. 4 as a function of intensity. The growth rate $(1/\tau)$ has a roughly linear relationship which may be summarised for this case by

$$(1/\tau) \simeq \Delta\omega_{sc}/4$$

where $\Delta\omega_{sc}$ is the well known space charge induced tune shift for filled parabolic distribution $-\phi_{max} \leq \phi \leq \phi_{max}$. The application of such a rule for other distributions is being explored.

The saturation amplitude exhibits a threshold at 0.8×10^{13} protons/bucket, corresponding to $\Delta\omega_{sc}/\omega_s$ of 0.05 and itself saturates at $\rho = 30°$. The latter is probably related to tune spread.

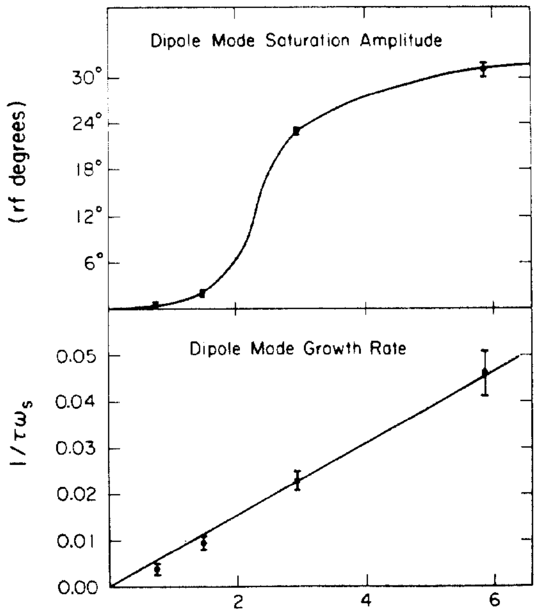

Fig. 4. a) normalized saturation amplitude vs charge/bunch. The tolerance for injection into a synchrotron may be $\sim 1°$, that for a fixed target facility $\sim 5°$. b) growth rate vs charge/bunch

Tolerances and Conclusions

The growth time for the process described here is faster than other relevant longitudinal instabilities. In the simulations no particles were lost nor came close to the bucket boundary. It would appear that a tolerance for the ring alone will be set by indirect mechanisms. An increased peak brightness increases the transverse tune shift. Generation of a halo in longitudinal phase space may couple into transverse phase space and increase foil traversals and scattering or exceed dynamic aperture constraints. The outer halo in Fig. 1c ($\rho = 5°$) is acceptable.

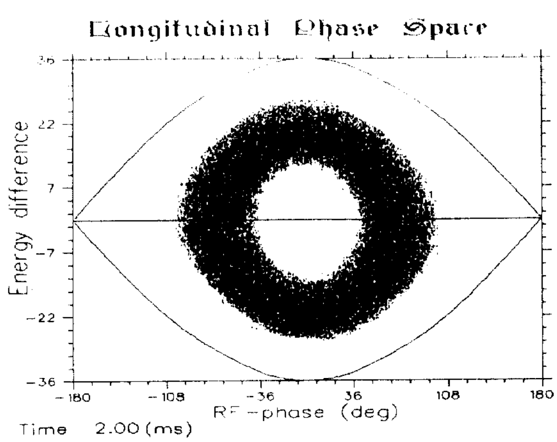

Fig. 5. With 1.6×10^{11} protons/bucket; corresponding more closely to one TRIUMF painting scheme, the dipole amplitude is barely apparent after 2049 turns.

If the accumulated beam is injected into a synchrotron then the requirements of the latter impose tighter tolerances on the accumulator. The frequency of the accelerating voltage is controlled, in part, by feedback from a detector of peak line density. The latter oscillates at ω_s. If this is imposed on the synchrotron RF the dipole amplitude will be damped but at cost of a longitudinal halo. A tolerance similar to the feedback resolution, of 1°, is appropriate. This is just met by the proposed TRIUMF accumulator painted from large to small longitudinal amplitude, Fig. 5. ACCSIM calculations including space-charge effects in all three planes show no significant transverse deterioration.

References

[1] D. Raparia, C.W. Planner, G.H. Mackenzie and J.R. Richardson, IEEE **NS-32**, 2456 (1985).
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[3] S. Koscielniak, paper submitted to European PAC (1988).
[4] S. Koscielniak, internal report, TRI-DN-88-20.
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