BEAM BREAK-UP IN THE TWO-BEAM ACCELERATOR

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Abstract

We have studied numerically beam break-up (BBU) in the drive beam of a Two-Beam Accelerator (TBA), using transverse wakes calculated by the AMOS code. We examine only cumulative BBU due to the wake of the linear induction accelerator cavities. We do not consider regenerative BBU due to the relativistic klystron (RK) cavities. We find growth lengths of order ~ 100 m for typical parameters.

Introduction

The Free Electron Laser (FEL)[1] and Relativistic Klystron (RK)[2] versions of the Two Beam Accelerator require the propagation of a relativistic electron beam of ~3 kA current, bunched on a scale of ~ 2 cm through a periodic lattice of rf structures (see Figure 1). Each period length is ~ 1 meter and at the end of each period the power generated in the rf structure (wiggler or cavity arrray) is extracted and fed into a parallel high gradient structure to accelerate a low current, high energy beam. Our concern in this paper is the beam break-up of the drive beam due to the transverse dipole wake of the LIA cavities that will be placed at intervals on the beamline. We briefly review the elements of the problem.

A high-current rf relativistic electron beam (REB) injected off-axis into the beamline of a TBA will have a dipole moment in its charge density. The axial current associated with this dipole moment will couple to the axial electric fields of the various structures along the beamline. The structures of most concern are the resistive pipe itself, the rf input and output ports, the klystron cavities (in the case of the RK TBA) and, the subject of this paper, the LIA modules.

Each slice of the beam will excite TM_{1n0}-like modes of the LIA cavity and the associated "v x B" force will give a kick to all slices to the rear. Once a slice is kicked farther from the axis, its dipole moment excites larger fields and an instability obtains.[3]

If the instability is sufficiently severe, the beam will eventually scrape the beam pipe wall and "beam break-up" or "pulse-shortening" will occur. However, even a small amount of BBU growth in the TBA is undesirable, since it will reduce the intended coupling of rf and beam in the FEL or RK, and thereby diminish the power out.

The lowest-order coupling of transverse beam centroid motion and the rf fields of a particular structure may be expressed in terms of the "transverse dipole wake potential, W(s).[4]

In this paper we use wakes, W(s), calculated by the AMOS[5] code for the ATA LIA cavity (see Figures 1 and 2). AMOS solves for the fields, in the time domain, of a bunch of gaussian axial profile traversing an azimuthally symmetric structure with mixed dielectric, conducting, and radiation boundary conditions.

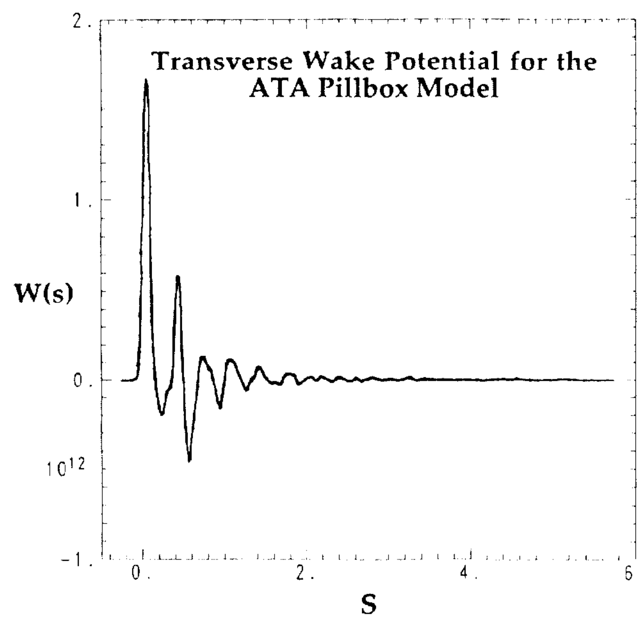

Figure 1. The wake of the "idealized ATA cavity," a pillbox terminated in an impedance. Units for W are V/C/m and s is in m (elsewhere units are cgs).

Models for the LIA Wake

We use two model numerical wakes. The first model wake, the "idealized ATA cavity wake," is that calculated by AMOS for a pillbox terminated in an impedance (the "idealized ATA cavity" or "Briggs Model").[6,7] This wake has been studied extensively in connection with BBU at the Advanced Test Accelerator at Lawrence Livermore National Laboratory (LLNL).[8]

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The second model wake, the "exact wake," is that calculated vis AMOS for the actual ATA geometry, with the approximation of azimuthal symmetry. It has the interesting feature of a new, "trapped" or high Q mode.

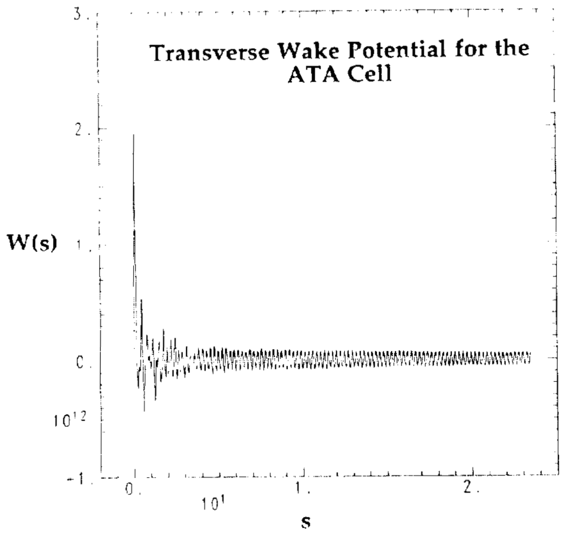

Figure 2. The wake of the ATA cell computed numerically via the AMOS code. Cylindrical symmetry is assumed."W(s)" in the figures is the "W(s)" of the text multiplied by L_{gap}. Units for W are V/C/m and s is in m (elsewhere units are cgs and s is in sec).

For analytic work, we use a third wake, the "dominant mode wake," due to a single, dominant mode of the LIA cavity[9]. It is given by

$$W(s) = W_0 \exp(-\nu s)\frac{\sin(\omega s)}{\omega}$$

where the notation is given in Table I and

$$W_0 = \frac{Z_\perp}{Q}\frac{\omega_0^3}{L_{gap}}$$

with $Z_\perp/Q$, the "surge impedance" of the mode. Note that for long λ_β, an average of this wake over the TBA period is appropriate, while for short λ_β, the wake is felt at intervals as a discrete kick.

Table I.
Notation and Typical Values for TBA Parameters

L_{gap}	=LIA gap length	~ 0.0254 m
L_p	=period length	~ 1 m
τ	=pulse length	~ 50 ns
f	=beam frequency	~ 17.14 GHz
I	=average or dc current	~ 1.5 kA
γ	= beam energy /rest energy	~ 27.4
λ_β	=betatron wavelength	~ 1.5 m
ω_0	=undamped dominant mode angular frequency	
ν	=damping rate	
Q	=cavity Q value	$=\omega_0/2\nu$
ω	=damped angular frequency	$=(\omega_0^2-\nu^2)^{1/2}$
s	=beam slice coordinate	$=t-z/v_z$
z	=axial position down beam line	
v_z	= beam axial velocity	~ c
c	= speed of light	

Basic BBU Particle Dynamics

The basic equation governing BBU is that of the motion of the center of mass of the beam slice at each s, $\xi(z,s)$, as it passes position z on the beam line. This displacement is driven by the Lorentz force associated with the fields created by the beam slices to the front at $s' < s$, and its response is governed by the focussing elements on the beam line:

$$\left(\frac{\partial}{\partial z}\gamma\frac{\partial}{\partial z}+\gamma k_\beta^2\right)\xi(s,z)=H(z)\int_0^s ds'\frac{I(s')}{I_A}W(s-s')\xi(s',z)$$

where $I_A = mc^3/e = 17.05$ kA is the Alfven current. As discussed below, we will replace the axial current profile I(s) with its average value, denoted "I." The function H(z) is 1 inside an LIA gap, and 0 outside.

We make several approximations. Consistent with our restriction to cumulative BBU, we assume the rf beam frequency does not excite appreciable resonances in the cavity (17 GHz is above cut-off in the beam pipe). Therefore, we may work with the average beam current. In addition, for analytic estimates, we assume λ_β is constant throughout each TBA period, since γ varies by only about 5%. We also neglect spreads in λ_β, i.e., Landau and BNS damping.

Analytic Estimates

Using the analytic wake exhibited above, it is straightforward to solve the BBU equation up to quadrature.[10] Laplace transforming in s, solving the simple harmonic oscillator initial value problem, and inverting the Laplace transform we have

$$\xi(s,z) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dp\,\frac{1}{p}\cos\left(z\sqrt{k_\beta^2-\frac{I}{\gamma I_A}w(p)}\right)$$

where w(p) is the Laplace tranform of the wake averaged over a TBA period and the integral is to the right of the origin in the complex p-plane. This result may be evaluated asymptotically via steepest descents. We have taken a step initial beam perturbation. Other options include a "tickled" pulse, tuned to the resonant mode, and a delta function pulse offset.

Two regimes should be distinguished: strong focussing and weak focussing. In each regime, a sufficiently high Q results in an absolute instability, while low Q results in a convection of the peak growth down the length of the beam toward the tail.

In the strong focussing regime, λ_β is short compared to the growth length, L_g. We may compute the asymptotic behavior via steepest descents and we have, for $\tau > (z/L_g)\,\nu^{-1}$,[11] a convective instability characterized by peak growth along the pulse with exponent ~ z/L_g, where

$$L_g = L_p \frac{ck_\beta}{\omega} \frac{\gamma I_A}{I} \frac{2}{Q} \frac{1}{cZ_\perp/Q}$$

For high Q, on the other hand, such that $\tau << (z/L_g)\, v^{-1}$, we have an absolute instability characterized by growth with exponent $\sim (z/L_g)^{1/2}$ where

$$L_g = \frac{k_\beta L_p}{c\tau} \frac{c^2}{\omega^2} \frac{\gamma I_A}{I} \frac{1}{cZ_\perp/Q}$$

In the weak focussing regime, λ_β is long compared to the growth length. Again, for low Q such that $\tau > (z/L_g)\, v^{-1}$, the instability is convective. Growth is characterized by exponent $\sim z/L_g$, where

$$L_g = \frac{2^3}{3^{3/4}} \left(\frac{L_p c}{\omega} \frac{\gamma I_A}{I} \frac{1}{Q} \frac{1}{cZ_\perp/Q} \right)^{1/2}$$

For high Q, such that $\tau < (z/L_g)\, v^{-1}$, the instability is absolute and the growth is characterized by an exponent $\sim (z/L_g)^{2/3}$ where[12]

$$L_g = \frac{2^{7/2}}{3^{9/4}} \left(\frac{L_p}{c\tau} \frac{c^2}{\omega^2} \frac{\gamma I_A}{I} \frac{1}{cZ_\perp/Q} \right)^{1/2}$$

and this applies provided $\tau >> (z/L_g)\, /2\omega$.

Numerical Results

The code, "TWA", running on the MFECC Crays, makes use of a fourth order Runge-Kutta advance in z, and a fourth order gaussian integration in s, assisted by a fourth order Lagrangian interpolation of the integrand. The wake is read in as an array provided, for example, by the AMOS code.

For runs discussed in this paper, λ_β and γ are constant throughout the beam. The wake is turned on for one step per period and normalized accordingly. Runs were performed for the idealized ATA cell and the realistic cell, for design parameters as in Table I.

We found negligible growth in 100 m for either wake. That this should be so may be seen from a simple estimate. Using $Q \sim 6$ - 8 and $Z_\perp/Q \sim 8\ \Omega$ for the idealized ATA cell, and the parameters of Table I, we obtain, in the strong focussing regime, a convective growth length of ~ 100 m for this example TBA design. This is in contrast to ATA parameters, which give growth lengths of order ~1 m.

In addition, we examined the effect of increasing I and λ_β beyond the values in Table I. (see Figure 3). These additonal runs provided a test of the weak focussing analytic results, which are, in general not applicable to the TBA, due to the long pulse length. We observed asymptotic growth with the $z^{2/3}$ dependence in the exponent, as expected.

Conclusions

Even in the absence of Landau or BNS damping,[13] we can expect BBU growth lengths on the order of ~100 m for typical TBA parameters and this is acceptable for a future TBA.

Future work should include: an examination of higher frequency modes not cut-off in the beam pipe (regenerative BBU); studies of BBU due to the RK cavity and the the rf input and output ports for the FEL TBA; and a BBU analysis for the SNOMAD II cavity currently the subject of experiments at ARC/LLNL.

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[10] An example of a similar calculation may be found in G.J. Caporaso, W.A. Barletta and V.K. Neil, Part. Acc. 11, 71 (1980)

[11] V.K. Neil, L.S. Hall and R.K. Cooper, Part. Acc. 9, 213, 1979. The actual restriction on this result is $L_g >> L_p/\sin(k_\beta L_p)$.

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[13] Landau damping refers to damping due to a spread in k_β within a slice at s,z, while BNS damping refers to that due to a spread in k_β, across the beam, i.e., $k_\beta(s)$, constant within each cross-section.

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