BBU IN MICROTRONS WITH SUBHARMONIC INJECTION*

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Abstract

The current in recirculating electron accelerators is limited by the recirculating regenerative type of beam breakup (BBU).[1] Existing calculations of the BBU threshold current, I_s, average over the relative phase between the beam bunch and the RF mode causing the blowup. This averaging is invalid if the BBU frequency is an integral or half-integral multiple of the beam frequency, in which case I_s may be substantially reduced. This effect is not important when the beam frequency equals the accelerating mode frequency, f_0, because the RF structure can be designed to avoid the harmonic condition. When the beam frequency is a large submultiple of f_0, this may not be possible. Calculations for the NIST RTM[2], which will inject at the 36^{th} subharmonic of f_0, are presented. Our calculations also include the effects of the reversed first return orbit and the variable return-path focusing of the RTM.

Introduction

The basic mechanism of recirculating regenerative BBU is illustrated in figure 1. In addition to the accelerating mode, most RF structures support many other modes, including some (e.g., TM_{11}- and TE_{11}-like modes) which can deflect the beam transversely, even if the beam is on axis. A beam deflected by this interaction will, in general, return to the accelerator on subsequent passes off axis, where the beam can exchange energy with the deflecting mode. If, on average, the mode extracts energy from the off-axis beam, the deflection will grow until the beam is lost, unless the energy in the mode is removed from the structure. The mechanism for energy removal in room temperature structures is resistive dissipation of the structure. Since the rate of energy input is proportional to beam current while the rate of dissipation is independent of current, recirculating regenerative BBU will exhibit a threshold current, I_s, above which the accelerator will not operate stably, and below which there is little, if any, perturbation of the beam.

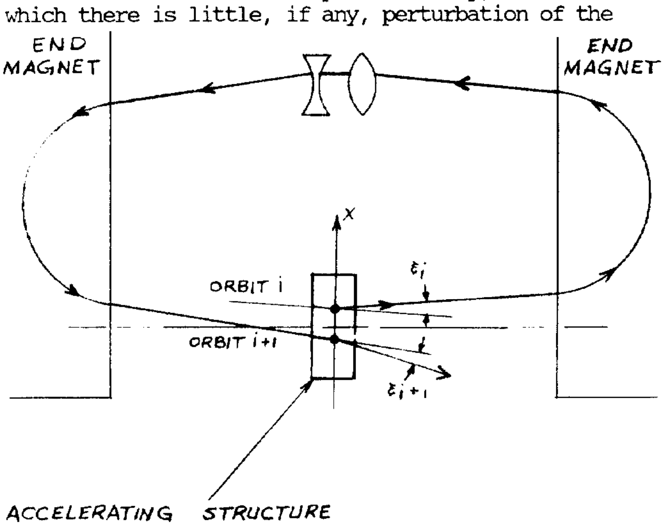

Figure 1. Schematic drawing of the basic mechanism of recirculating regenerative BBU.

The BBU phenomenon in recirculating accelerators has been studied extensively. In reference 1, it is estimated that I_s>0.83 mA in the NBS RTM. The original proposal[3] for the development of the NIST RTM envisioned an experimental study of BBU, but it was not believed to be a limiting factor in machine performance since the design maximum average current is 0.55 mA, well below the predicted threshold.

It is necessary to reexamine the BBU problem as it applies to the NIST RTM because of the plan to inject beam into the RTM at a subharmonic of the accelerating frequency, f_0. Subharmonic injection is used to increase the peak beam current and thus increase the gain of the Free Electron Laser which is the primary planned user of the RTM, without increasing the average current, which is limited by available RF power.[2] All existing calculations of recirculating BBU (to the best of our knowledge) make use of an average over the phases of the electron beam bunches relative to the blowup mode. This averaging process is valid in the every-bucket-filled case unless $f/f_0 = n/2 \pm O(1/Q)$, where f is the blowup-mode frequency, n is an integer, Q is the (loaded) quality factor of the blowup mode, and O indicates "of the order". If the beam is injected at subharmonic h of the accelerating frequency (so that the beam repetition frequency is f_0/h), the averaging process is invalid when

$$f/f_0 = n/(2h) \pm O(1/Q) \quad . \qquad (1)$$

Existing data on the mode pattern of the side-coupled structure used in the RTM indicates that the harmonic condition probably occurs for two modes in the TM_{11} band.[4] It is therefor necessary to include this possibility in the BBU calculations.

The goals of the present study are to predict the BBU threshold current for the RTM and to find practical methods to raise the threshold, if necessary. A realistic computer model of BBU must include:

1. the ability to calculate I_s when an harmonic condition, as defined by equation (1), exists;
2. the effect of the reversed-first-return geometry of the RTM;
3. the full effect of the focusing system of the RTM; and
4. the effect of the coupled-cell nature of the RF structure, which is expected to affect the width of the resonances as well as influence the value of I_s.

The model is being developed in stages to facilitate comparison with previous work, and to allow assessment of the importance of the various effects. As the computer model is developed, it will be used as a guide to the experimental program of determining I_s and to choosing operating conditions which will raise the BBU threshold.

Formulation of the Problem

The present computer model contains the following major approximations:

1. The accelerating section of the RTM is represented by a single, short cavity in which the energy gain per pass is ΔW. This cavity supports several BBU modes, such that the ratios of the BBU mode frequencies to the accelerating mode frequency are

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$\eta = f/f_0$. The different BBU modes are assumed to act independently on the beam, so that each has its own value of I_s. The lowest calculated I_s is taken to be the actual BBU threshold.

2. The focusing of the RTM is represented by a set of 2 X 2 transfer matrices, R_{jk}, representing the motion, in one of the transverse planes, between the center of the accelerating section on passes j and k. Together with approximation 1, this implies that the beam-accelerator interaction on pass j occurs at energy

$$W_j = W_0 + (j-1/2)\Delta W \quad . \qquad (2)$$

Transverse-longitudinal coupling, which can affect the transverse position of the beam centroid when focusing elements are used on the RTM return paths, is ignored. Subject to these approximations, I_s is calculated as outlined below.

The stored energy, U, in the cavity, <u>in the BBU mode</u> at time t is governed by the equation

$$\frac{dU}{dt} + \frac{2\pi f}{Q} U = P(t) \quad , \qquad (3)$$

where P(t) is the power input to the mode by the beam, averaged over a time of order $\tau = Q/(2\pi f)$, the decay time of the BBU mode (when P=0). U is related to the electric field amplitude, E, (in a pillbox cavity of length l) by the shunt impedance, R, given by

$$R = E^2 l^2 Q/(8\pi f U) \quad . \qquad (4)$$

The axial electric field, E_z, and the transverse magnetic field, B_y, are given by

$$E_z = -(\pi f E x/c) \sin 2\pi f t \qquad (5)$$

and

$$B_y = (E/2c) \cos 2\pi f t \quad , \qquad (6)$$

where x is the transverse coordinate measured from the cavity axis. The angular deflection of the beam centroid on pass j due to B_y is

$$\xi_j = -(eEl/2W_j) \cos 2\pi f t \quad . \qquad (7)$$

The beam centroid displacement on any later pass, k, due to ξ_j, is $x_k = (R_{jk})_{12}\xi_j$. The energy transferred from a pulse of electrons with charge q (which passes through the cavity at displacement x and at time t) to the cavity is $\Delta U(t) = q(t)\, l\, E_z(x,t)$.

We must sum the energy inputs over all passes for each pulse. The result is

$$\Delta U = \frac{\pi e l^2 f q E^2}{2c} \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk} \sin(\phi+\theta_k) \cos(\phi+\theta_j) \quad , \qquad (8)$$

where: J = number of passes through the cavity, $F_{jk} = (R_{jk})_{12}/W_j$, $\phi = 2\pi f t_1$ is the arbitrary initial phase of the beam pulse (on pass 1 relative to the phase of the blowup-mode RF field),

$\theta_j = 2\pi\eta \sum_{i=1}^{j-1} N_i/\beta_i$ is the beam phase advance relative to the blowup mode from pass 1 to pass j (θ_1=0 and β_i is the velocity of the beam on pass i, in units of the speed of light), and N_i is the circumference of the orbit on pass i in units of the accelerating mode free-space wavelength $\lambda = c/f_0$.

In obtaining equation (8) we have used the fact that the total transit time,

$f_0^{-1}\sum_{i=1}^{J-1} N_i/\beta_i$, is much less than τ, so that E can be taken to be time-independent. The beam consists of a series of infinitismally short pulses, each containing charge q, at the repetition frequency f_0/h, so that the time-averaged beam current is $I = qf_0/h$. Time averaging equation (8) leads to

$$P = e l^2 f I E^2 S(\phi)/(4c) \quad , \qquad (9)$$

where

$$S(\phi) = \frac{2}{M} \sum_{m=1}^{M} \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk} \sin(\phi_m+\theta_j) \cos(\phi_m+\theta_k) \quad . \qquad (10)$$

In equation (10), $\phi_m = \phi + 2\pi\eta h m$, and M is the number of beam pulses over which the average is taken. Substituting equation (9) in equation (3), and using equation (4) to eliminate E, we obtain

$$\frac{dU}{dt} = \frac{2\pi f}{Q} \left(\frac{I}{I_s} - 1\right) U \quad , \qquad (11)$$

where we have defined

$$I_s = \lambda/(\pi\eta R S) \quad . \qquad (12)$$

It is clear from the form of equation (11) that if $I > I_s$, the power in the blowup mode will grow exponentially, whereas if $I < I_s$, any excitation of the mode will damp.

<u>Harmonic vs non-Harmonic Case</u>

We next address the dependence of S on the initial phase, ϕ. Equation (10) can be rewritten as

$$S(\phi) = \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk}[\sin(\theta_k-\theta_j) + \langle\cos 2\phi_m\rangle \sin(\theta_k+\theta_j) + \langle\sin 2\phi_m\rangle \cos(\theta_k+\theta_j)] \quad , \qquad (13)$$

where < > indicates averaging over m. If ηh is any integer or half-integer, $\langle\cos 2\phi_m\rangle = \cos 2\phi$ and $\langle\sin 2\phi_m\rangle = \sin 2\phi$. For any other value of ηh, these averages vanish. In the latter (non-harmonic) case

$$S = S_0 = \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk} \sin(\theta_k-\theta_j) \quad , \qquad (14)$$

and $I_{s0} = \lambda/(\pi\eta R S_0)$. In the harmonic case,

$$S(\phi) = S_0 + S_1 \sin 2\phi + S_2 \cos 2\phi \quad , \qquad (15)$$

where

$$S_1 = \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk} \cos(\theta_k+\theta_j) \quad , \qquad (16)$$

and

$$S_2 = \sum_{k=2}^{J} \sum_{j=1}^{k-1} F_{jk} \sin(\theta_k+\theta_j) \quad . \qquad (17)$$

In the harmonic case, since the buildup of the blowup mode starts from noise, it will tend to assume that phase which minimizes I_s, $I_{smin} = \lambda/(\pi\eta R S_{max})$, where, from equation (15)

$$S_{max} = S_0 + |S_1 \sin\psi + S_2 \cos\psi| \quad , \qquad (18)$$

and $\psi = \tan^{-1}(S_1/S_2)$.

<u>General Form of the Starting Current</u>

In both harmonic and non-harmonic cases, I_s is very sensitive to the value of η because the phase, θ_j, reaches values of the order $2\pi\eta N_J J$. For the NIST RTM, N_1=101, N_2=204, $N_j = N_2+2(j-2)$, and J=15. The blowup mode frequency ratios, η, are in the range 1.5 to 2. Thus, θ_J is of order 2×10^4. A change of η by one part in 10^4 can change I_s drastically. This is clearly unphysical when the blowup-mode Qs are of order 10^4. In evaluating I_s, for each value of η we find the smallest value of $I_s(\eta')\exp[2Q(\eta-\eta')/\eta]^2$, and take this to be the starting current for the mode with frequency ratio .

The transverse tune of the RTM enters the calculation via the $(R_{jk})_{12}$. These could all be made zero, but this corresponds to a half-integer resonance condition, and is not practical, especially when the extended length of a real accelerating section is considered. However, transverse tunes, μ, in the range of 45 to 90 degrees of betatron phase per pass are possible. Thus, the F_{jk} can change sign several times in the summations, resulting in a large increase in the BBU starting current, compared to a weakly focused microtron.

From the form of equation (18), it is obvious that I_s for any given mode will be lower if the harmonic condition, equation (1), is satisfied for that mode. In our numerical studies for the NIST RTM, the harmonic-case threshold is lower than in the non-harmonic case by a factor of about two, typically, and occasionally by as much as a factor of five. However, when there are several blowup modes, one of the modes which does not satisfy the harmonic condition may have the lowest threshold. In such cases, subharmonic injection has no effect on the BBU threshold.

Results

A computer program has been written to calculate I_s in both the harmonic and non-harmonic cases. Two versions of the program are available. The more general one uses transfer matrices R_{jk} obtained from measurements or calculations. The simplified version of the program uses

$$(R_{jk})_{12} = (W_j/W_k)^{1/2}\beta \sin(k-j)\mu \,, \qquad (19)$$

where β and μ are constants. In this case we also use

$$N_j=N_1+\nu(j-1) \quad . \qquad (20)$$

Figure 2 illustrates the behavior of I_s in this simplified model as a function of μ. The parameters chosen for these sample calculations, W_0=5 MeV, ΔW=12 MeV, and J=15 are appropriate for the NIST RTM. We use equation (20) for N_j, with N_1=202 and ν=2.

In these sample calculations we use a model for the properties of the RF structure of the main accelerating section of the NIST RTM in which there are six blowup modes at frequencies near the intersections of the TM_{11} band frequencies (as a function of phase shift per cell) with a line representing the condition that the phase shift per cell be $\pm\pi\eta$ so that the beam encounters all cells at the same blowup-mode phase. This condition is used in the absence of a calculation of the effects of the finite extent of the accelerating structure, and should correspond to a lower BBU threshold than any other phase shift per cell (which would not be synchronous with the beam).

The six modes are all assigned a transverse shunt impedance of R=20 MΩ, and Q=10^4. The frequencies are known approximately from measurements made on the preaccelerator section of the NIST RTM.[4] Two of these frequencies are very close to satisfying the harmonic condition for 36th subharmonic injection, and were arbitrarily shifted to exactly satisfy equation (1). R and Q are estimates, since they have not been measured, but are believed to be conservative.

The calculations shown in figure 2 are in general agreement with the estimates in reference 1 for the magnitude of I_s and its trend to increase with stronger focusing (increasing μ and decreasing β). The decrease in blowup threshold for a subharmonically bunched beam is a new result.

The more general version of the program has been used to investigate the effect on BBU threshold of a number of particular features of the NIST RTM design. With the design parameters given above, in the particular case of a betatron tune of 45 degrees/pass, the effect of the reversed first return path (which changes N_1 from 202 to 101 leaving all other N_j unchanged and reduces β by a factor of two for the first pass only) is to reduce I_s by a factor of about two in both the subharmonic and every-bucket-filled cases. We next used a full set of calculated transfer matrices corresponding to the nominal design of the RTM. This design uses a betatron tune of approximately 90 degrees/pass on the first three passes and 45 degrees/pass on all others. This change increased I_s, to about the original value of the simplified version of the calculation for the non-harmonic case. In this particular numerical example, there is no decrease in I_s due to bunching at the 36th subharmonic. There would have been a reduction (of about 12%) if bunching were at the 32nd subharmonic. The tendency for stronger focusing on the early passes to increase the threshold should be quite general.

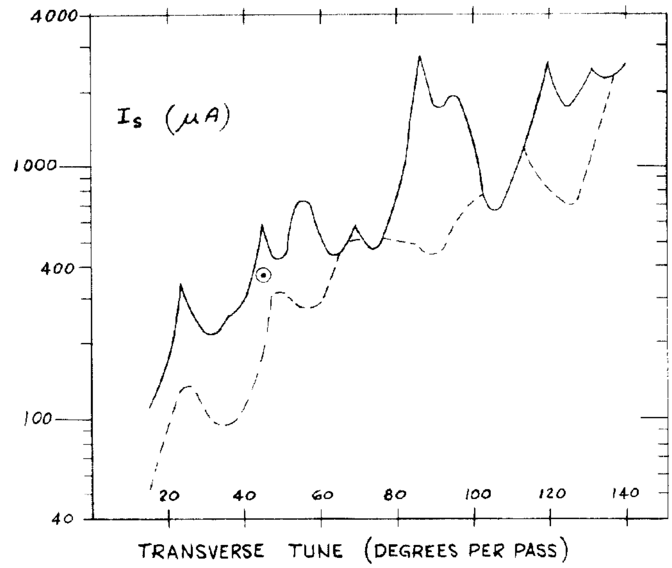

Figure 2. Predicted BBU threshold current as a function of transverse tune. The solid curve is the every-bucket-filled case, and the dashed curve is for a beam injected at the 36th subharmonic of the accelerating-mode frequency. Discontinuities in the slopes of the curves are due to changes in the mode having the lowest threshold. The circled dot at a tune of 45 degrees is the result using the more general formulation, which includes the calculated transfer matrices of the NIST RTM.

The most significant omission from these calculations is the effect of the 8-m length of the accelerating structure. However, since we have chosen blowup frequencies which are synchronous with the beam, and transfer matrices for vertical motion (which corresponds to the TM_{11} polarization which couples cell-to-cell by the coupling cells of the side-coupled structure), the predicted blowup thresholds are probably conservative. By choosing an appropriate transverse tune, the threshold is expected to be above 0.5 mA in the subharmonic beam case, and above 1.0 mA in the every-bucket-filled case.

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