MODELING FERRITE ELECTROMAGNETIC RESPONSE IN THE TIME-DOMAIN *

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Abstract

The behavior of ferrite loads commonly found in induction accelerators has important consequences for the performance of these accelerators. Previous work by the authors on modeling the electromagnetic fields in induction cavities has focussed upon use of a simple, phenomenological model for the process of magnetization reversal in these ferrite loads. In this paper we consider a model for magnetization reversal which is more deeply rooted in theory, and present a simulation of the reversal process based upon this model for an idealized set of boundary conditions.

Introduction

The subject of rapid magnetization reversal, in addition to its substantial theoretical importance, has acquired a practical significance as well, arising from the use of this process in linear induction accelerators (linacs). In these devices, the large electromagnetic fields generated by rapid flux reversal in ferrite toroids are used to accelerate charged particles. It is necessary to understand the flux reversal process in order to model these fields.

In general, the reversal process is extremely complicated because it results from the simultaneous influence of four different interactions: the magnetic, anisotropy, magnetoelastic, and exchange interactions. Because of this complexity, a *general* treatment of the process is, at present, hopelessly difficult. One can, however, by limiting one's attention to regimes in which one or two of the interactions dominate, reduce the complexity to manageable levels. Of these limiting regimes, there are two which may bear upon the flux reversal process in linacs. In the first, dominated by magnetic interactions alone, the flux reversal occurs by a process called *uniform rotation* [1]. In the second, the reversal occurs by *domain wall motion* [2], and is dominated by the combined effects of both anisotropy and exchange.

The authors are currently working to incorporate mathematical models for both regimes into a numerical algorithm which simulates the electromagnetic fields inside linac cavities. The focus of the present paper, however, will be upon the uniform rotation regime. We will briefly discuss the theoretical basis of the model for this regime, and present the results of a numerical simulation of the flux reversal process in an infinite cylinder with idealized boundary conditions.

Magnetization Reversal by Uniform Rotation

The fundamental supposition underlying the uniform rotation model is that there exist circumstances in which the reversal process is governed by magnetic interactions alone. The model neglects the effects of anisotropy, magnetostriction, and exchange. Neglecting anisotropy and magnetostriction is at least superficially plausible for the soft ferrites used in linac cavities, since the maximum anisotropy and magnetoelastic energy densities in such materials are both generally smaller (sometimes much smaller) than the magnetic energy density. However, the energy density associated with the exchange interaction can be several orders of magnitude *larger* than the magnetic energy density, if neighboring dipoles in the lattice have significantly different orientations. It can be shown, though, that if the orientation of the atomic dipoles does not change significantly over a distance on the order of 10^5 lattice spacings ($\approx 10^{-2}$ cm), then the exchange energy density is indeed negligible compared to the magnetic energy density. (This is a consequence of the short-range nature of exchange: a given dipole interacts via the exchange interaction only with its nearest neighbors, whereas the same dipole interacts *magnetically* with every other dipole in the sample.) Thus, the assumption that exchange may be neglected can be seen as equivalent to the assumption that throughout the reversal process, the magnetization remains a gradually varying function of position.

Consider a rigid, cubic lattice of magnetic dipoles. (The choice of a cubic lattice is a convenience; the following treatment may readily be extended to any other regular array). If $\vec{\mu}_i$ is the magnetic moment of the ith dipole in the lattice, then the torque $\vec{N}_i$ on that dipole is given by:

$$\vec{N}_i = \frac{\partial \vec{j}_i}{\partial t} = \vec{\mu}_i \times \vec{\mathcal{B}}_i, \tag{1}$$

where $\vec{j}_i$ is the angular momentum of the ith dipole, and $\vec{\mathcal{B}}_i$ is the magnetic induction at the ith lattice site due to all *other* dipoles in the lattice and sources outside the sample. It can be shown that $\vec{\mathcal{B}}_i$ is related to the *volume average* magnetic induction $\vec{B}$ and volume average magnetic moment density $\vec{M}$ by the expression [3]

$$\vec{\mathcal{B}}_i = \vec{B} - \frac{2\mu_o}{3}\vec{M}, \tag{2}$$

where μ_o is the free-space permeability.

The net angular momentum of an atomic dipole is related to its magnetic moment by $\vec{j} = \gamma^{-1}\vec{\mu}$, where γ, the gyromagnetic ratio, is a constant which is characteristic of the material. Using this and employing Eq. (1) we obtain:

$$\dot{\vec{\mu}}_i = \gamma \vec{\mu}_i \times \vec{\mathcal{B}}_i. \tag{3}$$

Multiplying by the number of dipoles per unit volume, and using Eq. (2), yields

$$\dot{\vec{M}} = \gamma \vec{M} \times \left(\vec{B} - \frac{2\mu_o}{3}\vec{M}\right) = \gamma \vec{M} \times \vec{B}. \tag{4}$$

This expression, essentially an equation of motion for the magnetization, was first introduced by Landau and Lifshitz [4], and it is rigourously correct provided that our assumptions are correct. Notice, however, that since $\dot{\vec{M}}$ is perpendicular to both $\vec{M}$ and $\vec{B}$, the equation implicitly conserves the magnetic energy of the sample; this, unfortunately, is at odds with the observed tendency of magnetic materials to dissipate magnetic energy (into lattice vibrations) whenever the magnetization state of the material changes. (Eq. 4 is deficient in this respect because we assumed that the lattice was perfectly rigid.) The dissipation mechanisms involved in magnetization reversal are poorly understood; despite this, we are forced to take account of the dissipation in some fashion, and we therefore make the following modification of Eq. (4) suggested by Gilbert [5,6]:

$$\dot{\vec{M}} = \gamma \vec{M} \times \vec{B} + \frac{k_d}{M}\vec{M} \times \dot{\vec{M}}, \tag{5}$$

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where M_s is the saturation magnetization of the material. The added term is a damping term, since it produces a component of rotation that is antiparallel to the torque exerted by the magnetic induction. The constant k_d is to be treated as an adjustable parameter. It should be emphasized that there is, at present, no sound theoretical basis for this characterization of the damping process.

The Solution for Simplified Boundary Conditions

Under the assumptions set forth in the previous section, the volume-average electromagnetic fields in a region of space containing ferrite are governed by the following set of equations:

$$\left.\begin{aligned} \nabla \times \vec{H} = \sigma \vec{E} + \dot{\vec{D}}, \qquad \nabla \times \vec{E} = -\dot{\vec{B}}, \\ \nabla \cdot \vec{B} = 0, \qquad \nabla \cdot \vec{D} = \rho, \\ \dot{\vec{M}} = \gamma \vec{M} \times \vec{B} + \frac{k_d}{M_s} \vec{M} \times \dot{\vec{M}}, \end{aligned}\right\} \tag{6}$$

provided that the ferrite remains in the uniform rotation regime. We now obtain solutions to this set of equations for a simple, one-dimensional problem. We choose for the sample geometry an infinite circular cylinder of radius a. This geometry is depicted in Fig. 1. (Note that this geometry does bear some resemblance to that of the ferrite toroids in linac cavities, since an infinite cylinder is equivalent to a toroid of *circular* cross-section in the limit as the inner radius of the toroid goes to infinity.) The applied field $\vec{B}_a$, i.e., the magnetic induction due to sources outside the cylinder, is assumed to be spatially uniform in the vicinity of the cylinder, and have a time variation given by:

$$\left.\begin{aligned} \vec{B}_a = B_a \hat{z}, \qquad (t < 0), \\ \vec{B}_a = -B_a \hat{z}, \qquad (t \geq 0). \end{aligned}\right\} \tag{7}$$

The applied field, then, is assumed to be reversed instantaneously at $t = 0$. If the system is initially in equilibrium, then at $t = 0$, $\vec{M} = M_s \hat{z}$; it follows that for $t \geq 0$, $\dot{\vec{M}} = 0$, and system remains in (unstable) equilibrium. This situation would never arise for any physically realizable set of boundary conditions since, among other things, one could never keep the applied field perfectly parallel to the magnetization during the process of reversing its direction. But for the idealized boundary conditions used here, it is necessary to assume some initial angular displacement in $\vec{M}$ in order for the reversal to occur. The choice we will make is:

$$\vec{M}(\vec{r}) = M_s \cos\delta \hat{z} + M_s \sin\delta \hat{\phi}, \qquad \delta \ll \pi. \tag{8}$$

It is to be emphasized that this rather artificial assumption of initial displacement will not be necessary when realistic problem geometry and boundary conditions are used.

Since the applied field, the initial magnetization state, and the governing equations all have cylindrical symmetry, it follows that the resultant motion of the magnetization will have the same symmetry. We therefore require that $\vec{E}$ and $\vec{B}$ exhibit only radial (ρ) variation, which forces $E_\rho = B_\rho = 0$, and yields the following relations:

$$\left.\begin{aligned} \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi) = \epsilon \dot{E}_z, \quad -\frac{\partial H_z}{\partial \rho} = \epsilon \dot{E}_\phi \\ \frac{\partial E_z}{\partial \rho} = \mu_0 \left(\dot{H}_\phi + \dot{M}_\phi\right), \quad -\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi) = \mu_0 \left(\dot{H}_z + \dot{M}_z\right). \end{aligned}\right\} \tag{9}$$

We apply the boundary conditions $E_\phi = \frac{\partial E_z}{\partial \rho} = 0$ at $\rho = 0$, and apply a radiation condition to the fields at $\rho = \infty$. Eqs. (5) and (9) can be integrated numerically using an explicit time integration scheme [7]. To carry out the integration, the field components are represented at discrete radial points, and at discrete time steps. The electric field is represented at integer time steps, $t = 0,\ \Delta t,\ 2\Delta t, \ldots$, where Δt is the time step size, and at radial positions $\rho = 0,\ \Delta\rho,\ 2\Delta\rho, \ldots$, where $\Delta\rho$ is the radial node spacing. The magnetic induction is represented at radial positions $\rho = \Delta\rho/2,\ 3\Delta\rho/2, \ldots$, and exists at half-integer time steps, i.e., $t = \Delta t/2,\ 3\Delta t/2, \ldots$. The parameters Δt and $\Delta\rho$ are chosen to satisfy stability criteria.

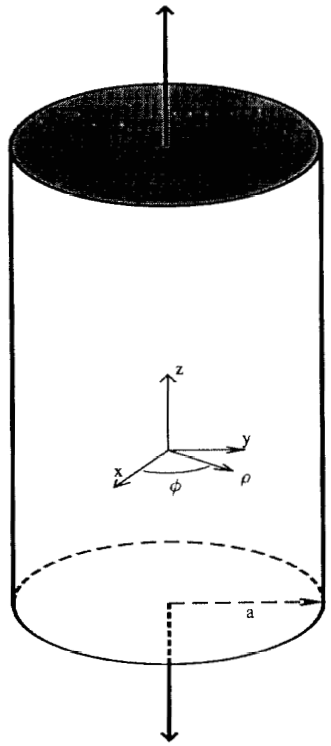

Fig. 1. Sample geometry. Cylinder has circular cross section of radius a, and is infinite in the axial (z) direction. Cylinder is surrounded by free space.

Fig. 2 depicts the solution for $M_z(t)$ averaged over all radii for $a = 10$ cm, $B_a = 9.42 \times 10^{-4}$ T, $M_s = 300$ KA/m, $\gamma = -.278$ rad·sec^{-1}·T^{-1}, and a dielectric constant of $\epsilon_r = 12$. These values are representative of the situation in the ATA (Advanced Test Accelerator) linac cavities. Nonuniformities in the induced magnetic field cause significant radial variations in $\vec{M}$ to develop during the switch, an effect that becomes more pronounced as the damping parameter is decreased. For the results presented in this paper the damping parameter was taken to be $k_d = 4$, and was chosen to produce a switching time in agreement with the values observed in the ATA. Fig. 3 shows the solutions for M_z, M_ϕ, and M_ρ at $\rho = a/2$. It may be observed that for our choice of parameters M_ρ is quite small, reaching a maximum of $\approx 10^{-3} M_s$ halfway through the switch. Therefore, to a first approximation, the rotation of $\vec{M}$, at a particular radius ρ, occurs tangent to the cylinder of constant ρ. The time-evolution of the flux of this tangent component of $\vec{M}$ is depicted in Fig. 4.

It is also worth noting that the switching time can be minimized with respect to the damping parameter. The minimum switching time for $B_a = 9.42 \times 10^{-4}$ T is approximately 1/4th of the value observed in the ATA, and is obtained for $k_d \approx 1$.

These results can be understood qualitatively in the following way: The magnetic energy density of the sample at $t = 0$ is given by:

$$W_s = -\frac{\mu_o}{6} M^2 + \mu_o M B, \tag{10}$$

where the first term represents the magnetic self-energy of the sample, and the second term represents the interaction energy between the sample and the applied field. After the flux reversal the magnetic energy density is

$$W_f = -\frac{\mu_o}{6} M^2 - \mu_o M B. \qquad (11)$$

So the magnetization reversal can be thought of as resulting from the tendency of the sample to seek out a state which lowers its interaction energy with the applied field. However, the magnetic self-energy of the sample (first term) is typically two orders of magnitude larger than its interaction energy with the applied field (second term). It follows that since the sample starts out in a state of minimum magnetic self-energy, it must, to first order, *remain* in states of minimum magnetic self-energy throughout the process of reversal. It can be shown that states of minimum magnetic self-energy are uniquely those states for which $\nabla \cdot \vec{M} = 0$ everywhere, and an examination of Fig. 4 reveals that the intermediate magnetization states are, indeed, states in which the divergence of $\vec{M}$ is very nearly zero (in fact, if we ignore the small radial component of $\vec{M}$, $\nabla \cdot \vec{M}$ is rigourously zero).

Conclusions

The Landau-Lifshitz-Gilbert equation can be used in conjunction with Maxwell's equations to model the magnetization reversal process in those situations in which the non-magnetic interactions in the material are negligible. We have seen that the resulting behavior, called uniform rotation, is characterized by intermediate magnetization states for which $\nabla \cdot \vec{M} \approx 0$, and may be described as a precession of the magnetization vector about the small demagnetizing field that is induced during the switch.

The theory underlying the uniform rotation model which we have presented is incomplete in two respects. The first results from the purely ad hoc nature of the model's characterization of the dissipation process. The second results from the inability of the model to predict its own threshold, that is, to specify the set of circumstances in which it will be valid. Although a general theoretical treatment of these issues has proven a formidable task, it should be possible to resolve them experimentally for any given material, should the need arise.

The work presented here is part of a broader attempt by the authors to simulate the magnetization reversal process in induction linacs. This work includes (1) the incorporation of the uniform rotation model, and a separate model for domain wall motion, into a general numerical algorithm for field simulation in linacs; (2) an attempt to gain a theoretical understanding of the thresholds and dissipation mechanisms of these regimes; and (3) an effort to verify experimentally the existence of both types of behavior.

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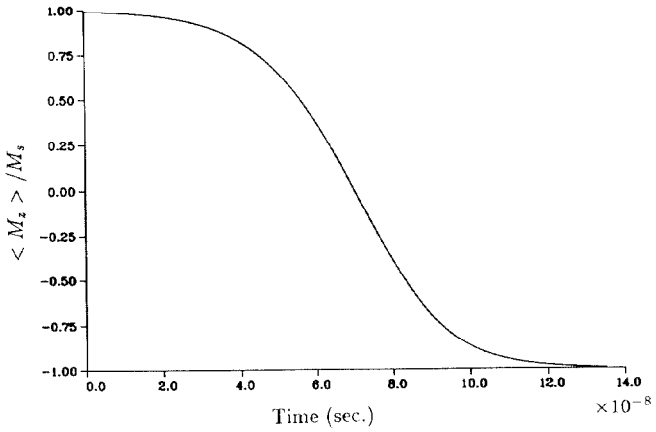

Fig. 2. $M_z(t)$ averaged over all radii for $\epsilon_r = 12$, $M_s = 300$ KA/m, $B_a = 9.42 \times 10^{-4}$ T, $k_d = 4$, $\gamma = -.278$ rad·sec^{-1}·T^{-1}, and $a = 10$ cm, which values are chosen to be representative of the ATA linac cavities.

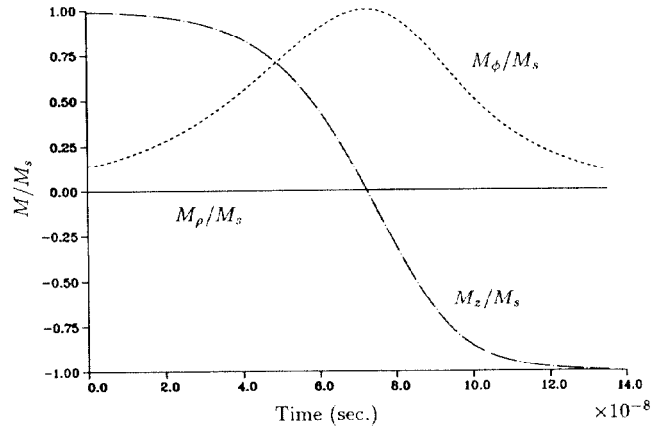

Fig. 3. Components of $\vec{M}(t)$ at radial position $\rho = a/2$. M_r, while small compared with M_s, produces a radial demagnetizing field that becomes comparable to the applied field.

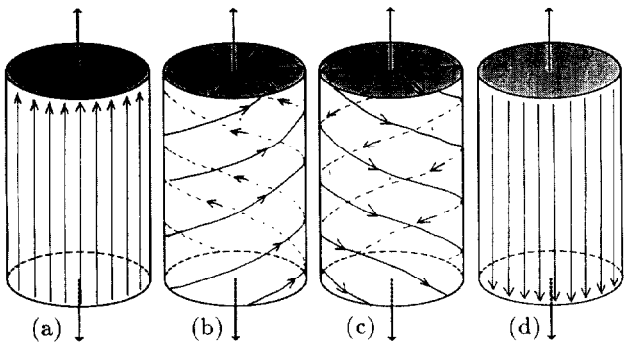

Fig. 4. Depicts the flux of $\vec{M}$ for a shell of constant ρ. (a) Initial state, consisting of $+z$-directed magnetization with a small azimuthal perturbation. (b) and (c) Intermediate states, in which the flux of $\vec{M}$ spirals around the cylindrical shell, while acquiring a small radial component. In effect, $\vec{M}$ precesses about the demagnetizing field associated with the small radial component M_r. (d) The final state.

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