EFFECT OF ENERGY SPREAD ON THE DIPOLE BEAM BREAK-UP INSTABILITY*

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Introduction

The cumulative beam break-up (BBU) instability has been studied extensively since its discovery at SLAC and its subsequent theoretical description by Panofsky and Bander[1] and others.[2-5] Its effects have been observed in both rf[1] and induction[2,6] linacs and it is widely believed to be a fundamental current limiting mechanism in these machines.

BBU growth can be controlled by reducing the Q's of the deflecting modes, by using strong focusing and by increasing accelerating gradients. More recently it has been suggested[7,8,9] that a small intra-bunch energy spread will also have a strong stabilizing effect, at least for a short bunch. Experimental confirmation of this effect in rf linacs has recently been obtained at SLAC.[10]

The quantitative effect of energy spread is rather different in rf and induction linacs, first because the focusing systems are usually different (quads in rf linacs and solenoids in induction machines) and second because the bunch length is short compared to the rf wavelength of the deflecting mode in an rf linac and long compared to that wavelength in an induction linac. While a small, systematic head-to-tail energy spread can have a dramatic effect on BBU growth in an rf machine, the same cannot be said of a solenoid focused induction machine, for reasons discussed below; a quadrupole focused induction linac would have an advantage in this regard.

The basic effect of energy spread is to introduce a relative betatron phase shift among the different 'slices' of a bunch, assuming that all particles in a slice have the same energy. It is possible, in principle, for a quadrupole focused system, to choose the distribution of energy in the bunch to cancel exactly the phase change due to the transverse defocusing force of the wake field;[7,8,11] the same is not true in a solenoid focused machine. In practice, a linear dependence of energy on distance from the bunch head, decreasing from head to tail, can effectively stabilize the BBU in short bunches in a quadrupole focused system.

Two-Particle Approximation

The two-particle approximation offers a simple method for investigating the effect of wake fields in short beam bunches. The method is best suited to problems where the bunch length is short compared with the distance to the first peak in the transverse dipole wake field. For long bunches the wake field is oscillatory within the bunch, and the two-particle model becomes a poor approximation.

In this approximation the bunch of charge q is treated as a consisting of two particles, a head particle (denoted "1") and a tail particle ("2"), each carrying a charge $q/2$, and separated by a distance z. The head particle undergoes a free betatron oscillation with wavenumber k, while the tail particle is driven by the dipole wake field due to the head particle, and can have betatron wavenumber $k + \Delta k$.

Bane[8] has analyzed this model for quadrupole-focused transport, with the result

$$\frac{x_2 - x_1}{x_0} = \left(1 - \frac{C}{2k\Delta k}\right) 2i \sin\left(\frac{\Delta k s}{2}\right) e^{i(k+\Delta k/2)s} \quad (2\text{-}1)$$

where

$$C = \frac{eqW(z)}{2E} \quad ,$$

s measures the distance along the linac and x_0 is determined from the initial conditions. Here $W(z)$ is the dipole wake field due to the head particle, evaluated at the location of the tail particle, and E is the particle energy (assuming $\Delta E/E$ is small). The solution corresponds to a beat wave between two sinusoidal oscillations. The amplitude of the beat wave will be zero if

$$\Delta k = \frac{C}{2k} \quad , \quad (2\text{-}2)$$

as discussed by Bane.

Without energy spread, the two particles are resonant, and the amplitude displays a secular growth, linear in s. With finite Δk the resonance is spoiled, and the solution is then purely oscillatory, as shown in Eq. (2-1), although the amplitude of the oscillation can still be unacceptably large. The condition given by Eq. (2-2) reduces the amplitude of the oscillation to zero. Although this analysis has been carried out in the two-particle approximation, the basic result that the instability can be eliminated with finite energy spread is also reproduced in an N-particle model.

For a solenoidal focusing system, as has been used in induction linacs such as the Advanced Test Accelerator at LLNL, the situation is very different from that described by Bane. In this focusing system the equations of motion are

$$\begin{aligned} \zeta_1'' - ik_c\zeta_1' &= 0 \\ \zeta_2'' - i(k_c + \Delta k)\zeta_2' &= C\zeta_1 \quad , \end{aligned} \quad (2\text{-}3)$$

where $k_c = \frac{eB_o}{pc}$ is the wavenumber associated with the cyclotron frequency, and $\zeta_1 \equiv x_1 + iy_1$ and $\zeta_2 \equiv x_2 + iy_2$. With the initial conditions ($s = 0$) specified as $\zeta_1(o) = \zeta_2(o) = x_o$, $\zeta_1'(o) = \zeta_2'(o) = ik_ox_o$, the solution may be expressed as

$$\begin{aligned} \frac{\zeta_2 - \zeta_1}{x_o} &= \left(1 - \frac{k_o}{k_c}\right)\frac{iCs}{k_c + \Delta k} \\ &+ \frac{k_o}{k_c}\left(\frac{C}{k_c\Delta k} - 1\right)\left(e^{ik_cs} - 1\right) \\ &+ \frac{1}{k_c + \Delta k}\left[k_o - \frac{C(k_o + \Delta k)}{\Delta k(k_c + \Delta k)}\right]\left(e^{i(k_c+\Delta k)s} - 1\right) \quad . \end{aligned} \quad (2\text{-}4)$$

If the particles are initialized on Larmor orbits, having zero canonical angular momentum, then $k_o/k_c = \frac{1}{2}$, and the solution contains a secular term, growing linearly with s. In this case there is no choice for Δk that will eliminate the effect of wake fields.

Alternatively, the bunch may be initialized on a cyclotron orbit which encircles the beam axis, *ie.* $k_o/k_c = 1$. In this case, with the bunch centered on the axis, the secular term will be absent for finite Δk, and the wake field influence can

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be eliminated (in the two-particle model) for $\Delta k = C/k_c$, which is essentially the same as the result obtained by Bane [Eq. (2-2)] with quadrupole focusing.

These results indicate that the solenoid-focussed accelerator should be less-affected by energy spread than is a quadrupole-focused accelerator.

Optimum Energy Profiles

To proceed beyond the two particle model one can consider a continuum model of BBU. For quadrupole focusing the equation governing the transverse position of the beam $x(s;\zeta)$ as a function of distance s along the accelerator and distance ζ back from the head of the beam is, neglecting acceleration,

$$\frac{\partial^2 x}{\partial s^2} + k_\beta^2 x = \frac{e^2}{m\gamma c^2}\int_o^\zeta d\zeta' f(\zeta') W(\zeta - \zeta') x(s;\zeta') \quad (3\text{-}1)$$

where the beam distribution $f(\zeta)$ is normalized to $\int_0^\infty d\zeta f(\zeta)$ = total number of particles in bunch. In Eq. (3-1), γ depends only on ζ; in the smooth approximation k_β also depends only on ζ, through its dependence on γ.

In analogy with the two particle model, one may ask whether there exists an optimum energy distribution within the bunch (that is, an optimum function $\gamma(\zeta)$) such that a bunch launched at $s = 0$ with initial conditions independent of ζ remains coherent ($\frac{\partial x}{\partial \zeta} = 0$) for $s > 0$. Such an ideal distribution would be expected to lead to no emittance growth in the presence of the transverse wake. As pointed out by Balakin,[11] the requirement that $\frac{\partial x}{\partial \zeta} = 0$ in Eq. (3-1) leads to the condition

$$k_\beta^2(\zeta) - k_{\beta 0}^2 = \frac{e^2}{m\gamma c^2}\int_o^\zeta d\zeta' f(\zeta') W(\zeta - \zeta') \quad . \quad (3\text{-}2)$$

where $k_{\beta o} \equiv k_\beta(0)$. If we write $k_\beta \approx k_{\beta o}\left(1 + \xi\frac{\gamma - \gamma_o}{\gamma_o}\right)$ where ξ is the lattice chromaticity, and if we treat $\frac{\gamma-\gamma_o}{\gamma_o}$ as small then Eq. (3-2) gives the ideal energy distribution as

$$\frac{\gamma - \gamma_o}{\gamma_o} \approx \frac{e^2}{2\xi k_{\beta o}^2 m\gamma_o c^2}\int_o^\zeta d\zeta' f(\zeta') W(\zeta - \zeta') \quad . \quad (3\text{-}3)$$

For a uniform bunch, $\gamma - \gamma_o$ is proportional to the integrated transverse wake function. If that function is dominated by a single damped mode, $W(\zeta) \propto \exp(\frac{-k\zeta}{2Q})\sin k\zeta$ then the ideal shape of the energy distribution is shown in Figure 3-1. For negative chromaticity, $\gamma(\zeta) \le \gamma_o$ for all ζ, in agreement with the two particle model and with the notion that trailing particles must be more strongly focused than the head particle, to compensate for the defocusing wake fields they experience.

It is interesting to note that no similar optimum energy distribution exists for a solenoid transport system. In this case, Eq. (3-1) is replaced by

$$\frac{\partial^2 z}{\partial s^2} - ik_c\frac{\partial z}{\partial s} = \frac{e^2}{m\gamma c^2}\int_o^\zeta d\zeta' f(\zeta') W(\zeta - \zeta') z(s;\zeta') \quad (3\text{-}4)$$

where $z = x + iy$ and k_c is the cyclotron wavenumber, $cB/m\beta\gamma c^2$. One sees immediately that if it is assumed that z is independent of ζ, and z is pulled out of the integral in (3-4) then the solution to the resulting differential equation in s, shows explicit, non-removable dependence of z on ζ, contradicting the original assumption. The basic physical reason is that while the dipole wake field force is proportional to transverse particle ***displacement***, the restoring force due to a longitudinal magnetic field is proportional to transverse particle ***velocity***; the two different forces cannot be combined to give a net force independent of ζ.

Examples

Quadrupole Focusing

For a long bunch, one which is at least several rf deflecting mode wavelengths long, we may illustrate the effect of linear dependence of energy on distance back from the head of the bunch on BBU, by numerical integration of either (3-1) or (3-4), modified to include acceleration.

For a quadrupole system, in the smooth approximation, we consider an induction linac which accelerates a 10 ns, 30 kA beam from 2.5 to 100 MeV with an average gradient of 1.5 MeV/m. The betatron wavelength is assumed to vary as $(\text{energy})^{1/2}$; the initial betatron wavelength is 0.5 m and the final value is 3.16 m. Nominal ATA cavity parameters, $Z_\perp/Q = 10\Omega$, $Q = 4$, are assumed, along with a deflecting mode frequency of 785 MHz; the beam is therefore 7.85 deflecting mode wavelengths long. The beam is initially offset by 0.5 mm with zero slope; an initial normalized emittance of 1 rad-cm is assumed. Figure 4-1 shows the normalized emittance of the beam versus s for (1σ) energy spreads of 0% and 5%; in the 5% case the energy increases linearly from head to tail; no significant difference was observed when the energy spread was the same magnitude, but the energy decreased from head-to-tail. A very significant reduction in BBU growth is observed in the 5% case.

Solenoid Focusing

With solenoid focusing the treatment of the BBU instability, in general, requires a model containing both transverse dimensions. The single exception is the case having zero energy spread with the beam initialized to have zero canonical angular momentum, $P_\theta = 0$. This case can be treated[2] as a one-dimensional problem by transforming to the Larmor frame, which rotates at half the cyclotron frequency. Since the cyclotron frequency is energy dependent, a finite head-to-tail energy spread on the bunch implies that each axial slice within the bunch rotates at a different frequency. The wake fields from earlier slices then cause an azimuthal kick, which destroys the conservation of P_θ.

The model of V.K. Neil[2] has been generalized to treat solenoid focusing, including finite energy spread and finite P_θ. The gaps are treated as regions of zero extent, where the bunch receives a transverse momentum kick, derived from the expression in V.K. Neil's paper. The impedance of the gap is prescribed in the model, and the wake field is assumed to be due to a single mode only. The particle motion between gaps is solved by numerically integrating the transverse equations of motion in the laboratory frame.

The transport system consists of 197 gaps, each with a transverse impedance $Z_\perp/Q = 5\Omega$ and $Q = 4$ for the deflecting mode, which is assumed to be at a frequency of 785 MHz. The gaps are separated by 0.33 m, and are immersed in a uniform solenoidal field of 0.5 T. The beam is injected at an energy of 2.5 MeV with an initial offset of 5 μm from the cavity axis. The beam bunch is 30 ns long and carries a current of 30 kA. In these calculations the bunch is divided into 400 evenly-spaced slices, which each carries the same charge. There is no acceleration in the gaps.

Figure 4-2 shows the average radius of the slices as a function of location in the transport system. The entire system is 65 m long. For this figure the beam slices have each been initialized to have $P_\theta = 0$. The figure has three cases, one with zero energy spread between slices, one where the tail energy is 20% *higher* than the head energy (denoted as +20% energy spread), and one where the tail energy is 20% *lower* than the head energy (denoted as - 20% energy spread). The head energy is the same in each of the three cases, and the slice energy varies linearly from head to tail.

For these parameters, the transverse beam amplitude grows by approximately three orders of magnitude over the length of the transport system. The effect of energy spread is rather small; negligible for +20% energy spread and less than a factor two improvement for -20% energy spread. These results contrast sharply with those obtained in a quadrupole focused system, described above. There energy spread has a significant stabilizing influence on the BBU. In the solenoid focused system, described here, energy spread does not remove the secular growth of the instability, and therefore has only a small effect. Negative energy spread is expected to have a greater effect than positive energy spread because it reduces the coefficient of the secular term in Eq. 2-4.

Summary

The behavior of the BBU instability in short bunches has been studied analytically, using the two-particle approximation, and in long bunches it has been analyzed numerically. The results in both cases show that quadrupole-focused systems are more sensitive to energy spread than are solenoid-focused systems. This result may be attributed to the presence of a zero-frequency mode in the solenoid system. The secular growth due to this mode persists even in the presence of energy spread, which therefore is less effective in stabilizing the growth of the instability.

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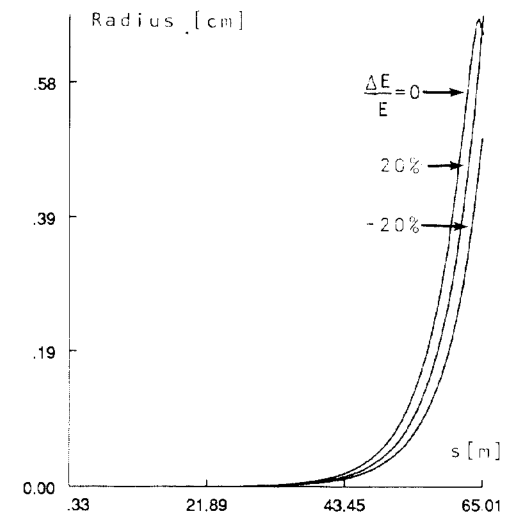

Figure 4-2: Average bunch radius vs. distance.

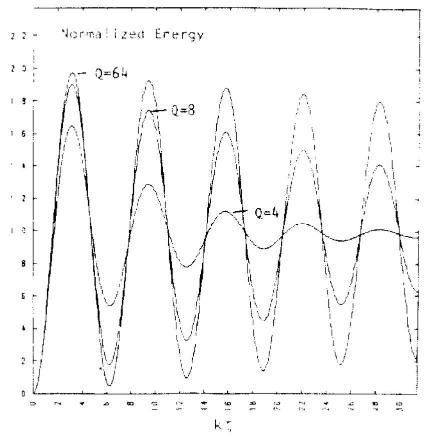

Figure 3-1: Optimum normalized energy profile for different values of Q.

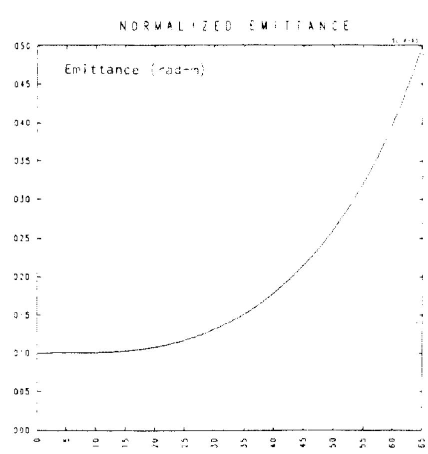

Figure 4-1a: Normalized emittance vs. distance for $\sigma_E/E = 0.05$.

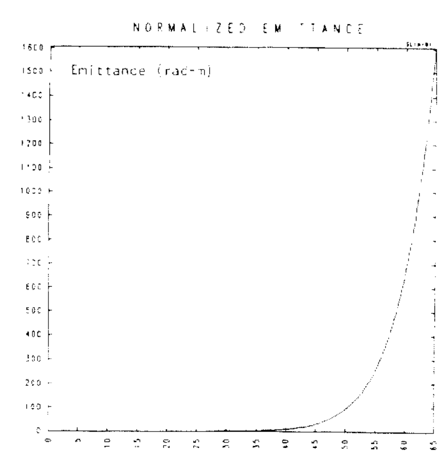

Figure 4-1b: Normalized emittance vs. distance for $\sigma_E/E = 0$.