High Average Beam Power Linac Waveguides

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All early microwave travelling-wave linacs were of constant impedance design (uniform geometry) for obvious reasons; insufficient engineering information existed describing shunt impedance and attenuation (or group velocity) for various apertures and modes (ie., longitudinal phase shifts per period) to do anything else.

Nevertheless, H. Leboutet (then of the CSF, Christ de Saclay laboratory) proposed about 1955 that the waveguides for the Ecole Normale Superieure (Orsay) 1-BeV linac be constructed in a step constant gradient design. His motives were discussed in an article published later.(1)

R.B. Neal, of the Stanford University Microwave Laboratory, visiting Orsay in 1956, recognized the advantages of constant gradient design without beam loading(2) for which he produced extensive analyses.(3) Such a design was consequently intended for the proposed Project M (SLAC) 10-BeV linac,(4) although this was later modified to provide a constant gradient field with ten percent beam loading.(5)

It is an obvious extension of the foregoing remarks to investigate the constant gradient condition with an intended beam loading. The motive for such an investigation rests on the observation, that having developed a method of heat removal from a length of waveguide, the whole waveguide can be operated advantageously at such a rate of RF heat deposition. Another consideration is that the two proposals (constant gradient with and without beam loading) are substantially different in theory.

The heat transfer process implies a maximum power attenuation per unit length (2IP), where 2I is the power attenuation coefficient (nepers/m).

Achievement of high beam power linacs further implies that this power attenuation ought to be used along the entire waveguide length. As a consequence, the waveguide design will automatically be constant gradient, since the field strength $E^2=2IPr$ and the shunt impedance per unit length is nearly constant.

Alternatively, the power diffusion equation with beam-loading,

$$\frac{dP}{dz} = -2IP - i\sqrt{2IPr} \qquad (1)$$

being quadratic, may be solved in the form

$$E = \sqrt{2IPr} = \sqrt{\left(\frac{ir}{2}\right)^2 - r\frac{dP}{dz}} - \left(\frac{ir}{2}\right)$$

Thus, the constant gradient condition ($dE/dz = 0$) implies that $d^2P/dz^2=0$, or that dP/dz is a constant. This latter condition may be expressed,

$$\frac{dP}{dz} = -\frac{P_o - P_L}{L} \qquad (2)$$

where P_o and P_L are the input and output power of a waveguide of length L.

The solution of eq (2) is, obviously,

$$\frac{P(z)}{P_o} = 1 - \left(1 - \frac{P_L}{P_o}\right)z/L \qquad (3)$$

The constant gradient condition requires, further, since $E^2 = 2IPr$ and r is nearly constant, that

$$\frac{1}{P}\frac{dP}{dz} = -\frac{1}{I}\frac{dI}{dz} \qquad (4)$$

or, from eqs. (2) and (3)

$$\frac{1}{I}\frac{dI}{dz} = \frac{(1-f)/L}{1-(1-f)z/L} \qquad (5)$$

where $f = P_L/P_o$, which, with initial conditions $I = I_o$, $r=r_o$, $z=o$ has the solution

$$I = \frac{I_o}{1-(1-f)z/L} \qquad (6)$$

It is, perhaps, noticeable that the characteristics of the structure have been specified without reference to the beam current. However, initial values of these parameters have not been determined. Referring again to eqs(1) and (2), at z=0,

$$2I_oP_o + i\sqrt{2I_oP_or} = \frac{P_o - P_L}{L}$$

When this equation is put in the form (multiplying through by L/P_o)

$$2I_oL = 1 - f - \eta \qquad (7)$$

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where $\eta = iV/P_o$ is the beam power conversion efficiency; thus the "design index" $(2I_oL)$ of the waveguide is determined by the fraction of input power the designer consents to "waste" (f) and the intended beam power conversion efficiency (η).

Noting that $E=\eta P_o/iL$, a second design constraint arises, which may also be written

$$E = (2I_oL)\,Pr/V \tag{8}$$

To determine the beam loading diagram (energy gain at other than design current) it is necessary to determine the power flux down the waveguide. The power diffusion equation, eq(1), and parameter specification, eq(6),

$$\frac{dP}{dz} = -\frac{2I_oP}{1-(1-f)z/L} - i\sqrt{\frac{2I_oPr}{1-(1-f)z/L}}$$

may be solved by substitution, $u^2 = P/(1-(1-f)z/L)$, the details of which are omitted for brevity; finally

$$\frac{\eta\sqrt{\dfrac{P(z)}{1-(1-f)z/L}} - i\sqrt{(2I_oL)ir}}{\eta\sqrt{P_o} - i\sqrt{(2I_oL)ir}} = \left[1-(1-f)z/L\right]^{-\frac{\eta}{2(1-f)}} \tag{9}$$

where f and η are the design values.

The electric field intensity along the guide then becomes

$$E = \sqrt{2IPr}$$
$$= \sqrt{\frac{2I_or}{\eta}}\left[\frac{\eta\sqrt{P_o} - i\sqrt{2I_oLrL}}{\left[1-(1-f)z/L\right]^{\eta/2(1-f)}} + i\sqrt{2I_oLrL}\right] \tag{10}$$

and the energy gain, generally,

$$V = \int_0^L E\,dz$$
$$= \frac{1}{\eta}\left[\frac{\eta\sqrt{2I_oP_orL} - i\,2I_oLrL}{(1-f-\eta/2)}\left(1-f^{\frac{2(1-f)-\eta}{2(1-f)}}\right) + \sqrt{i\,2I_oLrL}\right] \tag{11}$$

so that the no-load energy gain (V_o),

$$V_o = \frac{\sqrt{2I_oL\,P_orL}}{(1-f-\eta/2)}\left(1-f^{\frac{2(1-f)-\eta}{2(1-f)}}\right) \tag{12}$$

which, noting that $V\eta = irL(2I_oL)$ establishes the load line.

It can further be easily shown, in view of eq(6), that the group velocity

$$V_g/c = V_{go}/c\left[1-(1-f)z/L\right] \tag{13}$$

so that the fill time of the structure,

$$T = \int_0^L \frac{dz}{V_g} = -\frac{\ln f}{1-f}\,\frac{L}{V_{go}} \tag{14}$$

As an application of the foregoing analysis consider the design of a CW travelling wave linear accelerator intended for operation at 1A, having a 4MW, L-band RF power source, and where what is wanted is eighty percent beam power conversion efficiency $(\eta=.8)$, which therefore implies an energy gain per section of 3.2 MeV $(V=\eta P_o/i)$.

Heat transfer studies for disc-loaded waveguide (not discussed in this report) indicate that for L-band structures about 160 KW/m can be removed without "heroic" efforts, so that in this case the initial power attenuation coefficient $(2I_o)$ cannot exceed 0.04 nepers/m.

In Table I are listed various possible structures with the fraction of "waste" power f as independent variable; $2I_oL$ is calculated from eq(7). The steady state accelerating field(E) is calculated from eq(8), where it is assumed that for the proposed structure operated at 1300 mcs, $2\pi/3$ mode, $r/Q=\eta/2b$ ohms/m and Q=19,400. The waveguide length L=V/E from which it follows that $2I_o=2I_oL/L$. It can be seen that f=0.15 is an approximately correct solution. For the proposed structure, with a 1.28 cm disc thickness, an iris aperture 5.517 cm and a cavity diameter 18.75cm

r/Q=20.65 ohms/cm and r=40 megohms/m.

Table I

f	$2I_oL$	E	L	2Io
0.13	0.07nep	3.5MV/m	0.91m	.76nep/m
.14	.06	3.0	1.07	.056
.15	.05	2.5	1.28	.040
.16	.04	2.0	1.60	.026
.17	.03	1.5	2.13	.014

Essential to proceeding with constant gradient design is an investigation of the properties of the intended structure for various beam apertures. Table II presents the measured "cold-test" properties of L-band disc loaded waveguide having a fully radiused aperture (disc thickness=1.28cm, $2\pi/3$-mode)

Table II

2a	2b	I	Vg/C
5.7 cm	18.30cm	0.0358 nep/m	.0196
5.6 cm	18.28	.0380	.0185
5.5	18.2	.0404	.0173
5.4	18.22	.0433	.0162
5.3	18.20	.0460	.0152
5.2	18.17	.0497	.0141
5.1	18.15	.0527	.0133
5.0	18.12	.0569	.0123
4.9	18.10	.0609	.0115
4.8	18.08	.0655	.0107
4.7	18.05	.0708	.0099
4.6	18.03	.0772	.0091
4.5	18.01	.0830	.0084
4.4	17.99	.0919	.0076
4.3	17.97	.1004	.0070
4.2	17.95	.1105	.0063
4.1	17.93	.1229	.0057

There are several comments which may be of interest on structure properties:

(1) The r/Q of most structures approaches a limiting value of $\eta/2b$ as the aperture (2a) becomes smaller. (2) The attenuation is given closely by $I=35.7/(2a)^4$ nepers/m, where 2a is given in centimeters. (3) Normalized group velocity is given closely by $Vg/c=2(a/b)^4$

As an aside, the achievement of constant gradient waveguide under realistic conditions, where shunt impedance is not constant, involves a slightly more complicated approach. Since $E^2=2IPr$ and $2I=\omega/v_g Q$ it follows that the constant gradient condition (without beam loading is given by

$$\frac{r}{Q\ v_g/c} = \frac{r_0}{Q_0}\ \frac{1}{v_{g0}/c\ (1-2I_0 z)}$$

References

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(4) Hearings before the Subcommittee on Research and Development, Joint Committee on Atomic Energy, Eighty-sixth Congress, July 14-15, 1959. G.P.O., Wash. Proposed Two-mile Accelerator Project, and Status Report ML report no. 785 Stanford Univ. (1961)p14.

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