# BEAM DYNAMICS IN THE SPIRAL-LINE INDUCTION ACCELERATOR 

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#### Abstract

The PSI/SAIC spiral-line induction accelerator (SLIA) is a compact accelerator which accelerates electrons to several hundred MeV. The geometry of this device consists of a series of straight and curved sections with externally imposed longitudinal guide fields, stellerator (strong-focusing) fields and vertical (bending) fields. In this paper, the 3-D electrostatic particle/field simulation code SPIRAL is used to trace electron trajectories through a $180^{\circ}$ bend. In particular, vertical (bending) field performance is evaluated as well as the ability of the stellarator coils to tolerate energy mismatch in the bends.


## Introduction

There has been considerable interest in recent years in compact, high-current electron accelerators. ${ }^{1-5}$ Due to the need for compactness, most designs, to minimize peak voltage requirements, recirculate the beam through the same acceleration region. This requires that the system be closed magnetically as the beam is steered back around.

In the spiral-line induction accelerator (SLIA), ${ }^{5}$ the beam is injected into a strong axial guide field, is accelerated by induction modules in a straight section, then is bent (using vertical fields) and brought to another straight section with an induction module, etc. This scheme is repeated as many times as required with the beam exiting along the same strong axial guicle field. Thus, this design has a desired open-ended feature, and can retain a somewhat compact configuration. Another positive consequence of the open-ended system is that. the vertical magnetic field strength for any particular bend remains constant throughout the acceleration process, since beam pulses passing through this region will always be at the same energy.

To address the second issue of energy mismatch intolerance with respect to the vertical field strength, the SLIA incorporates strong-focusing, twisted quadrupole magnetic field coils. It has been found that the addition of twisted quadrupole (stellarator) windings placed about the beampipe along bends can allow devices to tolerate a beam energy mismatch of up to about $50 \%^{2}$ (up from about $2-3 \%$ ). In the SLIA, the strong-focusing twisted quadrupole windings have not only been included along the bends, but also along the straight sections. Including the stellarator windings along the straight section allows minimum beam disruption while traversing from a straight section into a bend.

Since the SLIA project involves an experimental effort as well as an analytical study, the design studied in this paper is that of the actual system of coils. Consequently, the coil design simulation code SPIRAL is used to study the effect of coil design on single-particle behavior as the beam proceeds from a straight section, through a $180^{\circ}$ bend, and into another straight section (see Fig. 1). SPIRAL is a three-dimensional, single-particle magnetic field coil design simulation code that simulates the effects of overall coil design, coil placement tolerance, terminations, and feeds on single-particle behavior. In
particular, two issues are addressed. 1) The effect of vertical field coil termination design on particle behavior. 2) The effect on beam energy mismatch tolerance due to the inclusion of the stellarator windings.


Fig. 1: Coil configuration and geometry.

## Vertical Field Coil Design Performance

In this section, the effect on the behavior of single-particle motion due to the vertical field coil design is presented. The major concern for finite arc vertical fields is that the vertical fields must terminate in such a way as not to disrupt the beam as it passes through termination coils.

In the experiment, designing the vertical field to have a desired field index of $1 / 2$ over a sufficient minor radius ( 3 cm ) was accomplished ${ }^{6}$ with 20 coils wrapped in a "cosine $\phi$ " (where $\phi$ is the poloidal angle) configuration about the beampipe along the bend. To design termination coils to connect to those windings required use of a formalism by Laslett et. al. ${ }^{7}$ In this procedure, the termination of the windings are determined such that the field quality integrated along the beamline through the coil is conserved. However, the paper by Laslett et. al. presented the formalism for vertical field coils wrapped in a "cosine $\phi$ " configuration about a straight pipe. Since the SLIA uses vertical fields for a bent beampipe, the results from the formalism were modified by Bailey and Wake ${ }^{6}$ to include this effect.

The termination coil design by Bailey and Wake along with the $\cos \phi$ winding data were inputted into SPIRAL. As scen in Fig. 1, the particles are initialized at 100 cm along the beamline and the 50 cm radius, $180^{\circ}$ vertical ficld coil section begins at a beamline position of 136 cm . Nine particles were simulated, one at the center (reference orbit) and eight uniformly spaced at a radius of 0.2 cm , all with an axially directed beam energy of 800 KeV . The minor radius of the vertical coil windings is 6.069 cm and the major radius is 50 cm . The matched field strength is 80.543 G , requiring a current in each winding of 155.67 A . To test the effectiveness of the terminations, the particles orbits are simulated without any solenoidal or stellarator field. The particles begin 36 cm before the bend (axial position 100 cm ), go through the bend, and the simulation is stopped 36 cm from the end of the bend. What was
found was that the centroid of the particles, by the end of the simulation, had moved inward (toward the center of the torus) about 0.4 cm . However, the uniform circular particle placement remained intact.

A simulation was made of the above system with the terminations removed. It was found that the centroid now shifted about 5.5 cm inward. Since the beampipe radius is 3.2 cm , this centriod shift is not acceptable.

## Results with Stellarator and Guide Field Coils

In this section, the second result of the paper is presented. First, the system is simulated with just the vertical field coils (with terminations from now on) and a full set of solenoidal guide field coils. This will reveal the result if the system were designed traditionally. Secondly, the stellarator coils are then included, with gaps in the appropriate axial positions, and a matched beam energy ( 800 KeV ) is used. Finally, the same set of coils is again used, but the particle simulation energy is increased $10 \%$.

To investigate the case of including just the solenoidal field and the vertical field, the solenoidal coils are superimposed with the vertical field coils (with terminations). The solenoidal field coils are simulated as the stack of rings placed about the beamline. As shown in Fig. 1, the solenoidal coils begin at 50 cm along the beamline ( 86 cm before the bend) and end 86 cm after the bend. The coils are at a radius of 5 cm and are separated by 1.389 cm along the beamline. the current in the coils is $3,876.9 \mathrm{~A}$, giving a field strength at axial position $z=100 \mathrm{~cm}$ of $3,500 \mathrm{G}$. Just as with the simulations presented earlier, the particles were initialized at $z=100 \mathrm{~cm}$ along the beamline at an energy of 800 KeV . However, since the beam will be formed in a field-free diode, the constraint that canonical azimuthal angular momentum be zero is used. Thus the offaxis particles are initialized with an angular velocity necessary to achieve this in the $3,500 \mathrm{G}$ field.

The results of this simulation (solenoidal and vertical fields only) are shown in Fig. 2. Figure 2. a shows the particle traces for $u$ versus $z$ as the beam moves through the system, where $u$ is the transverse coordinate which points inward in the bending region and $z$ is the axial position. Please note, as shown in Fig. 1, that the beginning and end of the bending section are at $z=136 \mathrm{~cm}$ and $z=293 \mathrm{~cm}$, respectively. In Fig. 2.a, this corresponds to where the center particle trace (the particle which began on the axis) first moves inwardly (to more positive values of $u$ ) about 0.09 cm , and then out wardly back to the original orbit. The orbit shift is not unexpected since the $u$ coordinate is with respect to a pre-defined beam axis which does not lie along the constant flux surface which the particles tend to follow. Figure 2.b st, ows the transverse particle traces. It can be seen that the overall beam size and position is not altered severely. It turns out that each particle orbit which began at a radius of 0.2 cm merely shifted inwardly the 0.09 cm as it traversed through the bend and then returned to the original position, all without affecting the cyclotron orbiting radius of the particle.

To investigate the cases which also include the stellarator field with the solenoidal and vertical field, the stellarator coils are superimposed over the previous case. As shown in Fig. 1, the stellarator coils begin at 46 cm along the beamline ( 90 cm before the bend) and end 90 cm after the bend. The coils are at a radius of 4.154 cm and have a pitch length of 18 cm (axial length for which the coil subtends $360^{\circ}$ azimuthally in the negative sense). The current in the coils is $13,826 \mathrm{~A}$, giv-
ing a maximum transverse field strength gradient of $370 \mathrm{G} / \mathrm{cm}$. Again, in both the matched and mismatched simulations, the particles were initialized at $z-100 \mathrm{~cm}$. The particles in the matched energy case have energy 800 KeV and the mismatched case has a $10 \%$ higher energy ( 880 KeV ). In both of these cases, the constraint of zero canonical angular momentum is used.


Fig. 2: Plots of a) $u$ vs. $z$ and b) $v$ vs. $u$ for the solenoidal/ vertical field case.

It is important to note that, for the gap here of about 5.43 cm (which is larger than the winding radius of 4.154 cm ), it was found in previous studies that the best results came from having the stellarator winding of one set not be rotated with respect to the other (adjacent) set, and this is the orientation used here.

The results for the matched energy case are presented in Fig. 3. Comparing Fig. 3.a with the respective plot in Fig. 2.a, it can be seen that the overall motion in the two transverse directions has not been effected in any appreciable way. However, comparing Fig. 2.b with Fig. 3.b, a dramatic, but expected, difference in detailed transverse particle behavior can be seen.

In Fig. 4, the results are presented for the case utilizing the same coil system, but now the particles are initialized with a $10 \%$ higher energy ( 880 KeV ). For a simple solenoidal/vertical field coil system, it is known that a $10 \%$ energy mismatch can be quite fatal, allowing the particles to hit the wall due to the weak-focusing vertical field. However, in the present case, it can be seen in Fig. 4 that the beam would not have hit the wall at radius 3.2 cm (see Fig. 4.b). The extreme radius of any particle in the 0.2 cm radius beam is about 0.5 cm from the


Fig. 3: Plots of a) $u$ vs. $z$ and b) $v$ vs. $u$ for the matched energy ( 800 KeV ), stellarator/solenoidal/vertical field case.
beampipe center axis. It should be noted that, at any one time, the overall beam radius actually never grew to more than about 0.22 cm . Figures $4 . \mathrm{a}$ and $4 . \mathrm{b}$ show this fact and also indicate how dramatically the beam centroid wanders about. However, a centroid movement about 0.2 cm from the beampipe axis is not serious for a beam inside a beampipe with a 3.2 cm radius.

## Conclusion

In this study two suecific issues have been addressed. First, the vertical field coils were tested for their design quality by tracking single-particle orbits through the fields resulting from the coils. One important point was that the field index was relatively constant to about 3 cm from the reference orbit position which indicated that the "cosine $\phi$ " coil positioning was well designed. The other point was that the proper placement of the vertical field terminations significantly reduce beam disruption as the beam traverses the transition into the vertical coils. The second issue addressed was that of the effect of stellarator fields on beam quality due to energy mismatch. It was found that the bean was not seriously degraded by an energy mismatch of up to $10 \%$, and that the beam centroid motion due to the mismatch was tolerable.


Fig. 4: Plots of a) $u$ vs. $z$ and b) $v$ vs. $u$ for the matched energy ( 880 KeV ), stellarator/solenoidal/vertical field case.

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