INTENSE ELECTRON BEAM PROPAGATION ACROSS A MAGNETIC FIELD

X. Zhang, C.D. Striffler, R.L. Yao, W.W. Destler, and M. Reiser
Electrical Engineering Department
and
Laboratory for Plasma Research
University of Maryland, College Park, MD 20742

Abstract

In this paper we consider the propagation of an intense electron-ion beam across an applied magnetic field. In the absence of the applied field, the beam system is in a Bennett equilibrium state that involves electrons with both large axial and thermal velocities and a cold stationary space-charge neutralizing ion species. Typical parameters under consideration are $V_o \sim 1$ MV, $I \sim 5$ kA, $T_e \sim 100$ keV, and beam radii $\sim$ 1 cm. We find that in the intense beam regime, the propagation is limited due to space-charge depression caused by the deflection of the electron beam by the transverse field. This critical field is of the order of the peak self-magnetic field of the electron beam which is substantially higher than the single particle cut-off field.

Introduction

The objective of the present investigation is to provide a theoretical understanding of the experiments performed at the University of Maryland on electron-ion beam propagation across a transverse magnetic field[1],[2]. The configuration investigated is shown in Fig. 1. Nominal diode parameters are 1 MV, 20 kA, and a FWHM of 30 nsec. The beam is injected through a hole into an evacuated drift tube. In order to obtain effective beam propagation to the end of the drift tube, a localized gas cloud near the injection hole is employed and collectively accelerated ions from the gas cloud provide a channel of neutralization that allows electrons injected late in the pulse to propagate. Under optimum conditions 20 kA can be propagated to the Faraday cup with no applied axial magnetic field.

In order to investigate the effects of a transverse magnetic field on this co-moving electron-ion beam, a Helmholtz coil is placed about half-way down the drift tube. If no gas is puffed into the system, that is, no ions are supplied, about 1 kA is observed at the Faraday cup with zero transverse magnetic field. As the transverse field is increased, the observed current steadily decreases to zero at about 200 gauss. When the gas cloud is present, about 5 kA are detected at the Faraday cup with zero transverse field. As the transverse field is increased, the entire 5 kA is propagated up to about 200 gauss with a steady but slow decrease in collected current above 200 gauss. The detected current goes to zero around 500 gauss. The purpose of these studies is to evaluate the properties of co-moving electron and ion beams for use in collective field accelerators and for beam propagation into free-space.

Previous theoretical studies have shown the existence of a self-consistent downstream Bennett equilibrium for the electrons and ions when no applied magnetic field is present[3]. We have related these downstream properties to the diode voltage, the transmitted electron beam current, and the ion properties in the localized gas cloud region. For our experimental parameters, $V_o = 1$ MV, $I = 5$ kA, the downstream self-pinched equilibrium state is composed of cold ions with low axial speed (< 0.05 c), and electrons with a temperature of about 60 keV and an axial speed of about 0.84 c. The nearly charge-neutral beam system has been shown to effectively propagate up to the diode current of 20 kA where the electron temperature is predicted to be about 200 keV. The 5 kA case was chosen for examining the effects of a transverse magnetic field on the electron-ion beam system, mainly because of experimental reproducibility.

Other work in the area of beam propagation across transverse magnetic fields usually consider a charge and current neutralized ion beam [4]–[8]. These systems have application to ion beam heating of dense plasmas, where an intense ion beam is charge and current neutralized by electrons so that it can penetrate the large applied fields in a fusion plasma. In these cases a two layer polarization model is adopted in the rigid beam limit. The polarization electric field allows the beam to propagate across the magnetic field by means of an $\mathcal{E} \times \mathcal{B}$ drift.

Model, Basic Equations, and Results

Consider a rigid beam model of the Bennett equilibrium system discussed in the previous section. Electron and ion beams of equal radii are incident along the z-axis onto a transverse magnetic field region. To simplify the analysis the profile of the Helmholtz coil is replaced by a constant magnetic field B_T in the y-direction over an axial distance $L_T \sim 20$ cm. In the parameter regime of interest, the electron gyroradius is comparable to L_T which is much larger that the effective Bennett beam radius a and the ion beam is assumed to be stationary and centered on the z-axis. The electron beam is confined by the self-magnetic field. The rigid electron beam is deflected by the transverse field B_T but feels a restoring force due to the ion beam. In this model, we examine only the effects of transverse forces on the rigid electron rod. Longitudinal forces and/or effects that arise as the beam front end enters the transverse magnetic field region are ignored.

When there is no transverse field, $B_T = 0$, the electron and ion components of the beam system are centered on the z-axis, are charge neutral, $n_{io} = n_{eo} = n_o$, and have identical Bennett profiles of the same effective radius, $a_i = a_e = a$. That is, the density profiles are

$$n_\alpha(r) = \frac{n_{\alpha o}}{[1 + (r/a)^2]^2}, \alpha = i, e \tag{1}$$

and the electron beam self-magnetic field due to the mean axial electron velocity V_{zo} is

$$B_{S\phi}(r) = -\frac{e\mu_o}{2} V_{zo} \frac{n_{eo} r}{1 + (r/a)^2}. \tag{2}$$

The ions are stationary and assumed infinitely massive. The equations of motion for the center of the electron beam are

$$\frac{dp_x}{dt} = eB_T V_z + F_{ei}(x) \tag{3}$$

*This work is supported by AFOSR and DOE.

CH2669-0/89/0000-1035$01.00©1989 IEEE

$$\frac{dp_z}{dt} = -eB_T V_x, \quad (4)$$

where $p_x = m\gamma V_x$, $p_z = m\gamma V_z$, and $\gamma = [1-(V_x/c)^2-(V_z/c)^2]^{-1/2}$. The averaged ion electric force acting on the rigid electron beam when the electron rod is displaced a distance x is

$$\begin{aligned} F_{ei}(x) &= -\frac{e}{Ne}\int E_{xi}(r)n_e(r)dS_e \\ &= \frac{-e^2 n_o a}{8\epsilon_o}\left[\frac{\bar{x}/2}{1+(\bar{x}/2)^2} + \frac{1}{(\bar{x}/2)}\right. \\ &+ \left.\frac{\sinh^{-1}(-\bar{x}/2) - \sinh^{-1}(3\bar{x}/2+\bar{x}^3/2)}{\bar{x}^2[1+(\bar{x}/2)^2]^{3/2}}\right] \quad (5) \end{aligned}$$

where $\bar{x} = x/a$.

In general, Eqns. (3) and (4) have to be solved numerically. However, if the displacement distance x is small compared to the beam radius, $\bar{x} \ll 1$, the ion electric restoring force can be approximated as

$$F_{ei}(x) \simeq \frac{-e^2 n_o}{6\epsilon_o}x. \quad (6)$$

Furthermore, assuming the relativistic factor γ remains constant, $\gamma = \gamma_o = [1-(V_{zo}/c)^2]^{-1/2}$, we find the solutions to Eqns. (3) and (4) to be

$$x(t) = \frac{\omega_c V_{zo}}{\omega^2}(1-\cos\omega t) \quad (7)$$

and

$$V_z(t) = V_{zo}\left[1 - \left(\frac{\omega_c}{\omega}\right)^2(1-\cos\omega t)\right], \quad (8)$$

with

$$\omega^2 \equiv \omega_c^2 + \frac{\omega_p^2}{6},\ \omega_c = \frac{eB_T}{m\gamma_o},\ \omega_p^2 = \frac{e^2 n_o}{\epsilon_o m\gamma_o}, \quad (9)$$

where ω_c, ω_p are the electron cyclotron and plasma frequencies. These solutions are similar to the results obtained by Peter and Rostoker[8], who used the two-layer polarization model. Since the oscillatory behavior of the electron rod is due to the deflecting applied transverse magnetic field and the restoring ion beam electric field force, we see that the frequency of these oscillations is a function of both ω_c and ω_p. Averaging out the oscillations in Eq. (8), we obtain for the time-averaged mean axial velocity of the electron beam

$$\bar{V}_z = V_{zo}\left(1 - \frac{\omega_c^2}{\omega^2}\right) \equiv V_{zo}(1-1/\epsilon), \quad (10)$$

where $\epsilon = 1 + \omega_p^2/6\omega_c^2$. This result is identical to that in reference [8] with the modified definition of ϵ.

Returning to Eqns. (3) and (4), we have solved these equations numerically for a given beam current as a function of the applied transverse magnetic field amplitude. The results for a 5 kA beam are displayed in Fig. 2. From reference [3], the self-consistent mean electron axial velocity is $V_{zo} = .84c$ for a beam radius of 1.0 cm. The peak self-magnetic field for this beam system is about 500 G [from Eqn. (2)]. The results displayed are typical for beams in the "intense" regime. Specifically, we have plotted in Fig. 2a the maximum beam displacement versus the transverse field amplitude, B_T, shown as the solid line. In Fig. 2b, the minimum axial velocity of the electron beam, which occurs at maximum displacement, is plotted versus B_T (solid line). As shown, the maximum displacement continually increases with B_T until a value is reached such that the axial velocity goes to zero. This value of the magnetic field at which $\beta_{z,MIN} \to 0$, is defined as B_{TC} and the displacement amplitude for this field x_c. For values of B_T above B_{TC}, the beam will blow up in its own space-charge. We see that this critical value is very close to the peak self-magnetic field of the Bennett beam.

The values of B_{TC} and x_c can be related from the constants of motion. From Eqns. (3) and (4), these constants are the total energy and the axial canonical momentum, which can be written as

$$(\gamma-1)mc^2 - e\phi_e = (\gamma_o - 1)mc^2 \quad (11)$$

and

$$p_z + eB_T x = p_{zo} = m\gamma_o V_{zo}, \quad (12)$$

where the effective potential is given as $\phi_e = (1/e)\int_o^x F_{ei}(x')dx'$. From Eqn. (12), when $p_z = 0$, then $x \to x_c$ and $B_T \to B_{TC}$, that is

$$B_{TC}x_c = \frac{p_{zo}}{e} = m\gamma_o eV_{zo}. \quad (13)$$

In the intense beam regime, when β_z goes to zero at B_{TC} we also have $\beta_x = 0$. That is, at this critical magnetic field $\gamma(x_{MAX} = x_c) = 1$, all the electron beam energy goes into potential energy. From Eqn. (11), with $\gamma = 1$, we obtain

$$e\phi_e(x_c) = \frac{eI}{4\pi\epsilon_o V_{zo}}\ell n\left[1 + \frac{1}{3}\left(\frac{x_c}{a}\right)^2\right] = (\gamma_o - 1)mc^2 \quad (14)$$

Thus, from Eqn. (14) we can determine x_c for a given injected beam, and determine the critical magnetic field from Eqn. (13). That is,

$$B_{TC} = \frac{m\gamma_o V_{zo}}{ex_c} = \frac{m\gamma_o V_{zo}}{e\sqrt{3}a}\left[\exp\left(\frac{17}{I(kA)}(\gamma_o - 1)\beta_{zo}\right) - 1\right]^{-1/2}, \quad (15)$$

the critical magnetic field for beam propagation in the intense beam regime.

If the beam current is sufficiently low, $I < 4kA$, we find that when $\beta_z \to 0$, β_x is not zero. In the single particle limit, this is obvious since as $\beta_z \to 0, \beta_x \to \beta_o$, since $\gamma = \gamma_o$ along the entire trajectory. In this case $x_c = p_{zo}/eB_{TC} = V_{zo}/\omega_c = R_e = L_T$, the electron Larmor radius. For beam currents between the single particle limit and the intense beam regime discussed above, Eqn. (11) tells us that part of the initial beam kinetic energy goes into potential energy as the beam is deflected. A summary of these results is shown in Fig. 3 where we have plotted the critical magnetic field B_{TC} versus beam current I. This graph is generated for the case of a 1 MV diode[3] and the self consistent downstream Bennett equilibrium. The mean axial velocity V_{zo} depends on the transmitted current as shown in reference[3] and this is included in the plot in Fig. 3. We see that the single particle cutoff field is about $250G[V_o = 1MV, L_T = 18\text{cm} = R_e]$. As the current is increased the critical field remains at the single particle limit until the beam current reaches a value such that its peak self-magnetic field is near the single particle cut-off field. Further increases in the beam current result in entering the intense beam regime where the critical field is a result of potential depression ($\gamma \to 1$ as $\beta_z \to 0$).

In order to explicitly compare our results with those of the experiment, we must normalize our uniform field model with a Helmholtz coil profile. We have displayed in Fig. 2a and 2 b the numerical results of solving Eqns. (3) and (4) for a

5 kA electron beam transversing a Helmholtz coil configuration. The coil field amplitude is the peak on-axis value. From conservation of axial canonical angular momentum and energy conservation, the relationship between the critical uniform field B_{TC} and that of the Helmholtz coil field profile is

$$B_{TC} = \frac{1}{x_c} \int_o^{x_c} B_{HC}\left(x', z(x')\right) dx'. \tag{16}$$

Discussion

For a given injected electron beam that is charge neutralized but not current neutralized by a stationary ion beam and has a Bennett equilibrium profile, we have determined the critical transverse magnetic field above which the beam will not propagate. In the intense beam regime, this limit is a result of space-charge depression caused by the deflection of the electron beam from the center of the ion channel. The value of the critical field is approximately given by the peak self-magnetic field of the electron beam.

A concern about the rigid beam model is the individual particle confinement in the Bennett equilibrium. The application of a transverse field results in a non-symmetric force about the beam axis. We have examined the individual particle trajectories for typical Bennett particles and have found the non-symmetry has substantial individual particle effects as B_T approaches $B_{s\phi}(a)$. We believe that this will lead to fractional loss of beam current as the beam transverses the field region. This fractional loss is observed in the experiment as B_{TC} is approached.

References

1. W.W. Destler, C.D. Striffler, Z. Segalov, R.L. Yao, X. Zhang, and J. Rodgers, Proc. O-E/LASE '88, Sym. Innovative Sci. & Tech., paper 873-33, Jan. 10-15, 1988.
2. W.W. Destler, J. Rodgers, Z. Segalov, C.D. Striffler, R.L. Yao, X. Zhang, and J. Guillory, 7th Inter. Conf. on High-Power Particle Beams, Beams '88, July 4-8, 1988, p. 185.
3. C.D. Striffler, R.L. Yao, and X. Zhang, Proc. of the 1987 IEEE PAC, page 975.
4. G. Schmidt, Phys. Fluids 3, 961 (1960).
5. K.D. Sinel'nikov and B.N. Rutkevich, Sov. Phys. - Tech. Phys. 12, 37 (1967).
6. E. Ott and W.M. Manheimer, Nuclear Fusion 17, No. 5, 1057 (1977).
7. F. Wessel and S. Robertson, Phys. Fluids 24, 739 (1981).
8. W. Peter and N. Rostoker, Phys. Fluids 25, No. 4, 730 (1982).

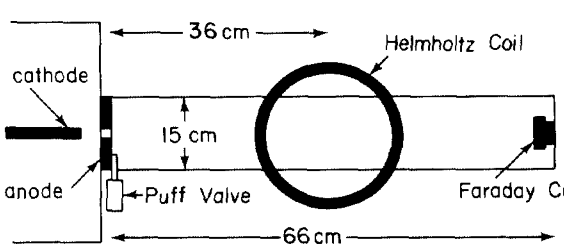

Fig. 1. Experimental configuration used for beam propagation studies across a magnetic field.

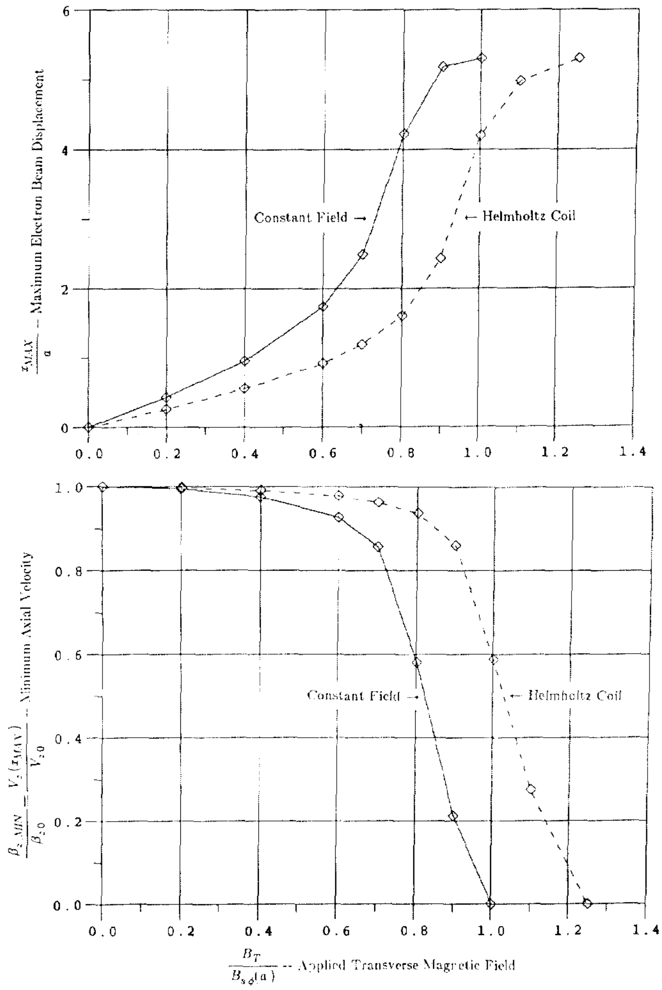

Fig. 2. Typical results in the Intense Beam Regime. (a) Maximum electron beam displacement and (b) minimum beam axial velocity versus the applied transverse magnetic field amplitude. The "solid" line represents the constant field model and the "dashed" line a Helmholtz coil configuration (peak on-axis amplitude). Fixed beam parameters are; $I = 5$ kA, $\beta_{zo} = 0.84$, $a = 1$ cm, and $B_{s\phi}(a) \simeq 500$ G.

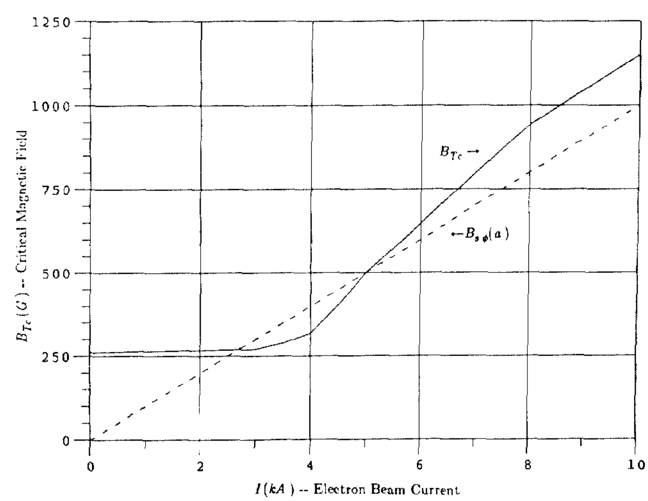

Fig. 3. Critical transverse magnetic field, defined by $\beta_z = 0$, versus electron beam current. The "dashed" line represents the peak self-magnetic field of the electron beam. The beam radii are 1 cm and the diode voltage is 1MV. See reference [3].