PONDEROMOTIVE EFFECTS ON CHARGED PARTICLE BEAM LIMITING CURRENT

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The physical mechanism responsible for the space charge limiting current in a wave cavity and in presence of electromagnetic fields is addressed. It is found that the ponderomotive energy alters radically the build up of beam electrostatic energy. As a consequence, the limiting current can be greatly decreased or enhanced depending on whether the ponderomotive energy is positive or negative, respectively. This effect could be observed experimentally and it is anticipated to have important practical consequences for present high-current particle beam experiments.

The propagation of high-current particle beams in a wave cavity or drift tube and in presence of electromagnetic fields is a basic theoretical problem in physics. The latest revival in its theoretical interest reflects the recent experimental advances in high-power free-electron lasers[1], gyrotrons[2], collective-ion accelerations[3], and so on. A basic phenomena occurring in these experiments is that as the beam current is increased, an electrostatic potential depression builds up due to the beam's self space charge field and as a result, the beam current cannot be increased beyond a certain amount. This is the limiting current of the cavity and it is a theoretical limit on the current, and hence on the power, in devices like gyrotrons[2]. The problem of limiting current of particle beams in a cavity has been addressed before[4], but not taking into account the presence of electromagnetic fields in the cavity. However, in these experiments[1-3], the particle beam does propagate in the presence of electromagnetic fields which changes fundamentally the physical situation. The oscillating electromagnetic field creates a quasistatic ponderomotive potential[5] which affects the particle motion, and hence, the limiting current. The static magnetic field, on the other hand, introduces resonances due to the coupling of the gyromotion with the particle motion in the electromagnetic wave field. Again, the limiting current calculations should reflect this occurrence. The inclusion of all these issues results in a more complete and realistic treatment of the limiting current problem applicable to present experimental situations.

The practical consequence of this work is that experimentalists[1-3] will have at hand the right behaviour and value of the limiting current for their experiments, which, typically, can be considerably different from the value[4] found without considering the electromagnetic fields in the cavity. We find that the combined effects resulting from the oscillating electromagnetic field and the static magnetic field can greatly decrease or enhance the limiting current depending on whether the ponderomotive energy is positive or negative, respectively. Our calculations show that we can obtain, say, an order of magnitude reduction in the limiting current of a 100kV relativistic electron beam using only modest electro-magnetic fields, and consequently, a decrease in power produced by, say, a gyrotron. Physically, this occurs because a positive ponderomotive energy of some appropriate cavity mode enhances the beam space charge electrostatic energy. Alternatively, we can think that the positive ponderomotive energy reduces the potential energy associated with the beam accelerating system. This is reflected as a decrease in the beam kinetic energy with a consequent decrease in the current that is able to propagate through the cavity.

We will first derive the limiting current as a function of beam energy, waveguide field and the external magnetic field for solid and hollow beams. We will then present numerical results showing that the space charge limiting current can take rather different values when taking into account the presence of electromagnetic fields in the cavity.

To derive the limiting current, we impose the following energy conservation condition for the relativistic beam:

$$K = (\Gamma - 1)\, mc^2 + e\phi + K_2 = (\Gamma_0 - 1)\, mc^2 = e\phi_c, \qquad (1)$$

where ϕ_c is the potential of the accelerating system and K_2 is the ponderomotive Hamiltonian. The left-hand side of Eq. (1) is the particle guiding center Hamiltonian K which is the total particle beam energy inside the wave cavity. Here we are using the notation of Refs. 6 and 7. The Hamiltonian $K(R, \mu, U_\parallel)$ results from an averaging over the gyromotion and fast oscillations due to the waveguide electromagnetic fields. Both averages were done[6,7] using Lie transforms resulting in a Hamiltonian containing only the slow effects on the guiding center. The averaged dynamical variables have the following meaning: The first variable R is the radial position of the guiding center, where R = 0 is the center of the waveguide. The next variable μ is the magnetic moment and is related to the perpendicular guiding center drift $U_\perp$ through $\mu = mU_\perp^2/2B$, where B includes the external magnetic field, and the longitudinal and azimuthal self-fields. Finally, $U_\parallel = \Gamma V_\parallel$ is the parallel world velocity of the guiding center, where the relativistic factor is defined as

$$\Gamma = \left(1 + \frac{2\mu B}{mc^2} + \frac{U_\parallel^2}{c^2}\right)^{1/2} .$$

In Eq. (1), the term $(\Gamma - 1)\, mc^2$ is the kinetic energy of the beam particle, $e\phi(R)$ is the potential energy which, in this case, is the beam space

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charge electrostatic energy, and $K_2(R, \mu, U_\parallel)$ is the ponderomotive Hamiltonian, which, being velocity-dependent, is a generalization of the ponderomotive potential. We see from Eq. (1) that a positive ponderomotive energy K_2 enhances the potential energy $e\phi$ while a negative K_2 counterbalances $e\phi$. Alternatively, we can think of K_2 as reducing or enhancing $e\phi_c$, the accelerating potential.

Next, we shall state the appropriate expressions for the electrostatic potential $\phi(R)$ and the ponderomotive Hamiltonian for both solid and hollow beams. It is possible to show that when the ponderomotive energy is included in the calculation of the limiting current, the maximum energy depression of the function $e\phi(R) + K_2(R, \mu, U_\parallel)$ still occurs at $R = 0$ for solid beams and at $R = R_a$ where R_a is the inner beam radius, for hollow beams as in the case when $K_2 = 0$. Therefore, we evaluate the left-hand side of Eq. (1) at $R = 0$ and $R = R_a$ for solid and hollow beams, respectively.

The electrostatic potential for a relativistic solid beam is given by[4]

$$\phi(R = 0)^{SOLID} = \frac{I}{V_\parallel} \left(1 + 2\ell n \frac{R_c}{R_b}\right) , \qquad (2)$$

where I is the total beam current, and R_b and R_c are the beam and waveguide radii, respectively. For a hollow beam, one can show that

$$\phi(R = R_a)^{HOLLOW} = \frac{I}{V_\parallel} \left(1 + 2\ell n \frac{R_c}{R_b}\right) - \frac{2IR_a^2}{(R_b^2 - R_a^2)V_\parallel} \ell n \frac{R_b}{R_a}. \qquad (3)$$

To illustrate how this formalism works, we choose the TE_{11} mode for the oscillating electromagnetic field, since the maximum electromagnetic field occurs at the center of the beam, for a solid beam, and the TE_{01} mode, since the maximum electromagnetic field can be set at the inner beam radius, for a hollow beam. These are the beam radial positions where the beam electrostatic energy build up is maximum. The derivation of the expression for the relativistic ponderomotive Hamiltonian for magnetized particles is rather involved. In Ref. 6, a general formula for the ponderomotive Hamiltonian was obtained [Eqs. (45) and (46) in that paper]. In that derivation, it was allowed for arbitrary $k\rho$, $E_\parallel/E_\perp$ ω/Ω, $k_\perp/k_\parallel$ $v/c < 1$, polarization, and for slowly growing and spatially modulated waves, where ω and k are the mode frequency and wavenumber, respectively, and where $\Omega = eB/\Gamma mc$. However, in this work we need particular expressions for the ponderomotive Hamiltonian corresponding to $k\rho << 1$, and to TE_{01} and TE_{11} modes. Hence, we need to reduce Eqs. (45) and (46) of Ref. 5 by imposing the correct forms for the waveguide field amplitudes. We know that the parallel components of both fields is zero while the perpendicular component is only azimuthal for the TE_{01} mode, $|E_\perp|^2 = |E_\Phi|^2$, but azimuthal and radial for the TE_{11} mode, $|E_\perp|^2 = |E_\Phi^2| + |E_R^2|$. After some algebra we obtain the following expression for the ponderomotive Hamiltonian

$$K_2(R, \mu, V_\parallel) = \frac{e^2|E_\perp|^2}{\Gamma m\omega^2} \frac{(\omega - k_\parallel V_\parallel)^2}{(\omega - k_\parallel V_\parallel)^2 - \Omega^2} , \qquad (4)$$

Note that this expression is the same as Eq. (41) of Ref. 7, derived specifically for a TE_{01} mode. We note that $K_2 \gtrless 0$ for $|\omega - k_\parallel V_\parallel|/|\Omega| \gtrless 1$, as we needed to enhance or to counterbalance the beam space charge electrostatic energy.

The substitution of Eqs. (2), or (3), and (4) in Eq. (1) yields the following expression for the conservation of energy

$$[\Gamma + \frac{e|E_\perp|^2}{\Gamma m^2c^2\omega^2} \frac{(\omega - k_\parallel V_\parallel)^2}{(\omega - k_\parallel V_\parallel)^2 - \Omega^2} - \Gamma_0]\, V_\parallel$$

$$= \begin{cases} -\dfrac{eI}{mc^2}\left(1 + 2\ell n \dfrac{R_c}{R_b}\right), & \text{for solid beam} \\ -\dfrac{eI}{mc^2}\left(1 + 2\ell n \dfrac{R_c}{R_b} - \dfrac{2R_a^2}{R_b^2 - R_a^2} \ell n \dfrac{R_b}{R_a}\right), & \text{for hollow beam} \end{cases} \qquad (5)$$

To calculate the limiting current, we will assume $U_\perp/U_\parallel << 1$, which is the case in most applications. However, our calculations carries through straightforwardly for $U_\perp/U_\parallel \simeq 1$, or even for $U_\perp/U_\parallel >> 1$. It is also convenient to assume $k_\parallel V_\parallel/\omega << 1$, though not required. In fact, in our derivation, we did not make this assumption. However, this assumption would be physically reasonable since it is experimentally desirable to set up a non-propagating, or slow propagating wave field so that the power spent in maintaining the field is minimum. Hence, from Eq. (5), we find the expression for the current in the wave cavity. The result is

$$I(\Gamma) = I_G(\Gamma^2 - 1)^{1/2}\left(\frac{\Gamma_0}{\Gamma} - 1 + \frac{\kappa}{\eta^2 - \Gamma^2}\right), \qquad (6)$$

where $\kappa = e^2|E_\perp|^2/m^2c^2\omega^2$, $\eta = eB/\omega mc$,

$\Gamma = (1 + U_\parallel^2/c^2)^{1/2}$, and I_G is the Alfven current with the geometric factor. For a solid beam, I_G is defined as

$$I_G^{SOLID} = \frac{mc^3}{e} \left(1 + 2\ \ell n \frac{R_c}{R_b}\right)^{-1} \qquad (7)$$

and for hollow beam as

$$I_G^{HOLLOW} = \frac{mc^3}{e} \left(1 + 2\ \ell n \frac{R_c}{R_b} - \frac{2R_a^2}{R_b^2 - R_a^2} \ell n \frac{R_b}{R_a}\right)^{-1}. \qquad (8)$$

To determine the limiting current, we maximize $I(\Gamma)$ as given by Eq. (6). The maximization condition, for both geometries, is

$$\Gamma^3 - \Gamma_0 - \frac{\kappa\Gamma^3(\eta^2 + \Gamma^2 - 2)}{(\eta^2 - \Gamma^2)^2} = 0 . \qquad (9)$$

The solution of Eq. (9) gives a root $\Gamma = \Gamma^*$, which, when substituted in Eq. (6), yields the limiting current $I_{lim} = I(\Gamma^*)$. We select the correct root Γ^* by knowing that, when $\kappa = 0$, Γ^* reduces to the well-known[4] standard limiting current calculation without the rf waveguide mode. For this case, the maximization condition gives $\Gamma^* = \Gamma_0^{1/3}$, which implies, from Eq. (6), that

$$I_{lim}^{\kappa=0} = I_G(\Gamma_0^{2/3} - 1)^{3/2} . \quad (10)$$

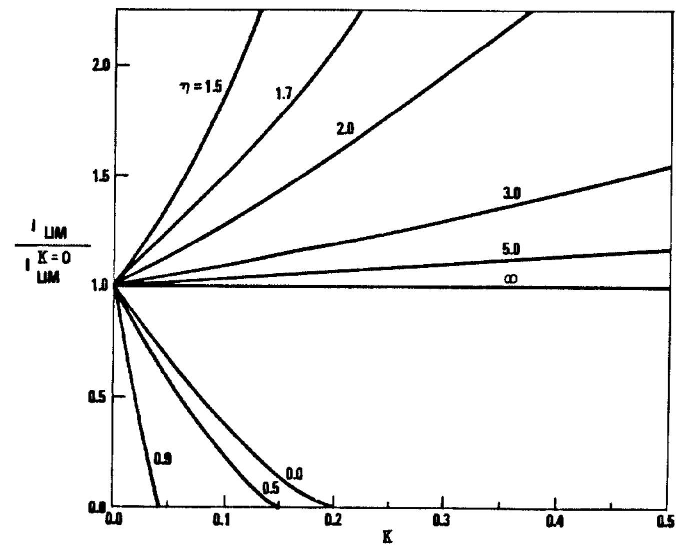

Fig. 1 The ratio $I_{lim}/ I_{lim}^{\kappa=0}$ versus κ for various η and $\Gamma_0 = 1.2$.

To realize the extent to which the presence of electromagnetic fields in the wave cavity alters the limiting current, we solve Eqs. (9) and (6) numerically for typical parameters and plot the results in Fig. 1. In Fig. 1, we plot $I_{lim}/I_{lim}^{\kappa=0}$ versus κ, for various η at fixed $\Gamma_0 = 1.2$ (100kV electron beam). We vary κ between κ=0, when $I_{lim}/I_{lim}^{\kappa=0} = 1$, and κ=0.5, which corresponds to an oscillating electric field of amplitude $E_\perp \approx 120$kV/cm if $\omega \approx 10^{10}$ rad/sec, well below the breakdown field. We observe that the electromagnetic fields in the wave cavity can increase ($I_{lim}/I_{lim}^{\kappa=0} > 1$) or decrease ($I_{lim}/ I_{lim}^{\kappa=0} < 1$) the limiting current as compared with a cavity without fields in it. An increase occurs for $\eta > 1$ (in fact, $\eta > \Gamma^*$, equivalently, $\Omega > \omega$), which corresponds to a negative ponderomotive energy, while a positive ponderomotive energy ($\eta < 1$) decreases the limiting current. For example, from the plot, for $\kappa = 0.2$ and $\eta = 1.7$ we obtain a 95% enhancement while for the same κ but for any $\eta < 1$ there is a complete current cut-off in the cavity.

In conclusion, we developed a theory to treat the propagation of a charged particle beam in a wave cavity when electromagnetic fields are present. This physical situation is typical in present experiments. We find that the limiting current problem is modified considerably to account for these fields.

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