THEORY OF WAKEFIELD EFFECTS OF RELATIVISTIC ELECTRON BEAMS

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Abstract

In recent years, there is a research program on the wakefield effects of a relativistic electron beam in a diffuse plasma for ion focused regime propagation. A brief theoretical description of the wakefield effects is presented in this article. In terms of the plasma density and beam parameters, this basic theory determines several critical wakefield quantities, including the oscillation frequency ω, the wavelength λ, the radial profile of the induced axial electric field and its strength.

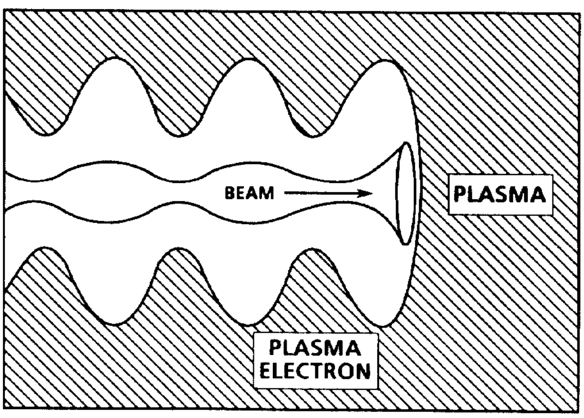

Fig. 1 Schematic presentation of wakefield effects.

Introduction

When a relativistic electron beam propagates through a tenuous neutral background plasma, it expels the plasma electrons from its vicinity leaving behind ions which will effectively neutralize the electric field generated by the beam itself. The plasma electrons move out to the charge neutralization radius R_n where the beam charge is the same as the total enclosed ion charge. However, in reality, when the plasma electrons are expelled by the beam, they will overshoot the charge neutralization radius and will oscillate at the frequency ω which is usually very close to the electron plasma frequency of the tenuous background plasma. This plasma electron oscillation near the charge neutralization radius essentially produces the wakefield which is electrostatic in nature and has an associated electric field components in the radial and axial directions. Particularly, the axial electric field may modulate the beam electron energy along the beam pulse. Shown in Fig. 1 is a schematic presentation of the wakefield effects. The ARTIC code at the Mission Research Incorporate was the first to see the presence of the wakefields and the FRIEZR code at the Naval Research Laboratory was the first to show the energy exchange and modulation of the beam pulse. Thus, in this article, we present an analytical theory of the wakefield effects on a relativistic electron beam propagating through an ambient plasma background.

Identification of Wakefield Wave Frequency

The wakefield phenomenon may be explained by the two-stream interaction between the plasma electrons and the beam electrons. A relativistic electron beam with radius R_b is propagating through a vacuum hole (its radius R_n) created inside the ambient plasma. In a typical present experiment, the beam radius R_b is much less than the charge neutralization radius R_n. Making use of the linearized macroscopic cold fluid model, we can show that the eigenvalue equation of the two-stream instability in this beam-plasma system is expressed as

$$\frac{1}{r}\frac{d}{dr}\left\{ r\left[1-\left(1-\frac{\omega V_b}{c^2 k}\right)\frac{\omega_{pb}^2 k^2}{\nu_b^2 p^2}\right]\frac{d}{dr}\delta E_z\right\}$$

$$-\,p^2\left[1-\frac{\omega_{pb}^2/\gamma_b^2}{\left(\omega-kV_b\right)^2}\right]\delta E_z = 0, \qquad (1)$$

for $0 < r < R_b$,

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d}{dr}\delta E_z\right] - p^2\delta E_z = 0, \qquad (2)$$

for $R_b < r < R_n$, and

$$\frac{1}{r}\frac{d}{dr}\left[r\left(1-\frac{\omega_{pe}^2 k^2}{\nu_e^2 p^2}\right)\frac{d}{dr}\delta E_z\right]$$

$$-\,p^2\left(1-\frac{\omega_{pe}^2}{\omega^2}\right)\delta E_z = 0, \qquad (3)$$

for $R_n < r < \infty$. In Eqs. (1)-(3), $\omega_{pb}^2 = 4\pi n_b e^2/\gamma_b m$ is the beam plasma frequency-squared, ω_{pe} is the background plasma frequency, k is the axial wavenumber, $\delta E_z(r)$ is the axial component of the perturbed electric field, γ_b is the relativistic mass ratio of beam electron, the parameter p is defined by

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$$p^2 = k^2 - \omega^2/c^2, \qquad (4)$$

and ν_b and ν_e are beam and plasma electron vortex frequencies, respectively.

The two-stream dispersion relation is obtained from Eqs. (1)-(3) and is given by

$$\frac{\xi^2 K_0(\xi)}{2 + \xi^2 K_0(\xi)} \frac{\omega_{pe}^2}{(\omega - k\beta_b c)^2 \gamma_b^3 (1-f_c)} + \frac{2}{2+\xi^2 K_0(\xi)} \frac{\omega_{pe}^2}{\omega^2} = 1. \qquad (5)$$

where $K_0(x)$ is the modified Bessel function of the second kind of order zero, f_c is the line charge neutralization factor in the beam, $\beta_b c$ is speed of the beam electrons and the parameter ξ is related to the Budker's parameter ν of the beam by

$$\xi^2 = 4(1-f_c)\nu. \qquad (6)$$

After a careful examination of Eq. (5), we find that the maximum growth rate of the two-stream instability occurs at the wavenumber

$$k = \frac{\omega}{\beta_b c} \qquad (7)$$

and the corresponding real oscillation frequency is given by

$$\omega = \frac{\omega_{pe}}{\left[1+\frac{1}{2}\xi^2 K_0(\xi)\right]^{1/2}} \left(1-\frac{1}{2}\left[\frac{\nu K_0(\xi)}{\gamma_b^3}\right]^{1/3}\right). \qquad (8)$$

Note that Eq. (8) is valid only for the case of $\nu K_0(\xi) << \gamma_b^3$. Obviously, Eqs. (7) and (8) determine frequency and wavenumber of the wakefield wave in terms of beam parameters and the ambient plasma density. For a typical present experiment characterized by $\xi << 1$, the wakefield frequency is simplified to

$$\omega = \omega_{pe}, \quad \xi << 1 \qquad (9)$$

which is identical to the background plasma frequency.

Wakefield Strength

Within context of the linearized macroscopic cold fluid model, the perturbed radial velocity δV_r of an fluid element of plasma electrons is expressed as

$$\delta V_r = -\frac{ec^2 k}{m\,\omega\,\omega_{pe}^2} \frac{d}{dr} \delta E_z(r) \qquad (10)$$

for $r > R_n$, where the eigenvalue equation (3) is expressed as

$$\frac{1}{r}\frac{d}{dr}\left[r \frac{d}{dr} \delta E_z(r)\right] + \left(\frac{\omega^2}{c^2} - k^2 - \frac{\omega_{pe}^2}{c^2}\right) \delta E_z(r) = 0. \qquad (11)$$

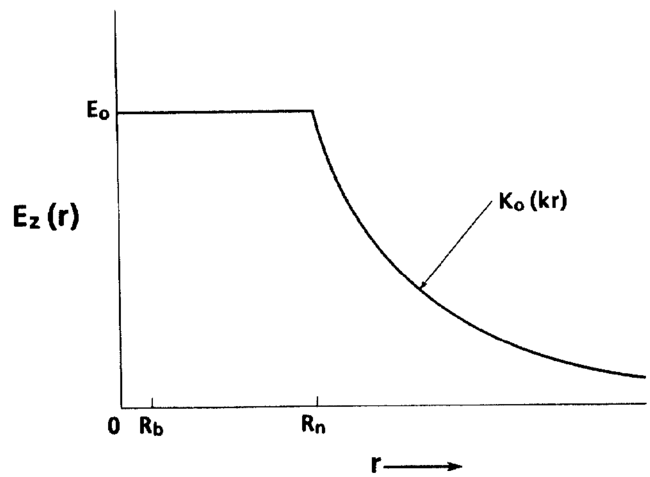

Fig. 2 Profile of the axial electric field generated by the wakefield.

For the case of $\xi << 1$ and $\omega = \omega_{pe}$, the eigenfunction $\delta E_z(r)$ is expressed as

$$\delta E_z(r) = \begin{cases} E_0, & 0 < r < R_n, \\ E_0 K_0(kr)/K_0(\xi), & R_n < r < \infty, \end{cases} \qquad (12)$$

where the axial electric field decreases monotonically from E_0 to zero as the radius r increases from $r=R_n$ to infinity. Figure 2 is the schematic presentation of the axial electric field profile. Substituting Eq. (12) into Eq. (10) and making use of Eqs. (7) and (9), we obtain

$$E_0 = \left(\frac{\delta V_r}{c}\right)_{r=R_n} \frac{mc}{e} \omega_{pe} \frac{K_0(\xi)}{K_1(\xi)}, \qquad (13)$$

where use has been made of the relationship $\omega_{pe} R_n = \xi c$.

Plasma electrons are beginning to move radially out when beam electrons arrive. To make the calculation analytically tractable, we assume that when plasma electrons move out to charge neutralization radius R_n the inner most electrons consist of those which are located at $r = R_b$ at the beginning. Thus, the radial electric field E_r, which exerts on the inner most electrons located at r, is obtained from

$$2\pi r E_r = 4\pi e \left[(1-f_c) N_b - \pi r^2 n_p\right] \quad (14)$$

where N_b is the number of beam electrons per unit axial length and n_p is the plasma electron density. Therefore, the radial equation of motion of the inner most electrons is expressed as

$$\frac{d}{dr} \gamma = \beta \frac{d}{dr} (\gamma\beta) = \frac{1}{2} \left(\frac{\xi^2}{r} - \frac{1}{2} \frac{r}{\delta^2}\right), \quad (15)$$

where γ is the instantaneous relativistic mass ratio of the inner most plasma electron defined by $\gamma^2 = \left(1 - \beta^2\right)^{-1}$, $dr/dt = \beta c$ is the radial velocity,
$\xi c = \omega_{pe} R_p$ and $\delta = c/\omega_{pe}$ is the plasma skin depth. After solving Eq. (15), the radial velocity of the fluid element of plasma electrons at $r=R_n$ is given by

$$\beta\left(r=R_n\right) = \left(\frac{\delta V_r}{c}\right)_{r=R_n} = \frac{(2\epsilon + \epsilon^2)^{1/2}}{1 + \epsilon}, \quad (16)$$

where

$$\epsilon(\xi) = \frac{1}{2} \xi^2 \left[\ell n \left(\frac{R_n}{R_b}\right) - \frac{1}{2} \left(1 - \frac{R_b^2}{R_n^2}\right)\right]. \quad (17)$$

Substituting Eq. (16) into Eq. (13), the maximum axial electric field is given by

$$E_0 = \frac{mc}{e} \omega_{pe} \frac{(2\epsilon + \epsilon^2)^{1/2}}{1 + \epsilon} \frac{K_0(\xi)}{K_1(\xi)}. \quad (18)$$

As shown in Eq. (18), the maximum axial electric field is proportional to the background plasma frequency ω_{pe} and increases from zero to $E_0 = (mc/e)\ \omega_{pe}$ as the beam current increases from zero to infinity.

Discussion

As shown in Eq. (7), the phase velocity of the wakefield waves is the same as the beam velocity. Thus, the perturbation created by the wakefield is a standing wave in the frame of reference moving with the electron beam. The axial electric field at the beam is also a standing wave in the beam frame, thereby removing energy from the beam in one half of the wavelength and adding energy to the beam in the other half. This mechanism modulates beam energy along the beam pulse, which is easily measurable. In reality, after a long range propagation, the portion of the beam which loses energy eventually loses its equilibrium condition and beam electrons in this portion are lost. This effect will break the beam into a train of small beam segments with length $\pi\beta c/\omega$. The perturbed electric field with oscillation frequency ω can also be detected and identified by various diagnostic means.

Acknowledgement

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