DYNAMICS IN AN ADIABATIC FREE ELECTRON LASER

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Abstract

The trapping and detrapping of electrons plays a significant physical role in free electron lasers (FEL),[1] and the dynamics of trapping and detrapping particles is now understood in simple adiabatic systems.[2] It is possible that beam heating could be greatly reduced in an FEL with truly adiabatic dynamics. This would be useful, for example, in a beam recirculation scheme. Previous work[2] indicates that the rms energy spread in such a beam would scale like $\varepsilon\ln(\varepsilon)$, where ε is the adiabatic parameter. This work is being generalized to apply more directly to the single particle dynamics of an FEL.

Introduction

Short wavelength FEL's operate in the single-particle or Compton regime. Because these devices cannot efficiently extract energy from an electron beam during a single pass, a recirculation scheme of some sort is desirable. A very simple scheme would involve accelerating the beam after each pass through the FEL, then reinjecting it once the extracted energy has been restored. Because the FEL interaction is a resonant process, the injected electrons must all have an energy close to the resonant energy in order for lasing to occur. Unfortunately, beam heating will occur during each pass through the FEL, limiting the number of times a single beam can be recirculated, and thus limiting the efficiency of the laser.

Recent theoretical work[2] indicates that beam heating could be made small enough to allow a single beam to be recirculated many times, without making the FEL structure excessively long. Such an FEL might consist of three components. First, a wiggler in which the magnetic field strength is ramped up adiabatically, in order to trap the injected particles. Second, a standard tapered wiggler, in which the resonant energy (and, hence, the energy of all the trapped particles) is decreased adiabatically. (This is how energy is extracted from the beam.) Finally, a wiggler in which the magnetic field is ramped down adiabatically, allowing the particles to detrap. This would require a structure approximately three times as long as a standard wiggler.

Adiabatic invariance theory (AIT) is an averaging theory, and thus it requires a separation of time scales. In an FEL, the fast time scale is the synchrotron oscillation period of a particle in the wiggler, while the slow time scale is the time during which this particle sees a significant change in the wiggler parameters. AIT consists, essentially, of averaging over the synchrotron oscillations of a particle. The adiabatic parameter is the ratio of the fast time scale to the slow one. For an FEL, this parameter, $\varepsilon=L_{syn}/L_{pon}$, can be written as a ratio of length scales, where L_{syn} is the local synchrotron wavelength and L_{pon} is the local characteristic length scale of the ponderomotive potential.

Consider a relativistic electron and an electromagnetic wave traveling through a wiggler magnet. A standard first approximation[1] is to assume the electron is close to the axis of the device, where transverse gradients can be neglected. If we assume both the wiggler field and the signal field to be circularly polarized, this implies vector potentials of the form

$$\mathbf{A}_w = -A_w(z)\left[\hat{x}\cos\left(\int_0^z k_w(z)dz\right) - \hat{y}\sin\left(\int_0^z k_w(z)dz\right)\right], \quad (1a)$$

$$\mathbf{A}_s = A_s(z)\left[\hat{x}\cos\left(\int_0^z k_s(z)dz-\omega_s t\right) - \hat{y}\sin\left(\int_0^z k_s(z)dz-\omega_s t\right)\right], \quad (1b)$$

where the w subscript refers to the wiggler, the s subscript refers to the electromagnetic signal, and z is the coordinate along the axis.

The relativistic Hamiltonian for this system is

$$H = \sqrt{1 + (\mathbf{P} - \mathbf{A})^2}, \quad (2)$$

where $\mathbf{A}=\mathbf{A}_w+\mathbf{A}_s$, $\mathbf{P}=\mathbf{p}+\mathbf{A}$ is the canonical momentum, and $\mathbf{p}$ is the linear momentum. We have chosen units such that the electron rest mass, the electron charge and the speed of light are all unity. Because H is independent of the transverse coordinates, P_x and P_y are constants of the motion. For simplicity, we assume these constants are zero. Because there is no z-component of the vector potential, $P_z=p_z$. Thus, we obtain the Hamiltonian

$$H = \sqrt{1+A_w^2+A_s^2+p_z^2+2A_wA_s\cos\left(\int_0^z [k_w+k_s]dz-\omega_s t\right)}. \quad (3)$$

The cosine term is called the ponderomotive potential.

One can make a number of canonical transformations and further approximations to obtain[1]

$$H' = \frac{k_w+\delta k_s}{\gamma_r}(\delta\gamma)^2 - \frac{\omega_s A_w A_s}{\gamma_r}[\cos(\psi) + \psi\sin(\psi_r)], \quad (4)$$

where γ_r and ψ_r are the energy and phase of a synchronous particle, the phase, ψ, is the coordinate, and the deviation in energy from resonance, $\delta\gamma$, is the canonical momentum. Also, z is the independent variable, and $\delta k_s \equiv k_s-\omega_s$ is small. This Hamiltonian is only valid locally, because it requires an expansion about the local resonant energy, while Eq. (3) can describe the full length of the FEL.

The Model

We study a simple model in order to understand the essential dynamics of trapping and detrapping particles in systems described by Eq.'s (3) and (4). The Hamiltonian we use is

$$H(p,q,y) = \frac{1}{2}p^2 + A(y)\cos(q), \quad (5)$$

where $y\equiv\varepsilon(q+t)$, with ε the adiabatic parameter and A(y) taken to be gaussian in all numerical simulations. This represents a nonrelativistic particle interacting with a large amplitude wave packet that is traveling to the left with group velocity $v_g=-1$. The particle mass and the wavenumber of the dominant Fourier component are taken to be unity. A simple transformation will yield a stationary wave packet with dominant phase velocity $v_\phi=1$. Because AIT depends on the functional form of the momentum, p, rather than on H, it is not difficult to generalize Eq. (5) to the relativistic case.

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Figure 1a shows the potential described by Eq. (5), as well as its envelope A(y), for ε=0.12. Figure 1b shows all the separatrices in the associated phase space. A separatrix is a constant energy contour which separates trapped trajectories from untrapped trajectories. A separatrix always contains an unstable fixed point. Figure 1b also shows three particle trajectories, sketched in a schematic way, in order to show trapping, bound oscillations and detrapping. As the wave packet moves to the left, the separatrices on the left side of Fig. 1b grow in size, while those on the right shrink. Trapping and detrapping always involves a particle trajectory crossing a separatrix.

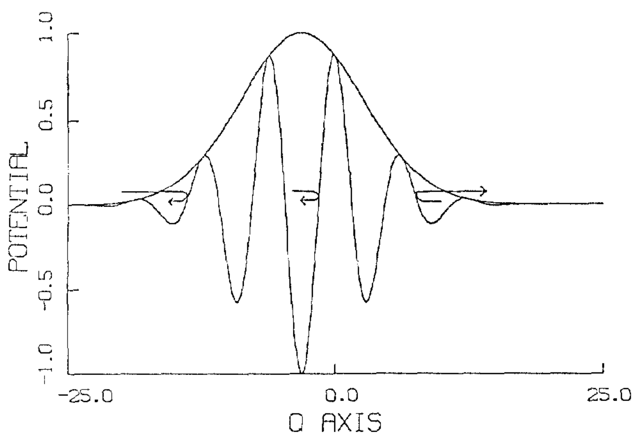

Fig. 1a The potential of Eq. (5) and its gaussian envelope; ε=0.12.

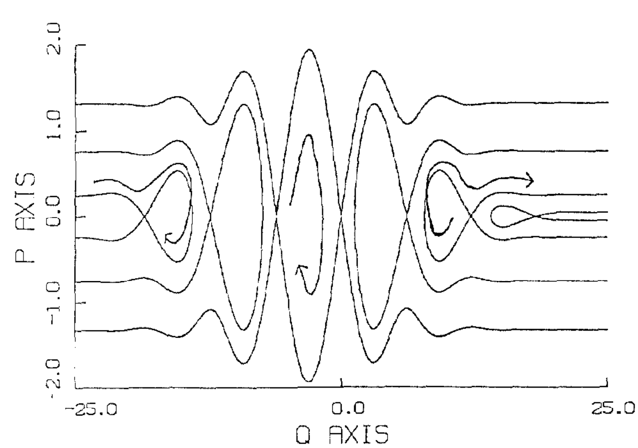

Fig. 1b The phase space corresponding to the potential of Fig. 1a.

Adiabatic Invariance Theory

Given a Hamiltonian of the form H(p,q,y), with $y \equiv \varepsilon(q+t)$, there exists a straightforward scheme[2] to calculate an approximate invariant J. First, a canonical transformation is used to write the momentum as a new Hamiltonian: P(E,y,q)=p, where E is the value of the old Hamiltonian, and q is the new independent variable. This makes it more convenient to average over the fast variations in q. Next, we write J as an asymptotic series:

$$J = J_0(E,y,q) + \varepsilon J_1(E,y,q) + \varepsilon^2 J_2(E,y,q) + \ldots \tag{6}$$

We require that dJ/dq vanish order by order in ε. This yields an infinite hierarchy of equations for the J_n. We can solve for J_0, J_1, and so on in turn, by requiring that each of the J_n be periodic in q.

In Hamiltonian systems of the form $H(p,q,\varepsilon t)$, J_0 is any function of the action.[2] The action, $I(E,\varepsilon t)$, is just the phase space area enclosed by a trajectory during a single oscillation; it is written as a loop integral: $I(E,\varepsilon t)=\oint dq\, P(E,\varepsilon t,q)$. The phase dependent first order term J_1 has been calculated in general and in a number of specific cases (see Ref. 2 and references therein).

AIT has only recently been applied to systems like Eq. (5). We have obtained the following analytic results. For untrapped particles, J_0 and J_1 are given by

$$J_0(E,y) = 4\sqrt{2(E+A)}\,E_c(k)\,\mathrm{sign}(p) + 2\pi E, \tag{7a}$$

$$\begin{aligned}\frac{A}{4A_y}J_1(E,y,q) &= [E_c(k)F_i(x,k) - F_c(k)E_i(x,k)]\,\mathrm{sign}(\pi-q)\\ &- \frac{\pi}{2}\left[\sqrt{2(E+A)}\,E_i(x,k) - \frac{2E}{\sqrt{2(E+A)}}F_i(x,k)\right]\mathrm{sign}(\pi-q)\,\mathrm{sign}(p)\\ &+ x\left[\sqrt{2(E+A)}\,E_c(k) - \frac{2E}{\sqrt{2(E+A)}}F_c(k)\right]\mathrm{sign}(\pi-q)\,\mathrm{sign}(p),\end{aligned} \tag{7b}$$

$$x \equiv \frac{1}{2}(\pi-q)\,\mathrm{sign}(\pi-q)\,. \tag{7c}$$

For trapped particles, J_0 and J_1 are given by

$$J_0(E,y) = 8\sqrt{A}\left[E_c(k^{-1}) - \frac{A-E}{2A}F_c(k^{-1})\right], \tag{8a}$$

$$\begin{aligned}\frac{A}{8A_y}J_1(E,y,q) &= (\pi-q)\sqrt{A}\left[E_c(k^{-1}) - \frac{1}{2}F_c(k^{-1})\right]\\ &+\left[E_c(k^{-1})F_i(\alpha(x),k^{-1}) - F_c(k^{-1})E_i(\alpha(x),k^{-1})\right]\mathrm{sign}(\pi-q)\,\mathrm{sign}(p),\end{aligned} \tag{8b}$$

$$x \equiv \frac{\pi}{2} - \frac{1}{2}(\pi-q)\,\mathrm{sign}(\pi-q)\,. \tag{8c}$$

We have used the following notation in Eq.'s (7) and (8). F_c and E_c are complete elliptic integrals of the first and second kind, as defined in Ref. 3. F_i and E_i are the corresponding incomplete integrals. The function *sign* yields the sign of its argument with unit magnitude, $k=\sqrt{2A/(E+A)}$, $\alpha(x)=\sin^{-1}|k\cos(x)|$, and $A_y \equiv \frac{dA}{dy}$. Whenever we write q in these equations, we actually mean (q mod 2π).

The first term in Eq. (7a) is the action. Thus, the lowest order adiabatic invariant for untrapped particles in this system is $J_0=I(E,y)+2\pi E$, not just the action. This is a fundamental departure from previously studied systems. The only term in Eq. (8a) is the action, so we have obtained the expected result for trapped particles.

Figures 2a and 2b show the result of numerically integrating a particle through the system described by Eq. (5). Figure 2a is a plot of the particle energy and local wave amplitude as a function of time. The crossing of these two curves indicates where trapping and subsequent detrapping occurs. Figure 2b is a plot of J_0 and $J^1 \equiv J_0+\varepsilon J_1$ for the same particle. There are discontinuities in both of these curves at trapping and detrapping, because the adiabatic invariant is a different function for trapped and untrapped particles. The first order corrected invariant, J^1, undergoes much smaller oscillations than J_0, except near the separatrix, where it diverges logarithmically. While the energy undergoes large excursions during the wave-particle interaction, the adiabatic invariant remains constant to a good approximation, except during trapping and detrapping.

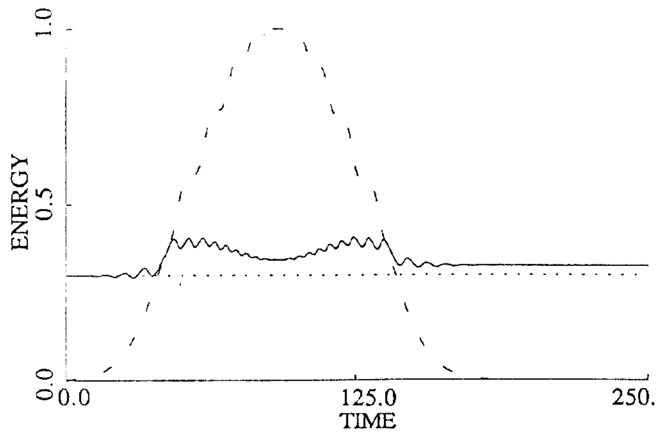

Fig. 2a Particle energy (solid) and local wave amplitude (dashed).

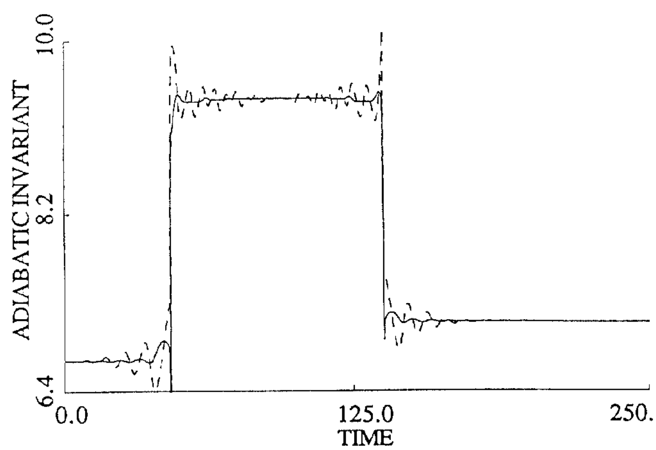

Fig. 2b J^1 (solid) and J_0 (dashed) for the particle in Fig. 2a.

<u>Separatrix Crossing Theory</u>

Separatrix crossing theory[2] (SCT) is used to calculate, through first order in ε, the change in the adiabatic invariant J during trapping and detrapping (i.e. when the particle trajectory crosses a separatrix). Such a theory is necessary, because AIT breaks down near a separatrix, so that J is not conserved for trapping and detrapping particles. This can be seen in Fig. 2b, which shows a net change in the adiabatic invariant of a particle that traps and then detraps. SCT proceeds by using AIT when a particle is far from the separatrix, then using perturbation theory (where $E-E_{sx}$, with E_{sx} the energy on the separatrix, is the new small parameter) when the particle is near the separatrix, and finally matching the two solutions in intermediate regions. This is an asymptotic theory, which has been shown[2] to work well in simple systems when $\varepsilon \lesssim 0.1$

SCT has shown[2] that, during a separatrix crossing, J remains constant to lowest order, but undergoes a phase-dependent change in first order. For systems with a symmetric potential, this change is $O(\varepsilon)$, while for asymmetric systems, it is $O[\varepsilon \ell n(\varepsilon)]$. Previously, SCT had only been applied to systems of the form $H(p,q,\varepsilon t)$. It is currently being applied to the system of Eq. (5). Although, the analytic results have not yet been fully tested, it is clear that the change is $O[\varepsilon \ell n(\varepsilon)]$.

Until now, we have been discussing a single particle. Given a beam of noninteracting particles, trapping and detrapping results in an rms spread in the adiabatic invariant of the same order as the change for a single particle[2]. The same is true for the energy. Furthermore, when many interactions are involved, this spread accumulates as in a random walk process. Thus, after N passes through an adiabatic FEL, an initially monoenergetic beam of particles would have an rms energy spread of order $\sqrt{N}\varepsilon \ell n(\varepsilon)$. If $\varepsilon \equiv L_{syn}/L_{pon} \lesssim 0.1$, the beam could be recirculated $N \gtrsim 18$ times before beam heating would prevent further operation.

<u>Conclusions</u>

Theoretical work in progress indicates that an FEL with a recirculating beam of electrons might be a practical device, as long as the wiggler parameters change slowly during trapping and detrapping of the beam, so that excessive beam heating does not occur. This could greatly increase the efficiency of a short wavelength FEL operated in the Compton regime. Standard tapered wigglers are adiabatic for the trapped particles, but these particles are significantly heated each time they enter and exit the wiggler.

<u>Acknowledgments</u>

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