A LATTICE WITH NO TRANSITION AND LARGE DYNAMIC APERTURE

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1. Summary

In the case of a one-ring high-energy scheme for an advanced hadron facility, beam losses can be reduced if the ring lattice accomodates the beam from injection to maximum energy without crossing the transition. Since there is no synchrotron booster in such a scheme and the injection energy is relatively low, this requirement implies a negative compaction factor and an imaginary transition energy. This can be achieved by making the horizontal dispersion negative in some regions of the arcs so that the average value taken in the dipoles is globally also negative. Such a modulation of the dispersion may result in an increasing difficulty to obtain a large enough dynamic aperture in the presence of sextupoles. A careful optimisation is therefore necessary and the possibility of modifying the linear lattice in order to include the requirements associated with chromaticity adjustments has to be studied. This paper summarizes the work done along this line and based on previous searches [1,2] for a race track lattice that can be used in a hadron facility main ring. It describes an alternative lattice design, which tends to minimize the effects of the nonlinear aberrations introduced by sextupoles and to achieve a large dynamic aperture, keeping the betatron amplitudes as low as possible.

2. Concepts for the arc linear optics

As in the existing proposals for main ring lattices [2,3], a race track configuration is retained with two long dispersion-free straight sections and two 180-degree arcs. The arc optics-design described below is intended to optimise the phase advances between sextupoles, minimize their strengths and reduce geometric aberrations, while keeping the transition energy negative. The dynamic aperture associated with the proposed design has to be checked in the presence of sextupoles cancelling the natural chromaticities, as required for achromatic extraction, for example.

In order to reduce the main ring dimensions, it seems appropriate to refer to the idea [1] of cancelling the dispersion in the straight sections by having an integer number of oscillations in each arc, rather than adding dispersion suppressors at every arc extremity. This can be achieved with geometrically identical cells in the curved section, filled with dipole magnets. It is, however, necessary to group the FODO cells and organize the quadrupole strengths in such a way that a dispersion perturbation and a negative compaction factor are obtained within one group. Moreover, each arc or curved section must contain such a number of groups of cells that the accumulated matrix be unity, i.e. the total phase advances per arc be a multiple of 2π. If the periodic solution for the horizontal dispersion in one or several groups of cells has initial and final values corresponding to zero slope but finite amplitude, the dispersion in the two curved sections of the complete ring, separated by two long straight sections, rises from zero at the arc entrance and makes oscillations of variable amplitude which vanish at the arc end. Indeed, when the phase advances through the long straight sections are not a multiple of 2π as expected from the condition that the total tunes of the ring are different from integers, the unique periodic solution for the horizontal dispersion has necessarily zero slope and amplitude at each arc extremity.

It is important to include at this stage the requirements that minimize sextupole nonlinear aberrations. It is known from previous work [4] that two nonlinear kicks of same strengths, due to two sextupoles, may cancel each other if the betatron amplitudes are the same, the phase separation is an odd multiple of π and if there are no other nonlinear perturbations in between. Since in practice the last condition cannot be satisfied, it is important to find positions where horizontally and vertically correcting sextupoles are not strongly coupled and to ensure cancellation over a distance as short as possible, i.e. over a phase separation corresponding to a small number of groups of cells. These requirements are somewhat in conflict with the negative compaction factor concept and this makes the lattice optimisation more challenging.

All the conditions described above can be expressed mathematically. Each arc is supposed to be made of N groups of k cells, the k cells being organized such as to achieve an imaginary transition energy or a negative integral of the dispersion. The nonlinear kicks due to sextupoles are then supposed to cancel over a phase separation corresponding to n groups of k cells. The average phase advances per cell inside one group are noted $\Delta\mu_x$ and $\Delta\mu_y$, while the minimum and maximum β-values which appear in the quadrupoles and will likely be almost equal to the values in the sextupoles are noted $\check{\beta}_x$, $\check{\beta}_y$ and $\hat{\beta}_x$, $\hat{\beta}_y$. The conditions for nonlinear kick cancellation and sextupole decoupling imply that the minima be much smaller than the maxima and that all maxima be approximately the same. With these notations, the requirements mentioned can be written as follows :

$$N \cdot k \cdot \Delta\mu_{x,y} = p \cdot 2\pi$$

$$\frac{1}{k\Delta\mu_x} \int_0^{k\Delta\mu_x} D_x(\mu_x)d\mu_x < 0$$

$$n \cdot k \cdot \Delta\mu_{x,y} = (2m+1)\pi \qquad (1)$$

$$\check{\beta}_x << \hat{\beta}_y,\ \check{\beta}_y << \hat{\beta}_x$$

$$\beta_{x,y}(\text{sextupoles}) \approx \hat{\beta}_{x,y} \approx \text{constant.}$$

The first relation (1) guarantees aberration cancellation in the two planes and a horizontal dispersion bump. In Eqs.(1), p and m are integers, the ratio N/n is even and k, n and m should be as small as possible. Table 1 gives some possible combinations that satisfy the first and third relations. The retained values of N correspond to a total number of cells close to the one used in recent proposals [2,3].

Among the possible solutions of Table 1, the first line appears very attractive with (k,n,m) = (3,1,0) and aberration cancellation over π or two times 3 cells. The combination (4,3,2) in the middle of Table 1 is the one retained for the TRIUMF proposal [3]; it implies cancellation over 5π or two times 12 cells (in both transverse planes).

3 . Design of the arc and ring completion

Let us concentrate on the most preferable case (3,1,0) of Table 1. Taking N = 8, it is possible to build an arc with 24 FODO cells, a cell length of 19.5 m and a dipole length of 7.25 m to remain near the existing proposal [2]. Starting with such groups of 3

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k	n	m	N	p	$\Delta\mu_{x,y}$(degree)
3	1	0	8	4	60
	2	1	8	6	90
	4	2	8	5	75
	4	3	8	7	105
4	2	1	8	6	67,5
	3	2	6	5	75
5	1	1	4	6	108
	2	2	4	5	90
6	1	1	4	5	90
	2	2	4	5	75
	2	3	4	7	105

Table 1
Possible combinations satisfying equations (1)

cells, the approach was to match an optics with $\Delta\mu_x = \Delta\mu_y = 60^0$ (value averaged over the 3 cells) and a negative compaction factor. It was practically impossible to satisfy these three requirements and it was necessary to define a compromise as explained hereafter. Since it appears more difficult to obtain a large acceptance vertically than horizontally [5], it was decided to keep the (3,1,0) combination in the vertical plane and relax the phase conditions in the horizontal plane. After a long search for a solution, the combination (3,4,3) of Table 1 with $\Delta\mu_x = 105^0$ was achieved for the horizontal motion. This solution is optimum in the vertical plane and has additional interesting features which compensate for the compromise made in the horizontal plane : i) inside two groups of 3 cells, the horizontal phase separation between the second and fourth or the third and fifth F-quadrupole is equal to π, which opens again the possibility of close aberration cancellation, ii) the β_x-values are about the same in each F-quadrupole and the β_y-values appearing on 2 in 3 of the D-quadrupoles are about equal to the β_x-values, even in the presence of the dispersion modulation.

The quadrupole pattern in the group of 3 cells proposed is then 1/2 QDA, QF, QD, QFA, QD, QF, 1/2 QDA and the integrated strengths are :

$$K\ell(QF) = 0.1431\ m^{-1} \qquad K\ell(QD) = -0.1258\ m^{-1}$$
$$K\ell(QFA) = 0.1955\ m^{-1} \qquad K\ell(QDA) = -0.1117\ m^{-1} \qquad (2)$$

The betatron-functions and matched dispersion ($\eta_x = D_x$) corresponding to this solution are given in Fig. 1. Putting together 8 groups of 3 cells, one arc through which the horizontal phase advances by $7 \cdot (2\pi)$ and the vertical one by $4 \cdot (2\pi)$ is obtained. Since the horizontal tune of the arc is an integer, the horizontal dispersion makes a closed bump. This is shown in Fig. 2 that gives the optics functions for half an arc to make clearer the main features of this alternative design [6].

The arc design proposed here is one possibility which satisfies most of the conditions enumerated in Sect. 2. Even if there may be other cases of interest, the present work brings in evidence that there are not many parameter intervals in which both requirements of absence of transition and minimum nonlinear aberrations are simultaneously satisfied. Hence, before looking for other similar solutions, it is important to investigate the quality of the present alternative lattice. For this purpose, an estimation of the dynamic aperture in the presence of sextupoles is required and implies to have a layout of the whole ring, including the straight parts of the racetrack. Long straight sections have been designed [6] in order to accomodate slow extraction scheme and direct H^- injection. The extraction insertion provides space for a massless preseptum, a wire-made electrostatic septum and a thick magnetic septum distant by approximately $\pi/2$ in vertical phase. The injection insertion includes a drift space long enough to accomodate four weak bending magnets, displacing the beam laterally. There is a double waist at the center of this drift, where a stripping foil is placed, to minimize the scattering effects on the beam. Each long straight section contains one of these insertions and additional cells for accelerating cavities as well as sets of collimators, and allows some tune adjustments [6]. Figs. 3 and 4 give the betatron functions of the two long sections, respectively.

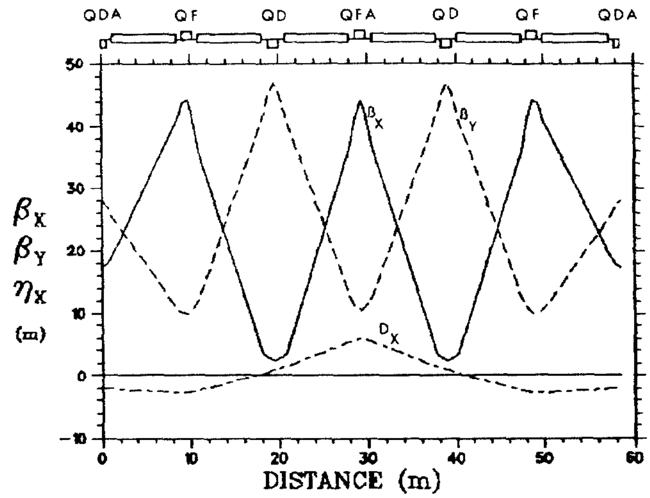

Fig. 1 - Optics functions for a group of 3 cells.

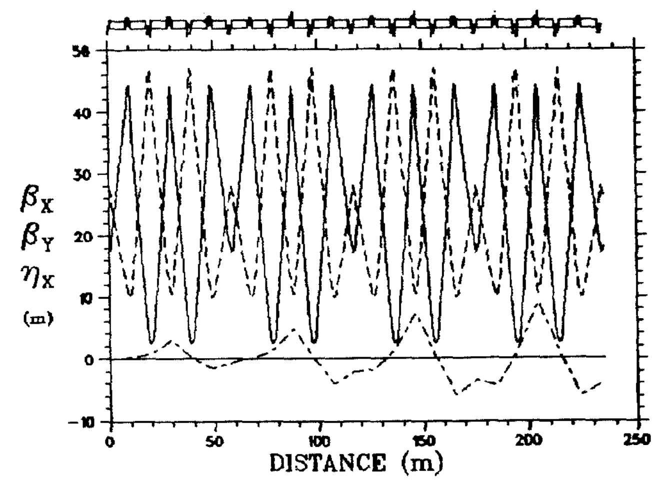

Fig. 2 - Optics and dispersion bump in half an arc.

3. Sextupole scheme and dynamic aperture

Referring to Fig. 1, the betatron amplitudes indicate that decoupling of transverse effects is rather effective if the sextupoles are positioned near QF, QFA and QD. In the horizontal plane, the QFA-position at the center of a group of 3 cells where dispersion is large and positive is particularly favorable. The QF quadrupoles, that may be distant by π from the QFA, are not retained initially even if they allow close aberration cancellation, because the dispersion is negative here and minimizing the sextupole strength is preferable. In the vertical plane, the two QD-positions are equivalent. Therefore, two families of sextupoles are appropriate; one called SF with 8 elements per arc near the QFA's and a D_x-sum equal to about 43.8 m and the other one called SD with 16 elements per arc near the QD's and a

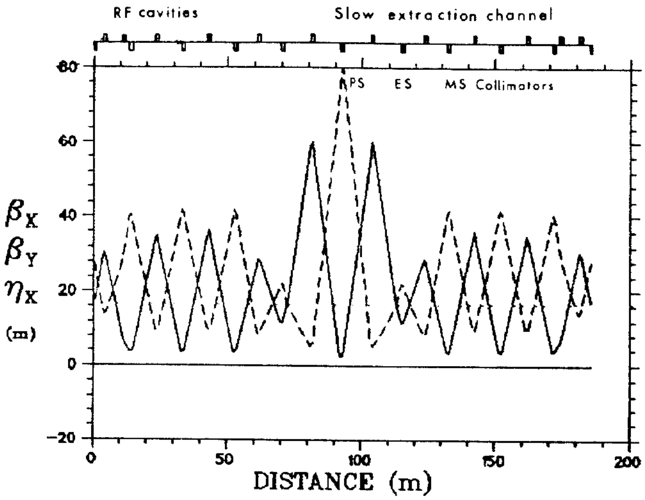

Fig. 3 - Straight section with extraction channel.

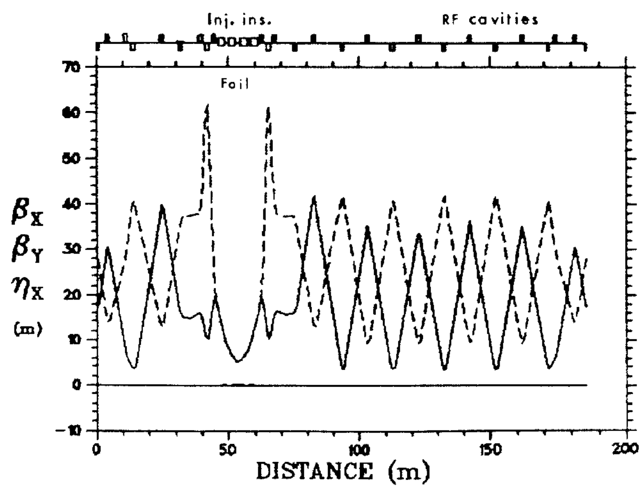

Fig. 4 - Straight section with injection insertion.

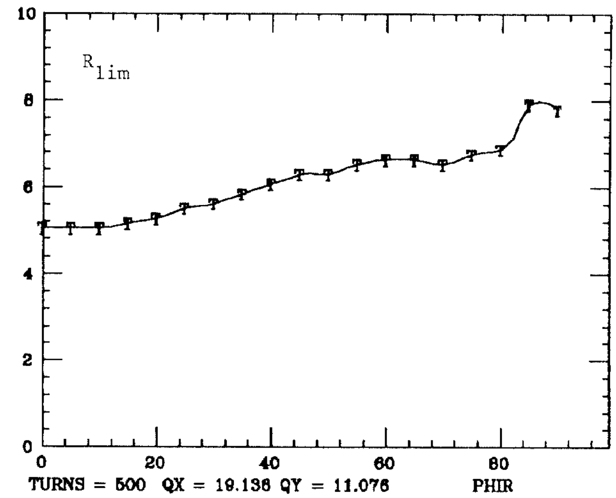

Fig. 5 - Maximum amplitude versus Φ_R.

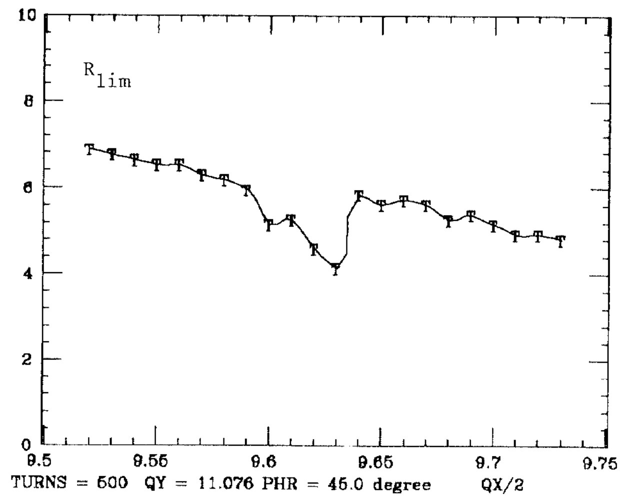

Fig. 6 - Maximum amplitude versus Q_x.

D_x-sum of about 20.9 m. The SF nonlinear aberrations are cancelled for pairs of sextupoles distant by 7π or 12 cells, which contain only 3 intermediate SF's and 8 SD's strongly decoupled. The SD aberrations are cancelled for pairs distant by π or 3 cells, which contain only two intermediate sextupoles, one SF and one SD.

In order to investigate the quality of this alternative lattice, it is necessary to estimate the dynamic aperture in the presence of sextupoles compensating the natural chromaticities [6]. The dynamic aperture is calculated by using the tracking option of the program DYNAP [7] applicable to a structure of drifts, quadrupoles and sextupoles in the case of two-dimensional betatron motion. Tracking is done over 500 turns, i.e. more than 1 ms, and gives the stability limit or maximum amplitudes $R_{lim} = (x_0^2+y_0^2)^{1/2}$ up to which the motion is stable over this time. Fig. 5 shows this quantity in cm for different initial conditions (in the real plane) defined by Φ_R = atan (y_0/x_0) and Fig. 6 the dependence of R_{lim} on the horizontal tune for $\Phi_R = 45^0$. Owing to the initial β-values of the structure, i.e. 5.25 m, the emittances can be calculated from the curve in Fig. 5 for the quoted tune values. The horizontal acceptance at $\Phi_R = 0$ is about 480 mm mrad and the vertical one at $\Phi_R = 90^0$ is even larger, about 1150 mm mrad. For $\Phi_R = 45^0$, the two maximum emittances are equal and reach the value of about 380 mm mrad. In comparing the dynamic aperture of this proposed structure and a previous lattice [2], one can notice that the acceptance [5] has been increased by a factor 7 to 8 at 45^0 and 90^0, and by 30% at $\Phi_R = 0$. These results reflect the fact that the amplitude limit in Fig. 5 does not drop when Φ_R varies from 0 to 90^0 because of the efficiency of the sextupole scheme in the vertical plane. The transitionless lattice, proposed in this paper, is a good solution for an advanced hadron facility main ring and deserves consideration. It profits indeed by a large dynamic aperture and satisfies the known requirements for such a facility.

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