EMITTANCE GROWTH IN A STORAGE RING DUE TO GROUND MOTION*

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For a synchrotron light source designed for a small natural emittance, ground motion could lead to a non-negligible growth in emittance. When the ground motion is of the plane-wave type, one can study this dynamical process analytically by employing a method that combines the previously developed transfer function technique with a normal mode analysis that takes into account the magnification of the motion due to magnet supports. The explicit relationship between the amplitudes of the ground vibration and the emittance growth as a function of the vibration frequency is presented. Numerical examples are included.

1. Introduction

In the design of the storage ring of a synchrotron light source, the natural emittance, ε_o, is a parameter of primary importance. Because the desired value of ε_o is very small (on the order of 10^{-9} m-rad for the APS), any time-dependent perturbations, such as ground motion, which will lead to closed orbit oscillations may potentially cause a considerable growth in ε_o. In order to evaluate this dynamic emittance growth due to ground motion, a three-step method is suggested below:

1. Ground motion is transmitted through the magnet supports to the magnets in the storage ring with a magnification factor A_1,

$$\delta_m = A_1 \cdot \delta_g, \qquad (1)$$

in which δ_g and δ_m are the magnitudes of the vibration of ground and the magnets, respectively.

2. When ground motion is of plane-wave type, the magnet vibrations are correlated and will result in a closed orbit oscillation with an amplitude δ_c,

$$\delta_c = A_2 \cdot \delta_m, \qquad (2)$$

where A_2 is the second magnification factor.

3. The closed orbit oscillation leads to a dynamic emittance growth,

$$\Delta\varepsilon/\varepsilon_o = 2\delta_c/\sigma, \qquad (3)$$

in which σ is the beam size.

In Section 2, we use a normal mode analysis to get the first magnification factor A_1. Section 3 employs a transfer function technique to calculate the second magnification factor A_2. Numerical examples and the vibration criteria, in terms of allowable horizontal and vertical vibration of the pertinent magnets in the storage ring, are given in Section 4. Section 5 is for discussion.

2. Vibration Magnification due to Magnet Supports

A simplified model of the magnet-pedestal system is a uniform girder supported by two elastic pedestals with stiffness k, see Fig. 1.

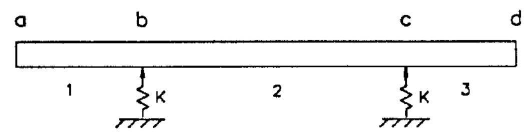

Fig. 1. A model of the magnet-pedestal system.

In the presence of a driving force and a damping force, the transverse vibration of this girder obeys the inhomogeneous Euler equation,

$$EI\,\frac{\partial^4 y}{\partial x^4} + c\,\frac{\partial y}{\partial t} + \rho\,\frac{\partial^2 y}{\partial t^2} = F(x,t), \qquad (4)$$

where
- E = Young's modulus,
- I = moment of inertia of the girder,
- c = the viscous damping per unit length, assumed to be a constant,
- ρ = mass density of the girder, and
- F = the driving force.

Let the solution y(x,t) of Eq. (4) be written as

$$y(x,t) = \sum_{p=1}^{\infty} Y^{(p)}(x) \cdot q^{(p)}(t), \qquad (5)$$

where $Y^{(p)}(x)$ is the p^{th} normal mode with normal frequency ω_p ($\equiv 2\pi f_p$), which can be obtained by solving the corresponding damping-free homogeneous Euler equation with proper boundary conditions, and $q^{(p)}(t)$ is to be determined through Eq. (4).

The boundary conditions that we use are as follows:

$$\begin{aligned} Y'' &= 0 \\ Y''' &= 0 \end{aligned} \qquad \text{for } x = x_a,\ x_d,$$

and

$$\begin{aligned} Y_1' &= Y_2' \\ Y_1'' &= Y_2'' \\ EI\left(Y_1''' - Y_2'''\right) - kY_1 &= 0 \qquad \text{for } x = x_b. \\ Y_1 &= Y_2 \end{aligned} \qquad (5A)$$

The same conditions apply to boundary c, with the subscripts 1 and 2 replaced by 2 and 3, respectively (see Fig. 1). Assume that the driving force is harmonic with a magnitude F_o and a frequency ω ($\equiv 2\pi f$)

*This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, under Contract W-31-109-ENG-38.

and acts on the girder through the two pedestals at b and c; in other words,

$$F(x,t) = F_o\, e^{i\omega t}\left[\delta(x-b) + \delta(x-c)\right], \tag{6}$$

and

$$F_o = k \cdot \delta_g. \tag{7}$$

Then the solution to (4) is

$$y(x,t) = A(x,f) \cdot \delta_g \cdot e^{i\omega t}, \tag{8}$$

where

$$A(x,f) = \sum_{p=1}^{\infty} Y^{(p)}(x)\left(Y^{(p)}(b) + Y^{(p)}(c)\right) \cdot \frac{k}{\rho(2\pi f_p)^2} \cdot M\left(\frac{f}{f_p}\right) e^{i\phi(f/f_p)} \tag{9}$$

in which

$Y^{(p)}(b)$ and $Y^{(p)}(c)$ are the values of $Y^{(p)}(x)$ at the supporting points b and c (Fig. 1), respectively;

$M(f/f_p)$ is usually called the p^{th} dynamic amplification factor and is equal to

$$1 \Big/ \left\{\left[1-\left(\frac{f}{f_p}\right)^2\right]^2 + \left(2\,\zeta_p \frac{f}{f_p}\right)^2\right\}^{1/2} ;$$

$\phi(f/f_p)$ is the p^{th} phase angle and is equal to

$$\tan^{-1}\left\{2\zeta_p \frac{f}{f_p} \Big/ \left[1-\left(\frac{f}{f_p}\right)^2\right]\right\} ;$$

ζ_p is the p^{th} damping factor, which determines the height and the width of the p^{th} resonant peak.

Both M and ϕ are universal functions and can be found in Ref. 1. Thus, by definition, the magnification factor A_1 is

$$A_1(f) = \max_x A(x,f) \tag{10}$$

In practical calculations of Eq. (9), the summation over the first four or five modes is, in general, good enough. The reader is referred to Ref. 2 for a detailed discussion of the above analysis.

3. Closed Orbit Oscillations Due to Correlated Magnet Vibration

When ground motion is of the plane-wave type and the storage ring is circular, the vibrations of the magnets in the ring are correlated and can be approximately computed using the so-called transfer function technique.[3] A uniform focusing around the ring is assumed. The resulting vertical closed orbit oscillation can be found in Eq. (2), with the amplification factor

$$A_2\left(\frac{f}{v}\right) = \sum_{s=-\infty}^{\infty} \frac{\nu_y^2}{\nu_y^2 - s^2}\, j^s J_s\left(\frac{2\pi f R}{v}\right) \tag{11}$$

in which ν_y is the vertical tune; R the radius of the storage ring; v and f the propagation velocity and frequency, respectively, of sound waves in the ground; j the square root of -1; and J_s the s^{th} order Bessel function. There is a similar expression for horizontal closed orbit oscillation, when replacing ν_y by the horizontal tune ν_x, and J by J', the derivative of J.

4. Numerical Examples and Vibration Criteria

Once we know the magnification factors A_1 and A_2, we can use Eq. (3) to estimate the emittance growth, $\Delta\varepsilon/\varepsilon_o$, for a given amplitude of ground vibration, δ_g. Alternatively, for a given emittance growth of, say, 10%, one may calculate the maximum allowable vibration amplitude. The latter is called the vibration criterion.

We have employed this three-step method to run a numerical simulation using representative parameters characterizing the magnets and support structures of the APS storage ring. The results are summarized in Table 1 and Figs. 2 and 3.

Table 1.
Maximum Allowable Vibration Amplitude
for an Emittance Growth of 10%*

Plane Wave (v in m/s, f in Hz)	$(\delta_m)_H\vert_{max}$, μm	$(\delta_m)_V\vert_{max}$, μm
f < 0.0072 v	16.1	4.4
0.0072 v < f < 0.015 v	16.1	4.4-0.28
0.015 v < f < 0.025 v	16.1	0.28
0.025 v < f < 0.044 v	16.1-0.90	0.28
f > 0.044 v	0.90	0.28

*$(\delta_m)_H$ and $(\delta_m)_V$ are the horizontal and vertical components, respectively, of magnet vibration.

5. Discussion

a. The effects of low-frequency vibrations (below 20 Hz) can be alleviated with feedback systems using steering magnets. Higher frequency vibrations that result in unacceptable emittance growth cannot be corrected in this manner and must be eliminated at their source or otherwise controlled.

b. Figure 3 shows that at certain normal frequencies of the magnet/support system (~41 Hz and ~128 Hz in this specific example), the inherent dynamic amplification drives the allowable amplitude of ground motion to extremely low values. This illustrates the importance of including the dynamics of the support structure in any vibration analysis.

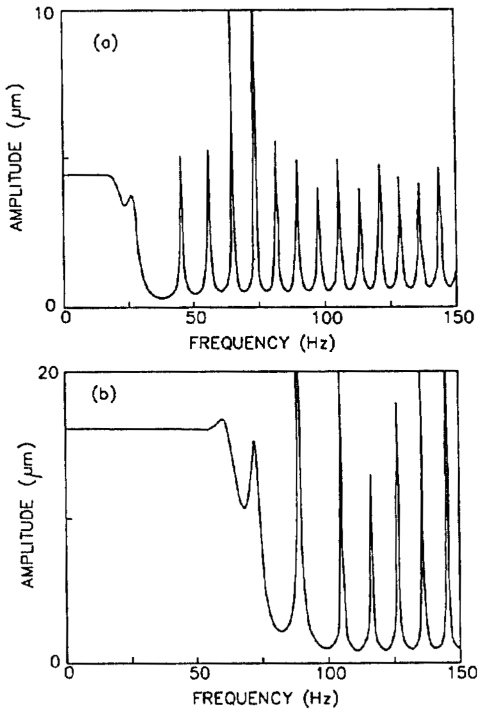

Fig. 2. Allowable amplitude of ground vibration for 10% growth in emittance when support effects are ignored: (a) vertical. (b) horizontal. (Propagation velocity of ground motion is assumed to be 2500 m/s.)

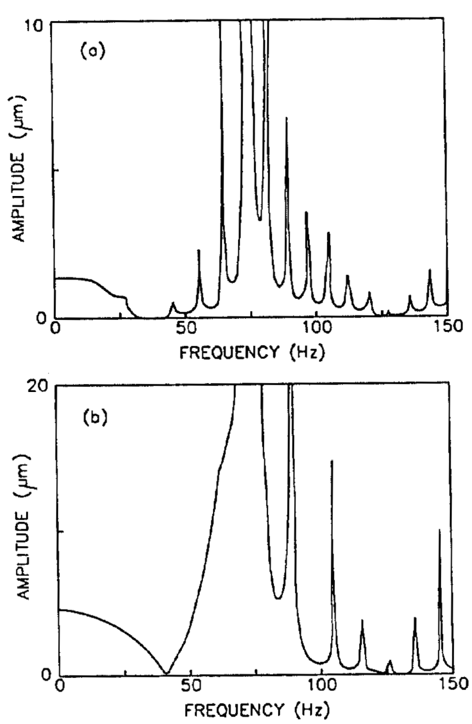

Fig. 3. Allowable amplitude of ground vibration for 10% growth in emittance when support effects are considered: (a) vertical. (b) horizontal. (Propagation velocity of ground motion is assumed to be 2500 m/s.)

c. The frequency dependence of the factor A_2 varies for different propagation velocity v (see Eq. (11) and Table 1), while A_1 is a function of mechanical structures only and is independent of v. Therefore, caution is needed in obtaining Fig. 3 for different velocities.

d. The plane wave model is applicable when vibration sources are far away from a storage ring. If, on the other hand, vibration sources are very close to the ring, ground motion may be of a spherical-wave type. In this case, our 3-step method can be employed with a modified Eq. (11).

e. The effects of concrete slabs are not included in this analysis.

References

1. W. C. Hurty and M. F. Rubinstein, Dynamics of Structures (Prentice-Hall, 1964), Chap. 8.

2. W. Chou, Argonne National Laboratory, Light Source Notes LS-77, LS-77A (1987), and LS-87, LS-88 (1988).

3. T. Aniel and J. L. Laclare, ESRF-SITE-86-04.