

CAPTURE AND CONTAINMENT OF THE POSITRON BEAM IN THE APS INJECTOR LINAC

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A nonlinear beam optics study of the positron-capture solenoid following the positron converter has been carried out. The design of the positron focusing structure along the 450-MeV positron linear injector for the 7-GeV Advanced Photon Source (APS) at Argonne National Laboratory is also considered.

Introduction

A 450 MeV positron linear-accelerator is planned to be built at Argonne National Laboratory. This accelerator, in conjunction with an accumulator ring and a 7 GeV booster synchrotron, will serve as an injector for the planned 7 GeV Advanced Photon Source storage ring. The positron linear accelerator is fed by the 200 MeV electron linac which produces a 1.25 A, 40 ns electron beam. This beam is then focused onto a 3 mm diameter spot at the 7 mm thick (two radiation length) tungsten target. By the process of multiple nuclear-scattering the target generates the positron particles. These positrons, in general, have a large diverging angle. In order to capture these rapidly diverging positrons, a strong focusing lens with large acceptance is placed close to the converter. In this case, a magnetic solenoidal lens has an advantage over the usual quadrupole lens because of its larger phase-space acceptance. This type of lens, however, has the disadvantage that for a given focal length it requires much more power than the quadrupole lens. When a solenoid requires high power, it is imperative to pulse it in order to reduce the time-averaged power.

The positron beam captured by the solenoidal lens passes through the main sections of the 450 MeV linear accelerator. Since the transverse focusing in a linac virtually disappears as the positron beam energy becomes relativistic, some means of the focusing has to be provided in order to transport the beam without any significant loss. The conventional method is to place FODO quadrupoles along the waveguide system of a linac.

In this paper, we present the result of our study for capture and confinement of the positron generated by the target. We first describe the positron-capture solenoid. We then discuss the pseudo-periodic FODO system of quadrupoles for the APS 450 MeV positron linear accelerator.

Positron Capture Double-layer Solenoid

For a double-layer solenoid, the field along the axis is given by [1]

$$B(0, z) = \frac{\mu_0 I N}{2(b-a)L} \left[\left(z - \frac{L}{2} \right) \ln \frac{b + \sqrt{b^2 + (z + \frac{L}{2})^2}}{a + \sqrt{a^2 + (z + \frac{L}{2})^2}} \right. \\ \left. - \left(z - \frac{L}{2} \right) \ln \frac{b + \sqrt{b^2 + (z - \frac{L}{2})^2}}{a + \sqrt{a^2 + (z - \frac{L}{2})^2}} \right]$$

where a and b are the inner and the outer radii in meters, L is the total length in meters, and N is the total number of windings on both layers (*ie.*, $N = N_1 + N_2$) of the solenoid. The above equation remains valid as long as the pitch of the solenoid is small compared to the total length of the lens. The field off the axis can then be obtained from the Taylor expansion of the field on the axis:

$$B_z(\rho, z) = B_z(0, z) - \frac{\rho^2}{4} \left(\frac{\partial^2 B_z(0, z)}{\partial z^2} \right) + \frac{\rho^4}{64} \left(\frac{\partial^4 B_z(0, z)}{\partial z^4} \right) - \dots \\ B_\rho(\rho, z) = -\frac{\rho}{2} \left(\frac{\partial B_z(0, z)}{\partial z} \right) + \frac{\rho^3}{16} \left(\frac{\partial^3 B_z(0, z)}{\partial z^3} \right) \\ - \frac{\rho^5}{384} \left(\frac{\partial^5 B_z(0, z)}{\partial z^5} \right) + \dots$$

The first- and second-derivatives of the field are:

$$\frac{\partial B_z}{\partial z} = C_a (\ln B_+ - \ln A_+) \\ + C_a \left(z + \frac{L}{2} \right)^2 \left[\frac{1}{B_+ \beta_+} - \frac{1}{A_+ \alpha_+} \right] \\ - C_a (\ln B_- - \ln A_-) \\ - C_a \left(z - \frac{L}{2} \right)^2 \left[\frac{1}{B_- \beta_-} - \frac{1}{A_- \alpha_-} \right]$$

and

$$\frac{\partial^2 B_z}{\partial z^2} = 3C_a \left(z + \frac{L}{2} \right) \left[\frac{1}{B_+ \beta_+} - \frac{1}{A_+ \alpha_+} \right] \\ - 3C_a \left(z - \frac{L}{2} \right) \left[\frac{1}{B_- \beta_-} - \frac{1}{A_- \alpha_-} \right] \\ - \frac{C_a \left(z + \frac{L}{2} \right)^3}{B_+ \beta_+^2} \left[\frac{1}{B_+} + \frac{1}{\beta_+} \right] \\ + \frac{C_a \left(z + \frac{L}{2} \right)^3}{A_+ \alpha_+^2} \left[\frac{1}{A_+} + \frac{1}{\alpha_+} \right] \\ + \frac{C_a \left(z - \frac{L}{2} \right)^3}{B_- \beta_-^2} \left[\frac{1}{B_-} + \frac{1}{\beta_-} \right] \\ - \frac{C_a \left(z - \frac{L}{2} \right)^3}{A_- \alpha_-^2} \left[\frac{1}{A_-} + \frac{1}{\alpha_-} \right]$$

where

$$C_a = \frac{\mu_0 I N}{2(b-a)L} \\ A_+ = a + \sqrt{a^2 + (z + L/2)^2} \\ \alpha_+ = \sqrt{a^2 + (z + L/2)^2} \\ B_+ = b + \sqrt{b^2 + (z + L/2)^2} \\ \beta_+ = \sqrt{b^2 + (z + L/2)^2} \\ A_- = a + \sqrt{a^2 + (z - L/2)^2} \\ \alpha_- = \sqrt{a^2 + (z - L/2)^2} \\ B_- = b + \sqrt{b^2 + (z - L/2)^2} \\ \beta_- = \sqrt{b^2 + (z - L/2)^2}$$

Motion of the Positrons through the Solenoidal Lens

Some parameters for the APS positron-capture solenoidal lens are listed in the following:

$$L = 4.6 \text{ cm}, a = 1.25 \text{ cm}, b = 2.25 \text{ cm}, N = 16 \text{ turns}, I \approx 5000 \text{ A}$$

These parameters are based on the DESY design [2]. We assume that the solenoid is placed at 5 mm distance from the target. With these parameters, in Fig. 1 we show the variations of B_z on the axis and off the axis. The off-axis field distribution was obtained from the fourth-order expansion of the field on the axis. This has been done with the aid of a

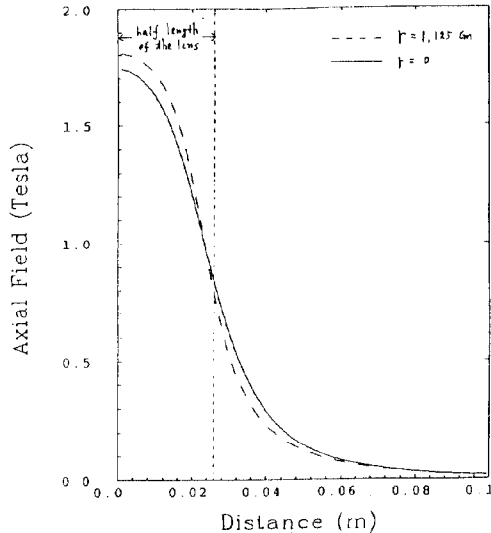


Fig. 1 Axial magnetic fields of the positron-capture solenoid. The solid curve denotes the axial field at $r=0$ cm. The dotted curve describes the axial magnetic field at $r=1.125$ cm.

computer. Fig. 1 indicates that the nonlinear effect is not significant. The focusing power of the solenoid can be written in the form [1]:

$$\frac{1}{f} = \left(\frac{e}{2M_0\gamma\beta c} \right)^2 \int_{-\infty}^{+\infty} B_z^2 dz$$

where $\beta = v/c$ and c is the speed of light. In the case of the APS positrons generated by the target, the central energy is assumed to be 8 MeV. This corresponds to $\gamma=16.6556$, $\beta=0.998196$. With these values, Fig. 2 summarizes the focusing power at different radii.

By using the fields obtained above, one can trace a number of positrons from the positron target to further downstream. First, we assume that the particles are distributed uniformly in radius and angle. The space charge effect was not taken into account in the calculation since the positrons are already relativistic (ie, space-charge force decreases as $1/\gamma^2$).

The positron-capture solenoid acts like a quarter-wave transformer. By this we mean that the role of the solenoid merely rotates the initial phase-space diagram by 90 degrees. That is, large angular divergence and small radial spread of the initial positrons are transformed into small divergence and large radial spread of the final positrons. That this really holds

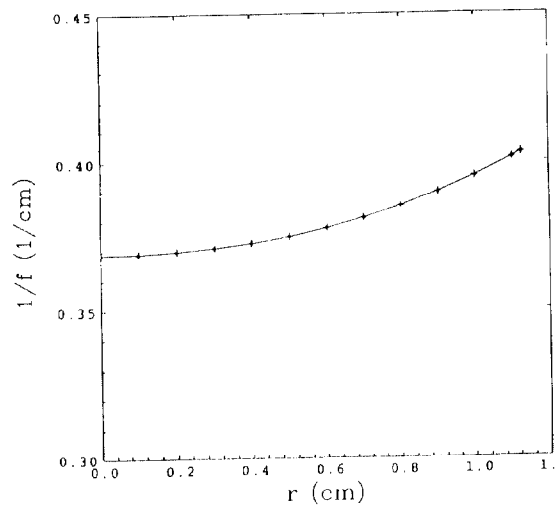


Fig. 2 The focusing power of the positron-capture solenoid as a function of radius.

in our case is demonstrated in Fig. 3 (a) and (b). In Fig. 3, (a) depicts the transverse phase-space diagram at the position of the target, and (b) describes the transverse phase-space diagram further downstream after the solenoid (ie, $z=10$ cm) where the field of the solenoidal lens is almost zero. In this figure, we take about 80 uniformly distributed extreme particles at the position of the target. The chosen initial conditions are $x' \approx \pm 210$ mrad. From the figure, one can clearly see the focusing property of the solenoid so that the initial large divergence ($1.5 \text{ mm} \times 210 \text{ mrad}$) is converted to ($7.5 \text{ mm} \times 42 \text{ mrad}$). The slight filamentation of the phase-space comes from the nonlinear magnetic field that we discussed before.

A further study indicates that the acceptance of the solenoid is about 360 mm mrad in both transverse planes when the current is optimized to 5000 amperes per turn (ie $B_0=1.81 \text{ T}$ at the center).

FODO System

With the given accelerating gradient, quadrupole length, initial beam energy, and maximum allowable beam size, the FODO system parameters (ie., quadrupole field gradient, quadrupole spacing) are uniquely defined in such a way that the maximum and minimum beam sizes remain constant throughout the linac. The quadrupole strength can be obtained by imposing the condition that with given quadrupole strength the acceptance of the linac should be a maximum. Assuming that the beam size is a maximum at the center of the defocusing quadrupole, the required field strength is then given by:

$$k = \frac{1}{\sqrt{2}(2\alpha - 1)l^2}$$

where l is the quadrupole half length and α is the solution of the following equation:

$$\alpha^3 - \alpha^2 - \alpha \frac{1 - \eta^2}{4\eta^2} + \frac{1}{6\eta^2} = 0$$

where

$$\eta = \frac{16l^2 \epsilon^2}{a^4}$$

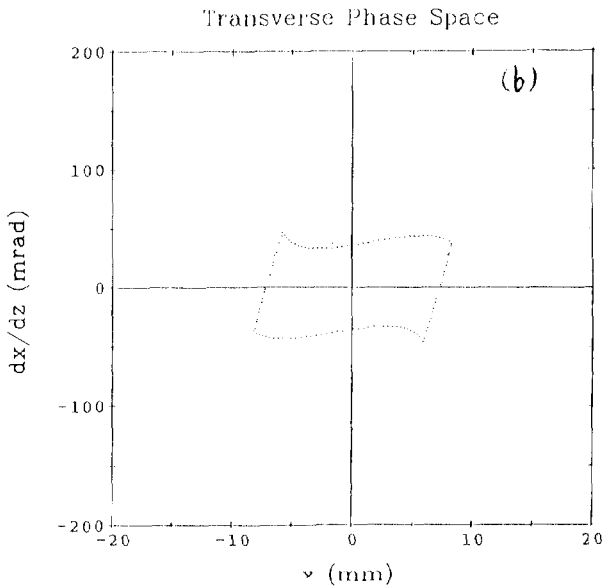
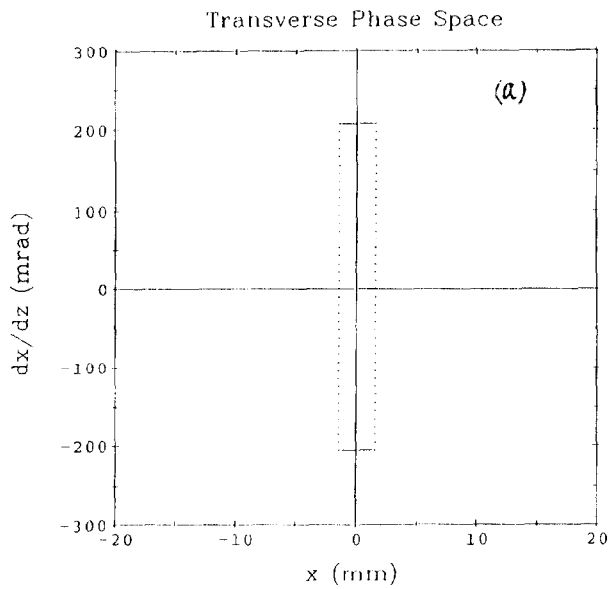


Fig. 3 The transverse phase space of particles. (a) is the transverse phase space at the position of target. (b) is the transverse phase space further downstream. A uniform distribution in angle and displacement is assumed. The initial energy spread is not included in these figures.

Here ϵ is the emittance of a beam and a is the maximum allowed beam size. The quadrupole spacing is determined from

$$L_n = L_1 \exp[(n-1)\lambda\gamma]$$

where

$$\lambda = \int_0^{L+2l} \frac{dz}{p} \approx \frac{L+2l}{p}$$

Parameters for the APS positron linac are, after the first two waveguides, initial energy=110 MeV, emittance at 110 MeV=27 $mm\text{mrad}$. Accelerating Gradient=18.9 MV/m. Fig. 4 shows the designed FODO system arrangement for APS positron

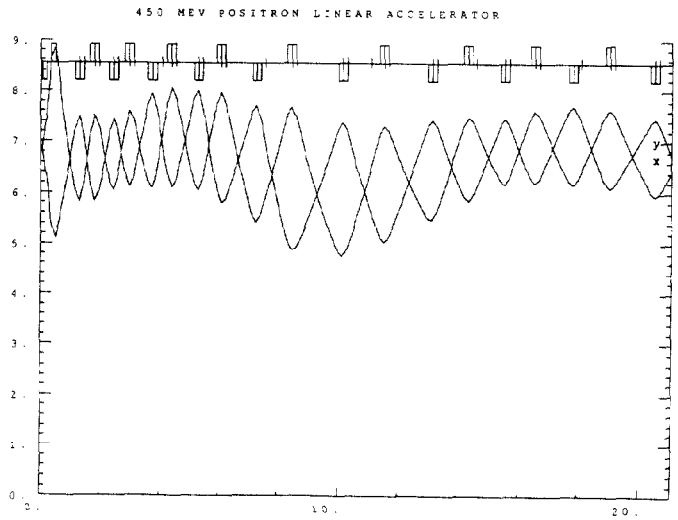


Fig. 4 FODO Quadrupole system along the positron linac for APS injector. The beam size is shown to be within 8 mm.

linac. Shown in the figure is the beam size in mm. It can be seen that the maximum beam size is limited to 8 mm. Except the first and the last two quadrupoles for matching, all the quadrupoles' field gradients are the same.

Summary

We described the positron-capture solenoid and FODO system for the 450 MeV positron injector linac of the 7 GeV Advanced Photon Source storage ring at Argonne National Laboratory. Based upon analytical expression of the magnetic fields, the phase space acceptance of the positron-capture solenoid was obtained. Detailed FODO structure of the positron-focusing quadrupoles was discussed also.

References

- [1] M. Yoon and G. Mavrogenes, LS-122, Argonne National Laboratory, June 1988
- [2] G. Stange, IEEE Trans. Nucl. Sci., Vol. NS-26, No. 3, June, 1979, 4145-4148

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