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COMPARISON OF ANALYTICAL AND COMPUTATIONAL ESTIMATES OF REDUCTION IN LINEAR APERTURE IN THE PRESENCE OF SEXTUPOLE FIELD*

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Summary

For an easy comparison of the expected improvement in the linear aperture when magnets are sorted according to various schemes, it is desirable to have a figure-of-merit which can be evaluated without time-consuming tracking calculations. Such a figure-of-merit is proposed here in terms of the sextupole distortion functions introduced by T.Collins.¹ A test lattice composed of 512 dipoles with random sextupole field component is used to calculate the figure-of-merit and the indicated improvement in the linear aperture is compared with the results from numerical tracking. As the linear aperture, two definitions are used, one by the SSC-CDG (smear 6.1%) and the other by CERN SPS (smear=1.75%).

Introduction

In large superconducting synchrotrons, one of the major causes of the aperture reduction is undoubtedly the sextupole field in dipoles, which cannot be reduced below a certain value without increasing the physical aperture of dipoles. Sorting of dipoles according to a predetermined procedure has been proposed as a simple and economical way to reduce the harmful effect.² In particular, the scheme proposed by R.L.Gluckstern and S.Ohnuma³ requires measuring and sorting magnets in small groups, enough to cover the betatron phase advance of 2π which is usually four to six FODO cells. The scheme has been further refined by R.Li and Gluckstern.⁴

The dynamic aperture of a given lattice is relatively easy to find by numerical particle tracking since it involves a few hundred revolutions and a simple question of particles "lost" or "not lost". In contrast, the transverse acceptance of the ring for a long-term (many hours) beam storage is difficult if not impossible because of the excessive amount of computing time required even with supercomputers. As an alternative answer to the question of long-term stability, a linear aperture is sometimes used in comparing the performance of different rings. The linear aperture is defined in terms of "smear", a measure of the deviation of transverse betatron oscillation amplitudes from their linear (invariant) values. Since the linear aperture is defined for a definite value of the smear,^{5,6} it is necessary to calculate the smear as a function of betatron oscillation amplitude with tracking, again a time-consuming task.

An analytical expression of the smear in terms of the distortion functions of T.Collins is proposed here for a quick comparison of the effectiveness of various sorting schemes of dipoles. The hope is to sift through many different arrangements without testing each case with elaborate tracking calculations. Analytical Expression of the Proposed Figure-of- Merit

 $R_i = \sqrt{A_{xi}^2 + A_{yi}^2},$

Define

where

$$A_{\boldsymbol{x}}^{2} = \frac{X^{2} + (\alpha_{\boldsymbol{x}}X + \beta_{\boldsymbol{x}}X')^{2}}{\beta_{\boldsymbol{x}}}, \qquad A_{\boldsymbol{y}}^{2} = \frac{Y^{2} + (\alpha_{\boldsymbol{y}}Y + \beta_{\boldsymbol{y}}Y')^{2}}{\beta_{\boldsymbol{y}}}.$$

with (X, Y) the displacements, and (X', Y') the angles of betatron oscillation, (β, α) the linear lattice parameters. Both A_x and A_y are invariant quantities when the oscillation is linear. In the presence of nonlinear multipoles, we have at the *i*-th turn,

$$A_{xi} = A_x + \delta A_{xi}, \qquad A_{yi} = A_y + \delta A_{yi}.$$

and

$$R_i = R + \delta R_i.$$

Using Collins' distortion functions, we find

$$\delta A_{x} = A_{x}^{2}(G_{1} + G_{3}) - A_{y}^{2}(2\bar{G} + G_{+} - G_{-}),$$

$$\delta A_{y} = -2A_{z}A_{y}(G_{+} + G_{-}),$$

 $G_{\alpha} = \mathcal{A}_{\alpha} \sin \phi_{\alpha} - \mathcal{B}_{\alpha} \cos \phi_{\alpha}$

where

and

$$egin{aligned} \mathcal{B}_{lpha}(arphi_{lpha}) &= rac{1}{2\sin\pi
u_{lpha}}\sum_{k=1}^{N}m_{lpha}\cos(|arphi_{lpha k}-arphi_{lpha}|-\pi
u_{lpha}), \ \mathcal{A}_{lpha}(arphi_{lpha}) &= rac{d\mathcal{B}_{lpha}(arphi_{lpha})}{darphi_{lpha}}. \end{aligned}$$

The subscript α represents five possible sextupole resonances and the corresponding quantities are listed in Table 1. Note that the Floquet phase φ runs from 0 to $2\pi\nu$ while the betatron phase ϕ increases by $2\pi\nu$ per turn.⁷

For a given arrangement of dipoles with sextupole components, distortion functions B_{α} and \mathcal{A}_{α} are uniquely determined at each point around the ring. Since the betatron phase ϕ_{α} increases by $2\pi\nu$ in each turn, G_{α} varies from turn to turn. We define for i = 1 to N turns,

$$\left(\frac{\delta R}{R}\right)_{rms} = \sqrt{\sum_{i=1}^{N} \frac{\left(R_i(\phi_0) - \dot{R}\right)^2}{N} / \dot{R}},\tag{1}$$

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Table 1: $S \equiv B''/(2B\rho), \ \phi \equiv \varphi - \phi_0$ (initial phase).

α	Floquet phase $arphi_{lpha}$	Betatron phase ϕ_{lpha}	tune $ u_{lpha}$	Strength m_{0}
1	$arphi_x$	ϕ_x	$ u_x$	$Seta_{\pi}^{rac{3}{2}}$ -1
2	$3\varphi_x$	$3\phi_x$	$3\nu_x$	$S eta_{x}^{rac{3}{2}}/4$
3	φ_x	ϕ_{x}	ν_x	$Seta_x^{rac{1}{2}}eta_y/4$
4	$2\varphi_y + \varphi_x$	$2\phi_y + \phi_x$	$2 u_y + u_x$	$S eta_{m{x}}^{rac{1}{2}} eta_{m{y}}/4$
5	$2arphi_y - arphi_x$	$2\phi_y-\phi_x$	$2\nu_y - \nu_z$	$Seta_x^{rac{1}{2}}eta_y/4$

where

$$\bar{R} = \sum_{i=1}^{N} R_i / N.$$

From the definition of R_i , we find

$$\left(\frac{\delta R}{R}\right)_{rms} = \sqrt{A_{\mathbf{r}}^2 (\delta A_{\mathbf{r}i})^2 + A_y^2 (\delta A_{yi})^2} / R^2.$$
(2)

Averaging over ϕ_0 which will be distributed uniformly between 0 to 2π ,

$$\left(\frac{\delta R}{R}\right)_{rms} = \sqrt{P\left(\frac{A_x}{R}\right)^4 + Q\left(\frac{A_y}{R}\right)^4 - 2U\left(\frac{A_xA_y}{R^2}\right)^2 A_x/R^2},$$

where

$$P = \frac{1}{2}(\mathcal{A}_{1}^{2} + \mathcal{B}_{1}^{2} + \mathcal{A}_{3}^{2} + \mathcal{B}_{3}^{2}),$$

$$Q = 2(\bar{\mathcal{A}}^{2} - \bar{\mathcal{B}}^{2}) + \frac{5}{2}(\mathcal{A}_{+}^{2} + \mathcal{B}_{+}^{2} + \mathcal{A}_{-}^{2} + \mathcal{B}_{-}^{2}),$$

$$U = (\mathcal{A}_{1}\bar{\mathcal{A}} + \mathcal{B}_{1}\bar{\mathcal{B}}).$$

For $A_x = A_y$, our figure-of-merit is

$$\left(\frac{\delta R}{R}\right)_{rms} = \sqrt{\frac{1}{4}(P+Q-2U)}A_{s}.$$
(3)

If $\beta_x = \beta_y = \text{const}$ and $\nu_x = \nu_y$, this quantity, when averaged over the entire ring, reduces to the figure-of-merit used by Li and Gluckstern.⁴ The figure-of-merit, Eq.(3), used in this study corresponds to the smear at one particular location of the ring.

Test Lattice

Our test lattice has 16 FODO cells followed by an insertion in each quadrant. The structure in each cell takes the form (QF(-k) - L - QD(-2k) - L - QF(+k) with QF and QD both a thin lens. The phase advance in each cell is 90°(both horizontal and vertical), L is a drift space of 28m. There are dipoles with normal sextupole error between quads, the distance between two adjacent dipole centers is 6m and it is 5m between dipole center and quadrupole. Altogether there are 512 dipoles and 4 insertions in the ring.

The tune excluding insertions is 16. We adjust ν_x and ν_y by choosing the proper phase advance per insertion. We chose $\nu_x = 17.68$, $\nu_y = 18.26$, a point near the resonance $3\nu_x = 54$.

Sorting Scheme

We divide 512 magnets into 16 gorups of 32 each, one group in approximately one betatron period. Within each group, the 32 magnets are arranged according to the amount of sextupole strength S. Three arrangements have been compared.

Arrangement (a); Highest S, 2nd lowest S, 3rd highest S, etc., then invert the scheme in every even number group, i.e. lowest S, 2nd highest S, 3rd lowest S, etc.. This is essentially the one suggested in ref. 3.

Arrangement (b); Highest S, (N/2 + 2)nd highest S, 3rd highest S, (N/2 + 4)th highest S, ..., (N/2 + 1)st highest S. 2nd highest S, (N/2 + 3)rd highest S, 4th highest S, etc.. Then in every even number group, do inverting inside the first 16 magnets and the remaining 16 magnets separately.

Arrangement (c); Random arrangement(unsorted).

<u>Results</u>

Both the smear and the figure-of-merit for five sets of random sextupole error fields have been calculated at the upstream end of one insertion. To find the smear, we used the tracking code TEAPOT⁸ which treats all magnets as thin lens. In Table 2, we list the improvement factors of the figure-of-merit and the linear aperture with two sorting schemes when they are campared with the unsorted arrangment. Note that the smear cited in ref. 6 (CERN-SPS) is 3.5%, but this value should be divided by two in order to make it consistent with the SSC-CGD definition.

Table 2: The improving factor of the figure-of-merit and the linear aperture.

Case	Sorting	Analytical results	Tracking smear 6.4%	Tracking smear 1.75°c
1	a	5.7	3.3	3.5
	ь	13.2	3.0	3.8
2	a	6.4	3.6	4.2
	b	14.0	2.6	4.3
3	a	3.6	2.1	1.6
	Ь	13.9	2.0	1.4
4	a	2.7	4.3	4.1
	ь	9.8	4.2	4.6
5	a	3.5	4.2	4.0
	ь	7.6	2.7	4.5

It is clear that, for sorting scheme (a), the proposed figure -of-merit is a reliable indicator of the actual improvement in the linear aperture whether the smear is taken to be 6.4% or 1.75%. The improvement factor of the figure-of-merit tends to be somewhat larger than the numerically evaluated improvement in the linear aperture but the difference is not significant. As for sorting scheme (b), it is perhaps not fair to make a quantitative comparison. This scheme is based on the minimizing of distortion averaged over the entire ring whereas the smear by tracking is evaluated at one specific location only. At the same time, scheme (b) ignores the strong focussing nature of the lattice (that is, the assumption $\beta_x = \beta_y = \text{const.}$) which may very well be one reason for the quantitative disagreement. It will be interesting to see if the agreement will be improved when the smear is also evaluated at all points in the ring.

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