COMPUTER SIMULATIONS OF DISRUPTION

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With the advent of linear colliders a decade ago, the dynamics of colliding positions and electron bunches became a subject of importance to the accelerator community.[1,2] There has recently been a resurgence of interest[3,4,5,6,7] because the self-consistent influence of one beam on the other will be even larger in future high energy linear colliders. When a charged beam moves through vacuum it generates a radial electric field and an azmithual magnetic field. If the beam is moving near the speed of light (relativistic) then the Lorentz and space charge forces from these self-fields are nearly in balance (to order $\frac{1}{\gamma_0^2}$). When an oppositely charged counterpropagating beam moves through the original beam, then the $\vec{v} \times \vec{B}$ force due to the other beam is in the same direction as the space charge force due to the other beam and the beam pinches. The same is true for the original beam, so it pinches as well.

Three phenomena which occur during such collisions have already received considerable attention. These are: 1) spurious events such as pair production[6] which are catalyzed by the large amplitude azimuthal magnetic field (The remnants of these events shower the detectors making it difficult to diagnose the experiment); 2) synchrotron radiation arising from the acceleration caused by the pinching forces (This is called quantum beamstrahlung[5] and can lead to non-trivial energy losses); and 3) the increase of the beam's densities that results from the self-pinching[1,2,3,4] (This enhances the luminosity (L) of the collider). The third is desirable while the first two are not. The third phenomenon is given the name, disruption, and this is the topic of this paper.

The degree to which disruption is important can be defined from two dimensionless parameters. These are D and $\frac{\lambda_d \sqrt{D}}{\sigma_o}$ where D is called the disruption parameter, λ_d is the Debye length in the transverse direction and σ_o is the beams transverse width. Both parameters have straightforward plasma physics interpretations. The square root of the disruption parameter, $\sqrt{D}$, is the number of plasma oscillations which occur during a collision $D = \frac{\omega_p^2 \sigma_z^2}{c^2} \sqrt{\frac{\pi}{2}}$ where σ_z is half the longitudinal length of the bunch. If the beams are Gaussian in shape with azimuthal symmetry, then it can be shown that $D = \frac{N r_e \sigma_z}{\gamma \sigma_o^2}$ where r_e is the classical electron radius. The physical significance of D can also be thought of in terms of beam optics. D is the ratio of the beams longitudinal length to the effective focal length caused by the beam pinching.

Since the Debye length is the distance an average particle travels during a plasma period then $\frac{\lambda_d \sqrt{D}}{\sigma_o}$ is the ratio of the average transverse distance a particle moves, to the beam size during a collision. It can be shown that $\frac{\lambda_d \sqrt{D}}{\sigma_o} = \frac{1}{(2\pi)^{3/2}} A$ where $A = \frac{\sigma_z \varepsilon}{\sigma_o^2} = \frac{\sigma_z}{\beta^*}$ and ε is the beam's emittance.

At present, within the published literature, there is both agreement and disagreement on how the luminosity enhancement, $H_D = \frac{L}{L_o}$, scales with the parameters D and A. Hollebeek[1] found H_D to saturate near 6 for $D \cong 5$ and to begin to decrease for $D \geq 20$. He argued on physical grounds that when $A < 1$ the effects of emittance could be ignored. He therefore modelled cold beams i.e., $\lambda = A = 0$. Fawley and Lee[3] found agreement with Hollebeek for $D \leq 10$. For $D \geq 10$ Fawley and Lee found H_D to continue to monotonically increase. The descrepancy between their work and Hollebeeks was attributed to possible differences in grid resolution. Their work was done for thermal beams with $A = 2$. Chen and Yokoya[4] claim that H_D depends sensitively on A for fixed D even when A is small (< 1). They find that H_D diverges as $\ln \frac{1}{A}$ for $A \rightarrow 0$ for Gaussian beams. Their $A = .2$ curve agrees with the $A = 0$ curve of Hollebeek (for $D \leq 10$) and the $A = .2$ curve of Fawley and Lee.

In all three studies the curves were generated from computer codes writted specifically to investigate disruption. Particle-in-cell techniques were used in the codes and the self-consistent fields were calculated by assuming azimuthal symmetry and highly relativistic beams. The particles were pushed using the paraxial ray approximation. At present, it is not understood why these codes give agreement in some aspects and disagreement in others. In an attempt to resolve the differences we have begun to carry out simulations of disruption using the well tested PIC codes, ISIS[8] and WAVE[9]. Both ISIS and WAVE are 2-D relativistic, electromagnetic particle-in-cell codes. ISIS is in cylindrical geometry while WAVE is in cartesian geometry. Since these codes employ fewer approximations they are more expensive to use; however they have the advantage of being well tested for a variety of particle beam and plasma problems. In addition, by gradually turning off different parts of the code it may be possible to isolate the causes of disagreement. In this paper we present preliminary results from a few simulations.

We chose three different values for D. These are $D = .45$, 2 and 10 and each is in one of the three regimes identified by Chen and Yokoya[4] The relevant simulation parameters are grid spacings $\Delta r = .0225$ and $\Delta z = .125 \sigma_z$; $\lambda_d = 0$; $\sigma_o / \sigma_z = 0.73$, .31, and .5 ; $\gamma = 550$, 10,000; and $D = .45$, 2, 10, respectively. We note that in order for the paraxial ray approximation to be valid $\frac{\sigma_o}{\sigma_z} D < 1$ and this is not satisfied in our $D = 10$ simulation. We plot the $\frac{d H_D}{dt}$ vs t curves for the three different values of D in Fig. 1. We note that the curves exhibit the characteristic shapes identified by Chen and Yokoya for the three regimes of D. When $D = .45$, disruption is in the weak-focusing regime. There is less than one plasma oscillation during the collision, or in otherwords the beams do not fully pinch when they overlap each other. There is not a second peak as there was for the $D = 0.5$ case in Ref. 4. When $D = 2$, the beams completely pinch just as they pass through each other since now there is a complete plasma oscillation (or the focal length is smaller than the bunch length). This results in the steep slope seen in Fig. 1b. This regime was called the transition regime by Chen and Yokoya. When $D = 10$ the beams quickly pinch down to a stable radius, σ_{of}. This causes the density of each beam to

increase to $n_0 \frac{\sigma_0^2}{\sigma_{of}^2}$. And since $L \propto n$ then the luminosity enhancement is $H_D = \frac{\sigma_0^2}{\sigma_{of}^2}$ in this "confined" regime. In order to determine H_D, in the "confined" regime it is therefore necessary to understand what determines the value of the final stable radius, σ_{of}. We will discuss this momentarily.

Although there is qualitative agreement in the shapes of the $\frac{dH_D}{dt}$ vs t curves between our runs and those by Chen and Yokoya, there is still quantitative disagreement. To make this clear we plot the results of Hollebeek, Fawley and Lee, Chen and Yokoya and the present work in Fig. 2.

For the selected values of D we plot H_D and label the points according to the value of A. The main area of disagreement appears to be that we find finite values for H_D even for $A = 0$. For example, when $D = 2$ we find $H_D = 11.8$ for $A = 0$. This is larger than Chen and Yogoya's $A = .05$ curve and it is a factor of 2 higher than Hollebeek and Fawley and Lee. Chen and Yokoya
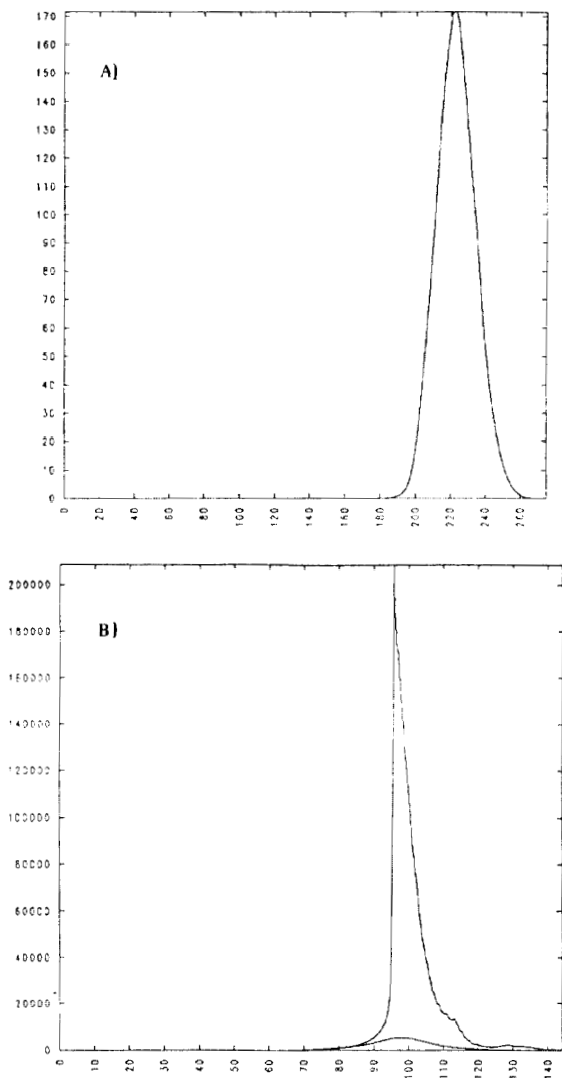

Fig. 1. We plot the $\frac{dH_D}{dt}$ vs t curves for simulations with a) $D = .45$, b) $D = 2$

suggest that other workers may have missed their $\ln \frac{1}{A}$ scaling of H_D because the transverse cell size may not have been large enough. Concerned by this, we reduced the radial cell size by a factor of three for the $D = 2$ run and found no changes in the simulation results.

Besides the resolution of the $\ln \frac{1}{A}$ scaling for H_D there is one other outstanding issue. It is still not understood what determines the equilibrium bunch radius (σ_{of}) in the confined regime. As
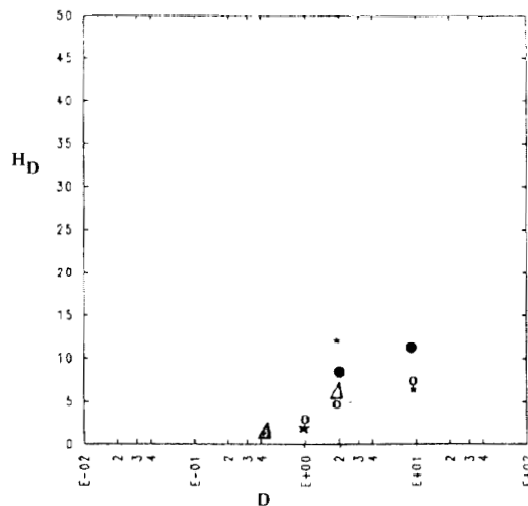

Fig. 2. We plot the luminosity enhancement factor H_D vs D from the published literature, Δ is from reference 1, $\bigcirc$ is from reference 3, $\cdot$ is from reference 4, and * is from current work.

pointed out earlier once the final beam radius is known then the luminosity enhancement is known since $H_D \approx \frac{\sigma_0^2}{\sigma_{of}^2}$. Therefore this question is fundamentally important to the understanding of the high D regime.

Some insight into this question can be gained by considering a few simple physical models. We begin by illustrating that the disruption process must cause the beams emittance to increase. At equilibrium

$$\sigma_{of}^2 = \epsilon \beta_f \quad \text{or} \quad k_{\beta f} \sigma_{of}^2 = \epsilon \quad (1)$$

If the beams remain Gaussian as they pinch then

$$k_{\beta f}^2 = \frac{4\pi N e^2}{\gamma m^2 c^2 \sigma_{of}^2 \sigma_z} = \frac{4\pi D}{\sigma_z^2} \quad (2)$$

From which it follows

$$\sigma_{of}^2 = \frac{\epsilon \sigma_z}{2\sqrt{\pi}\sqrt{D}} \frac{\sigma_{of}}{\sigma_o} \quad (3)$$

If ϵ did not change during disruption, then $\epsilon = \frac{\sigma_0^2}{\beta^*}$. So in this case[4]

$$\frac{\sigma_{of}}{\sigma_o} = \frac{\sigma_z}{2\sqrt{\pi}\beta^*\sqrt{D}} = \frac{A}{2\sqrt{\pi}\sqrt{D}} \quad (4)$$

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If this were true, then $H_D = \dfrac{4\pi D}{A^2}$. This predicts enhancement factors considerably larger than those observed in all simulations to date. It appears that this prediction is incorrect because emittance of the beam is not preserved during the disruption process. Significant emittance growth was observed in both our D = 2 and D = 10 simulations. Fawley and Lee[3] and Lee[2] had previously attempted to include the effect of emittance growth. However, their arguments lead to a maximum luminosity enhancement of only 3, well below what is observed in the simulations.

In order to qualitatively describe the approach to an equilibrium radius we offer two extreme scenarios and claim that the actual situation should be somewhere in between. If the second beam does not pinch, then it acts like a plasma lens. The first beam, therefore, continuously executes betatron oscillations. The beams' radius is then $\sqrt{2}$ times smaller on average compared to the initial radius. On the other hand if the second beam pinches down in the same manner, then we can model the value of the beam's radius by the differential equation

$$\ddot{\sigma} + \frac{F_r}{\sigma}\,\sigma = 0 \qquad (5)$$

where $F_r = \dfrac{2\pi N e^2}{\gamma m^2 \sigma_z \sigma}$. This describes a beam that pinches explosively toward the axis. Further work is required to determine the equilibrium radius.

Summary

Using the ISIS and WAVE plasma simulation codes we have examined the disruption of $e^+ e^-$ beams and found qualitative agreement with the $d\dfrac{H_o}{dt}$ curves of Chen and Yokoya in all three of the disruption regimes (weak focusing, transition and confined). Quantitatively, our results for H_D at A = 0 are higher than those of Hollebeek and Fawley and Lee and higher than the highest curves of Yokoya and Chen (A = .05), However, H_D remained finite at A = 0 in our simulation contrary to the prediction of Chen and Yokoya.

A possible explanation for the quantitative differences between our simulations and those of others is the low level of approximation in our codes. In particular, we do not take v_z to be c. For our simulation parameters this results in the beams interacting longer than would be the case for more realistic collider beam parameters (by about 20% for the D = 2 case).

The simulations presented here are preliminary. Further work is clearly necessary, particularly in understanding the pinched beam radii in the confined (high D) regime.

References

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